WHY ALGEBRA ?

gateway to future study and mathematically significant ideas, but it is often a wall that blocks the paths of many. Should algebra be made more accessible to more students by changing the amount or nature of what is taught? Many countries have already embarked on such changes, hoping to increase access and success. Alternatively, are these changes necessary: is algebra truly useful for the majority of people and, even if it is, will it be useful in the future?

An algebra curriculum that serves its students well in the coming century may look very different from an ideal curriculum from some years ago. The increased availability of computers and calculators will change what mathematics is useful as well as changing how mathematics is done. At the same time as challenging the content of what is taught, the technological revolution is also providing rich prospects for teaching and is offering students new paths to understanding. In the past two decades, a substantial body of research on the learning and teaching of many aspects of algebra has been established and there have been many experiments with adapting curricula and teaching methods. There is therefore a strong scientific basis upon which to build this study.

Outline of the program

The study has two aims: to make a synthesis of current thinking and lessons from the past which will help set directions for future work in the field, and to suggest guidelines for advancing the teaching and learning of algebra. Following the pattern of previous ICMI studies, this study will have two components: an invited study conference and a study volume to appear in the *ICMI Study Series*, which will share the findings with a broad international audience. A report will also be made at ICME-10 in 2004. The study conference program will therefore contain plenary and sub-plenary lectures, working groups and panels. At least two panels are planned. One will attempt to make explicit some perspectives on algebra, algebra activity, algebraic thinking or algebraic understanding. A second aims to highlight the significant differences in algebra education around the world and identify the main strands in the goals, content and teaching methods of this worldwide enterprise. A major part of the working time will be spent in working groups addressing different aspects of the study problem. Working groups are likely to be established to correspond with each of the sections listed below.

WHY ALGEBRA?

The technological future of a modern society depends in large part on the mathematical literacy of its citizens and this is reflected in the worldwide trend towards mass secondary education. For an individual, algebra is a gateway to much of higher education and therefore to many fields of employment. Educators also argue that algebra is part of cultural heritage and is needed for informed and critical citizenship. However, for many, algebra acts more like a wall than a gateway, presenting an obstacle that they find too difficult to cross. This section of the study is concerned with the significance of algebra for the broad population of secondary school students, recognising that regional and cultural differences may impact upon the answers in interesting ways. It addresses questions such as:

- Should algebra be taught to all ? There has been a call for algebra for all secondary students, but what aspects of algebra are of value to all ? What should comprise a minimal curriculum ? How do answers to these questions relate to regional or cultural differences ?
- What do we expect of an algebra-literate individual ? What are the values of algebra learning for the individual, especially in view of increasingly powerful computing capabilities ? Access to higher learning and employment are two values, but what are the more immediate values and how can they be achieved ?
- How can we reshape the algebra curriculum so that it has more immediate value to individuals ? Can we identify explicit examples in contexts meaningful to students in which algebraic ideas have clear, unambiguous value ? Are there undesirable consequences of such orientations to algebra ?
- How can we reshape the algebra curriculum so that specific difficult ideas are more easily accessed ?

APPROACHES TO ALGEBRA

Recent research has focused on a number of approaches for developing meaning for the objects and processes of algebra. These approaches include, but are not limited to, problem-solving approaches, functional approaches, generalisation approaches, language-based approaches, and so on. Problem-solving approaches tend to emphasise an analysis of problems in terms of equations and a view of letters as unknowns. Functional approaches support a different set of meanings for the objects of algebra; for instance, the use of expressions to represent relationships and an interpretation of letters in terms of quantities that vary. A somewhat different perspective is encouraged by generalisation approaches that stress expressions of generality to represent geometric patterns, numerical sequences, or the rules governing numerical relationships – such approaches often serving as a basis for exploring underlying numerical structure, predicting, justifying and proving. Some algebra curricula develop student algebraic thinking exclusively along the lines of one such approach throughout the several grades of secondary school; others attempt to combine facets of several approaches.

Synthesising the experience with and research on the use of various approaches in the teaching/learning of algebra leads to questions such as the following:

- What does each of these various teaching approaches mean ?
- What are the algebraic meanings supported by each ?
- What are the epistemological obstacles inherent in each ?
- Which important aspects of algebra are favoured/neglected in each approach ?
- What are the difficulties encountered by students in extending the meanings that are developed by each of these approaches to include the meanings inherent in other approaches ?