3.7 Generic c

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EXAMPLE 3.15. If $\lambda = 0$ and $\tau = \mathbf{1}$ is the trivial representation of W, the Verma module $M(0, \mathbf{1}) = \mathbf{C}[\mathfrak{h}]$. The action of $\mathbf{C}[\mathfrak{h}]$ is given by multiplication, that of $\mathbf{C}[\mathfrak{h}^*]$ is generated by the Dunkl operators and W acts in the usual way.

3.7 GENERIC c

Opdam and Rouquier have recently studied the structure of the categories $\mathcal{O}(H_c)$, $\mathcal{O}(eH_ce)$, and found that it is especially simple if c is "generic" in a certain sense. Namely, recall that for a W-invariant function $q\colon \Sigma \to \mathbf{C}^*$ one can define the *Hecke algebra* $\mathrm{He}_q(W)$ to be the quotient of the group algebra of the fundamental group of U/W by the relations $(T_s - 1)(T_s + q_s) = 0$, where T_s is the image in U/W of a small half-circle around the hyperplane of s in the counterclockwise direction. It is well known that $\mathrm{He}_q(W)$ is an algebra of dimension $|W|$, which coincides with $\mathbf{C}[W]$ if $q = 1$. It is also known that $\mathrm{He}_q(W)$ is semisimple (and isomorphic to $\mathbf{C}[W]$ as an algebra) unless q_s belongs for some s to a finite set of roots of unity depending on W (see [Hu]).

DEFINITION 3.16. The function c is said to be *generic* if for $q = e^{2\pi ic}$, the Hecke algebra $\mathrm{He}_q(W)$ is semisimple.

In particular, any irrational c is generic, and (more important for us) an integer valued c is generic (since in this case $q = 1$). We can now state the following central result:

THEOREM 3.17 (Opdam-Rouquier [OR]; see also [BEG] for an exposition). *If c is generic (in particular, if c takes non negative integer values), then the irreducible objects in $\mathcal{O}$ are exactly the modules $M(\lambda, \tau)$. Moreover, the category $\mathcal{O}$ is semisimple.*

We also have

THEOREM 3.18 ([OR]). *If c is generic then the functor F is an equivalence of categories.*

From Theorem 3.17 we can deduce

THEOREM 3.19 ([BEG]). *If c is generic, then H_c is a simple algebra.*

In the case $c = 0$, we get the simplicity of $\mathbf{C}[\mathfrak{h} \oplus \mathfrak{h}^*] \rtimes \mathbf{C}[W]$, which is well known.