12.2 Proof of Theorem 12.4

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LEMMA 12.5. *Let (Γ, d_Γ) be an* $\mathbf{R}$*-forest. Let ψ be a weakly bi-Lipschitz forest-map of (Γ, d_Γ). Let (K_ψ, f, σ_t) be the mapping-telescope of (ψ, Γ), equipped with a structure of forest-stack as defined in Section 2. Then the semi-flow $(\sigma_t)_{t\in\mathbf{R}^+}$ is a bounded-cancellation and bounded-dilatation semi-flow with respect to any horizontal d_Γ-metric (see Lemma 12.1).*

Proof. The horizontal metric $\mathcal{H}$ agrees with the metric d_Γ on all the strata $f^{-1}(n)$, $n \in \mathbf{Z}$ (see Lemma 12.1). Consider any horizontal geodesic g in the stratum $f^{-1}(0)$. If ψ is weakly bi-Lipschitz with constants μ_0 and K_0, then for any integer $n \geq 0$, we have $|[g]_n|_n \geq \frac{1}{\mu_0^n}|g|_0 - K_0(\frac{1}{\mu_0^{n-1}} + \frac{1}{\mu_0^{n-2}} + \ldots + 1)$. Since $0 < \frac{1}{\mu_0} < 1$, the sum tends to $\frac{\mu_0}{\mu_0 - 1}$ as $n \to +\infty$. Setting $\lambda_- = \frac{1}{\mu_0}$ and $K = K_0\frac{\mu_0}{\mu_0-1}$, this proves the inequality of item (1) for horizontal geodesics in $f^{-1}(n)$, $n \in \mathbf{Z}$, and an integer time t. For the case in which t is any positive real number and $g \in f^{-1}(r)$, r any real number, just decompose $\sigma_t = \sigma_{t-E[t]} \circ \sigma_{E[t-(E[r]+1-r)]} \circ \sigma_{E[r]+1-r}$. The map σ_t is a homeomorphism from $f^{-1}(r)$ onto $f^{-1}(r+t)$ for any $t \in [0, E[r]+1-r)$. That is, for any real r, $|[g]_{r+t}|_{r+t} = |\sigma_t(g)|_{r+t}$ for $t \in [0, E[r]+1-r)$. The monotonicity of the maps $l_{r,g}$ (see Lemma 12.1, item (2)) implies, for any r and $t \in [0, E[r]+1-r)$, that $|\sigma_t(g)|_{r+t} \geq \frac{1}{\mu_0}|g|_r$. The conclusion follows. □

LEMMA 12.6. *With the assumptions and notation of Lemma 12.5, if the map ψ is a (strongly) hyperbolic forest-map of (Γ, d_Γ) then the semi-flow $(\sigma_t)_{t\in\mathbf{R}^+}$ is (strongly) hyperbolic with respect to any horizontal d_Γ-metric.*

The proof is similar to that of Lemma 12.5. □

Proof of Theorem 12.4. By Lemmas 12.5 and 12.6, a mapping-telescope admits a structure of forest-stack $(\widetilde{X}, f, \sigma_t, \mathcal{H})$ with horizontal metric $\mathcal{H}$ such that the semi-flow $(\sigma_t)_{t\in\mathbf{R}^+}$ is a strongly hyperbolic semi-flow with respect to $\mathcal{H}$. Hence Theorem 4.4 implies Theorem 12.4. □

13. About mapping-torus groups

We first recall the definition of a *hyperbolic endomorphism* of a group introduced by Gromov [19].