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A final remark: Gauss' theory of binary quadratic forms led to two major developments: the theory of number fields on the one hand, and the theory of quadratic forms in more than two variables on the other. The arithmetic of forms of higher degree over $\mathbf{Z}$ seems to have been largely neglected. In modern times Shintani revived interest in the arithmetic of cubic forms by introducing a family of Dirichlet series that depend on class numbers of cubic forms, and have good analytic properties (analytic continuation and functional equations). This work has been reinterpreted in the language of adèles by Wright [16]. For a general introduction to arithmetic problems concerning forms of higher degree, see [9].

We would like to thank J. Hurrelbrink and S. Weintraub for helpful discussions concerning this work.

CONTENTS

2. BINARY QUADRATIC MAPPINGS

We shall assume throughout this section that the ground ring R is an integral domain of characteristic not 2. The fraction field of R will be denoted by K.

A *binary quadratic form* is a pair (M, q) such that M is a projective R-module of rank two and $q \colon M \to R$ is a mapping such that $q(a\mathbf{x}) = a^2 q(\mathbf{x})$, $a \in R$, $\mathbf{x} \in M$, and such that $b(\mathbf{x}, \mathbf{y}) := q(\mathbf{x} + \mathbf{y}) - q(\mathbf{x}) - q(\mathbf{x})$ is R-bilinear. The form q is said to be *primitive* if the ideal generated by $q(M)$ is R. A morphism $(M, q) \to (M', q')$ is an R-linear mapping $f \colon M \to M'$ such that $q = q' \circ f$. If $M = R^2$ is the free module, we will often omit reference to M.