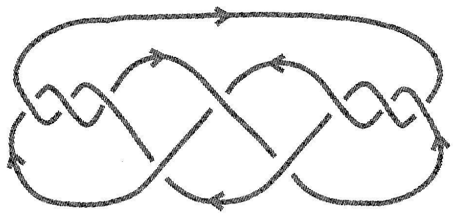
that the rational tangle has two components), the integer $u = 2K - q$ in the equation $q^2 = 1 + up$ cannot be zero. It follows then that the links of the type $N([2n])$, for $n \in \mathbf{Z}$, $n \neq 0$, with tangle fraction $2n/1$ are not invertible (recall the example in Figure 30). Note that, for $n = 0$ we have $T = [0]$ and $F(T) = 0/1$, and in this case Theorem 7 is confirmed, since $1^2 = 1 + u0$, for any u odd. See Figure 38 for another example of a strongly invertible link. In this case the link is $L = N([[3],[1],[1],[1],[3]])$ with $F(L) = 40/11$. Note that $11^2 = 1 + 3 \cdot 40$, fitting the conclusion of Theorem 7.



L = N([[3], [1], [1], [1], [3]])

FIGURE 38
An example of a strongly invertible link

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