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We now pass to a general L_q, $q \in Q_m$. We may assume that q is homogeneous of, say, degree d. By Corollary 3.3 we have that L_q is a non zero multiple of $(adL)^d(q)$. Since both q and L preserve Q_m, our claim follows. □

3.3 Differential operators on X_m

Now let us return to the algebra of differential operators $\mathcal{D}(X_m)$. Notice that $\mathcal{D}(X_m)$ contains two commutative subalgebras (both isomorphic to Q_m). The first is Q_m itself, the second is the subalgebra $Q_m^\dagger$ consisting of the differential operators of the form L_q with $q \in Q_m$. It is possible to prove

THEOREM 3.6 ([BEG]). *$\mathcal{D}(X_m)$ is generated by Q_m and $Q_m^\dagger$.*

Notice that by Corollary 3.3 we in fact have that $\mathcal{D}(X_m)$ is generated by Q_m and by L.

EXAMPLE 3.7. If $W = \mathbf{Z}/2$, $\mathfrak{h} = \mathbf{C}$ we get that $\mathcal{D}(X_m)$ is generated by the operators

$$x^2, \quad x^{2m+1}, \quad \frac{d^2}{dx^2} - \frac{2m}{x}\frac{d}{dx}.$$

Theorem 3.6 together with Proposition 3.4, imply

COROLLARY 3.8 ([BEG]). *$\mathcal{D}(X_m)$ is locally finite dimensional under the action of the Lie algebra $\mathfrak{sl}(2)$ defined in (8).*

This Corollary implies that our $\mathfrak{sl}(2)$ action on $\mathcal{D}(X_m)$ can be integrated to an action of the group $SL(2)$. In particular we have

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} q = L_q$$

for all $q \in Q_m$. This transformation is a generalization of the Fourier transform, since it reduces to the usual Fourier transform on differential operators on $\mathfrak{h}$ when $m = 0$.

EXAMPLE 3.9. If $W = \mathbf{Z}/2$, $\mathfrak{h} = \mathbf{C}$, we get that the monomials $\{x^{2i}\} \cup \{x^{2i+2m+1}\}$ are (up to constants) all lowest weight vectors for the $\mathfrak{sl}(2)$ action on $\mathcal{D}(X_m)$. x^n has weight $-n$. We deduce that $\mathcal{D}(X_m)$ is isomorphic as a $\mathfrak{sl}(2)$-module to the direct sum of the irreducible representations of dimension $n+1$ for n even or $n = 2(m+i)+1$, each with multiplicity one.