REMARK 5.4. We suggest that a similar statement might hold for the classical Teichmüller spaces and perhaps also for more general Kobayashi hyperbolic complex spaces. Hilbert geodesic rays from a point y that terminate on a line segment contained in the boundary may correspond to the Teichmüller geodesic rays defined by Jenkins-Strebel differentials that H. Masur considered when demonstrating the failure of CAT(0) for the Teichmüller space of Riemann surfaces of genus $g \geq 2$. The complement of the union of all line segments in the boundary ∂D may correspond to the uniquely ergodic foliation points on the Thurston boundary of Teichmüller space.

Using the arguments in [Ka01], see Proposition 5.1 of that paper, we obtain the following result as an application of Theorem 5.2:

THEOREM 5.5. *Let D be a bounded convex domain and $\varphi \colon D \to D$ be a map which does not increase Hilbert distances. Then either the orbit $\{\varphi^n(y)\}_{n=1}^{\infty}$ is bounded or there is a limit point $\bar{y}$ of the orbit such that for any other limit point $\bar{x}$ of the orbit it holds that $[\bar{x}, \bar{y}] \subset \partial D$.*

This theorem, which extends a theorem in [Be97], provides a general geometric explanation for a part of the main theorem in [Me01].

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