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$\operatorname{Re}(1/\kappa) + m > -1$ and bounded away from -1 as $\kappa \rightarrow -1/p$. For the γ_κ we can take *circular arcs* lying in the fourth quadrant, *orthogonal to the real axis* and running from 0 to $(1 - \kappa)/(1 + \kappa)$; these γ_κ will stay away from 1 while $(1 - \kappa)/(1 + \kappa) \rightarrow (p + 1)/(p - 1)$, i.e., while $\kappa \rightarrow -1/p$. Then $|1 - s|$ will be uniformly bounded away from 0 for s on these γ_κ , and the *integral* in (102) thus *remain bounded* as $\kappa \rightarrow -1/p$. At the same time, however, the *sum* in (102) will tend to ∞ like $1/((1/\kappa) + p)$. Hence $A(\kappa)$ will tend to ∞ as $\kappa \rightarrow -1/p$ in the manner described.

This being so for *each* of the points $-1/p$, $p = 2, 3, \dots$, $A(\kappa)$ can have *no analytic continuation from \mathcal{D}_+ into any neighbourhood of 0* , since any such continuation would have to coincide with the one just constructed on the intersection of the open second quadrant with the neighbourhood in question. That is what we needed to prove.

Before concluding, I must again thank my friend Victor Havin for having, during a conversation, expressed a thought which, indirectly, got me onto a path leading to the above argument.

BIBLIOGRAPHY

- [1] SCHIFF, L. I. *Quantum Mechanics*. McGraw-Hill, New York, 1955.
- [2] MARGENAU, H. and G. M. MURPHY. *The Mathematics of Physics and Chemistry*. Van Nostrand, New York, 1943.
- [3] HARTREE, D. R. The wave mechanics of an atom with a non-Coulomb central field. Part III. Term values and intensities in series in optical spectra. *Proc. Camb. Phil. Soc.* 24 (1928), 426–437.
- [4] SEATON, M. J. Quantum defect theory. *Reports Prog. Phys.* 46 (1983), 167–257.
- [5] GALLAGHER, T. F. *Rydberg Atoms*. Cambridge Univ. Press, 1994.
- [6] TITCHMARSH, E. C. *Eigenfunction Expansions*. Oxford, Clarendon Press, 1946.
- [7] HAM, F. S. The quantum defect method. In: *Solid State Physics*, edited by Seitz and Turnbull. Vol. 1 (1955), 127–192. Academic Press, New York.
- [8] MOTT, N. F. and H. S. W. MASSEY. *The Theory of Atomic Collisions*. Oxford, Clarendon Press; 2nd edition, 1949; 3rd edition, 1965.
- [9] FANO, U. and A. R. P. RAU. *Atomic Collisions and Spectra* Academic Press, Orlando, 1986.
- [10] RAU, A. R. P. and M. INOKUTI. The quantum defect: Early history and recent developments. *Amer. J. Phys.* 65:3 (1997), 221–225.
- [11] INCE, E. L. *Ordinary Differential Equations*. Dover, New York, 1956.

- [12] TITCHMARSH, E. C. *Eigenfunction Expansions*. Part II. Oxford, Clarendon Press, 1958.
- [13] SHUBOV, M. A. Asymptotic behavior of the quantum defect as a function of an angular momentum and almost normalization of eigenfunctions for a three-dimensional Schrödinger operator with nearly Coulomb potential. *Differential Integral Equations* 8: 8 (1995), 1885–1910.

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