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Autor: KOOSIS, Paul

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$\mathrm{Re}(1/\kappa)+m>-1$ *and bounded away from* -1 as $\kappa\to-1/p$. For the γ_κ we can take *circular arcs* lying in the fourth quadrant, *orthogonal to the real axis* and running from 0 to $(1-\kappa)/(1+\kappa)$; these γ_κ will stay away from 1 while $(1-\kappa)/(1+\kappa)\to(p+1)/(p-1)$, i.e., while $\kappa\to-1/p$. Then $|1-s|$ will be uniformly bounded away from 0 for s on these γ_κ, and the *integral* in (102) thus *remain bounded* as $\kappa\to-1/p$. At the same time, however, the *sum* in (102) will tend to ∞ like $1/((1/\kappa)+p)$. Hence $A(\kappa)$ *will* tend to ∞ as $\kappa\to-1/p$ in the manner described.

This being so for *each* of the points $-1/p$, $p=2,3,\ldots$, $A(\kappa)$ *can have no analytic continuation from* $\mathcal{D}_+$ *into any neighbourhood of* 0, since any such continuation would have to coincide with the one just constructed on the intersection of the open second quadrant with the neighbourhood in question. That is what we needed to prove.

Before concluding, I must again thank my friend Victor Havin for having, during a conversation, expressed a thought which, indirectly, got me onto a path leading to the above argument.

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Paul Koosis

Department of Mathematics & Statistics
McGill University
Montreal
Canada

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