

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 46 (2000)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ARITHMETIC OF BINARY CUBIC FORMS
Autor: HOFFMAN, J. William / MORALES, Jorge
Kurzfassung: Contents
DOI: <https://doi.org/10.5169/seals-64795>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 06.01.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

A final remark: Gauss' theory of binary quadratic forms led to two major developments: the theory of number fields on the one hand, and the theory of quadratic forms in more than two variables on the other. The arithmetic of forms of higher degree over \mathbf{Z} seems to have been largely neglected. In modern times Shintani revived interest in the arithmetic of cubic forms by introducing a family of Dirichlet series that depend on class numbers of cubic forms, and have good analytic properties (analytic continuation and functional equations). This work has been reinterpreted in the language of adèles by Wright [16]. For a general introduction to arithmetic problems concerning forms of higher degree, see [9].

We would like to thank J. Hurrelbrink and S. Weintraub for helpful discussions concerning this work.

CONTENTS

1. Introduction	61
2. Binary quadratic mappings	65
3. Cubic forms	73
4. A Lie algebra representation	77
5. Structure of the cubic C -forms	81
6. Cohomological interpretation	89
7. Explicit computations and cubic trace forms	91
References	93

2. BINARY QUADRATIC MAPPINGS

We shall assume throughout this section that the ground ring R is an integral domain of characteristic not 2. The fraction field of R will be denoted by K .

A *binary quadratic form* is a pair (M, q) such that M is a projective R -module of rank two and $q: M \rightarrow R$ is a mapping such that $q(ax) = a^2q(x)$, $a \in R$, $\mathbf{x} \in M$, and such that $b(\mathbf{x}, \mathbf{y}) := q(\mathbf{x} + \mathbf{y}) - q(\mathbf{x}) - q(\mathbf{y})$ is R -bilinear. The form q is said to be *primitive* if the ideal generated by $q(M)$ is R . A morphism $(M, q) \rightarrow (M', q')$ is an R -linear mapping $f: M \rightarrow M'$ such that $q = q' \circ f$. If $M = R^2$ is the free module, we will often omit reference to M .