Now it is evident that $\mathfrak{h} \otimes_{\mathbf{Q}} \mathbf{R}$ is isomorphic to either $\mathfrak{hei}_3(\mathbf{R}) \oplus \mathfrak{hei}_3(\mathbf{R})$, or $\mathfrak{hei}_3(\mathbf{R}[x]/(x^2))$, or $\mathfrak{hei}_3(\mathbf{C})$ depending on the sign of m. Thus, we have classified up to $\mathbf{Q}$-isomorphism all rational forms for these 3 real Lie algebras. By Theorem 2 these forms are non-isomorphic. The proof of the theorem is complete. □

REMARK. It is worth mentioning that the above three real Lie algebras are not pairwise isomorphic over $\mathbf{R}$. Indeed, the centralizer of any element in $\mathfrak{g}_- = \mathfrak{hei}_3(\mathbf{C})$ is even dimensional over $\mathbf{R}$ since this algebra can be viewed as a complex Lie algebra, whereas in both $\mathfrak{g}_+ = \mathfrak{hei}_3(\mathbf{R}) \oplus \mathfrak{hei}_3(\mathbf{R})$ and $\mathfrak{g}_0 = \mathfrak{hei}_3(\mathbf{R}[x]/(x^2))$ there are elements with 5-dimensional centralizers. In order to show that the last two algebras are not isomorphic we need some more information about elements with 5-dimensional centralizers.

The centralizer $C(x)$ will not be changed if we scale x by any $\lambda \neq 0$ or add to x any central element. This means that dimension of the centralizer is a well-defined function on the projective space $\mathbf{P}(\mathfrak{g}/[\mathfrak{g},\mathfrak{g}])$ where $\mathfrak{g}$ is either $\mathfrak{g}_+$ or $\mathfrak{g}_0$. Straightforward computations show that in $\mathbf{P}(\mathfrak{g}_0/[\mathfrak{g}_0,\mathfrak{g}_0])$ all points with 5-dimensional centralizer belong to a unique line whereas in $\mathbf{P}(\mathfrak{g}_+/[\mathfrak{g}_+,\mathfrak{g}_+])$ the points under consideration form two disjoint lines.

REFERENCES

[1] BOURBAKI, N. *Groupes et algèbres de Lie. Chap. 1, 3.* Hermann, Paris, 1972.

[2] CORWIN, L. J. and F. P. GREENLEAF. *Representations of Nilpotent Lie Groups and their Applications. Part I.* Cambridge Univ. Press, 1989.

[3] DIXMIER, J. Sur les représentations unitaires des groupes de Lie nilpotents III. *Canad. J. Math. 10* (1958), 321–348.

[4] RAGHUNATHAN, M. *Discrete Subgroups of Lie Groups.* Springer-Verlag, Berlin, 1972.

[5] MALCEV, A. I. On a class of homogeneous spaces. *Amer. Math. Soc. Translation 39* (1951); *Izvestiya Akad. Nauk SSSR. Ser. Mat. 13* (1949), 9–32.

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