L'Enseignement Mathématique, t. 48 (2002), p. 91–126

SEMISTABLE $K3$-SURFACES WITH ICOSAHEDRAL SYMMETRY

by Jan STEVENS *)

ABSTRACT. In a Type III degeneration of $K3$-surfaces the dual graph of the central fibre is a triangulation of S^2. We realise the tetrahedral, octahedral, and especially the icosahedral triangulation in families of $K3$-surfaces, preferably with the associated symmetry groups acting.

INTRODUCTION

A degeneration of surfaces is a 1-parameter family with general fibre a smooth complex surface. The case of $K3$-surfaces has attracted a great deal of attention. A nice discussion is contained in the introductory first paper [F-M] of the bundle [SAGS]. One usually allows base change and modifications to obtain good models. After a ramified cover of the base and resolution of singularities we may assume that the degeneration $f\colon \mathcal{X} \to S \ni 0$ is *semistable*: the zero fibre $X = f^{-1}(0)$ is a reduced divisor with (simple) normal crossings in the smooth manifold $\mathcal{X}$. Further modifications of a $K3$-degeneration lead to a minimal model, which falls into one of three types.

In a Type III degeneration of $K3$-surfaces the dual graph of the central fibre is a triangulation of S^2. In this paper I construct an example with my favourite triangulation, the icosahedral one. A substantial part is taken up by the tetrahedral case, which is easier to handle and allows more explicit results. A second purpose of this paper is to link general theory with concrete computations.

There are two obvious ways to realise a semistable degeneration with prescribed combinatorial type. The first is to start with a singular total space,

*) Partially supported by the Swedish Research Council (Vetenskapsrådet).