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3.8 The Levasseur-Stafford theorem and its generalization

Let us now recall a result of Levasseur and Stafford:

THEOREM 3.20 ([LS]). *If G is a finite group acting on a finite dimensional vector space V over the complex numbers, then the ring $\mathcal{D}(V)^G$ is generated by the subrings $\mathbf{C}[V]^G$ and $\mathbf{C}[V^*]^G$.*

As an example, notice that if we let $\mathbf{Z}/n\mathbf{Z}$ act on the complex line by multiplication by the n^{th} roots of 1, we deduce that the operator $x\frac{d}{dx}$ can be expressed as a non commutative polynomial in the operators x^n and $\frac{d^n}{dx^n}$, a non-obvious fact. We note also that this theorem has a purely "quantum" nature, i.e. the corresponding "classical" statement, saying that the Poisson algebra $\mathbf{C}[V \times V^*]^G$ is generated, as a Poisson algebra, by $\mathbf{C}[V]^G$ and $\mathbf{C}[V^*]^G$, is in fact false, already for $V = \mathbf{C}$ and $G = \mathbf{Z}/n\mathbf{Z}$.

One can prove a similar result for the algebra eH_ce. Namely, recall that the algebra eH_ce contains the subalgebras $\mathbf{C}[\mathfrak{h}]^W$, and $\mathbf{C}[\mathfrak{h}^*]^W$.

THEOREM 3.21 ([BEG]). *If c is generic then the two subalgebras $\mathbf{C}[\mathfrak{h}]^W$ and $\mathbf{C}[\mathfrak{h}^*]^W$ generate eH_ce.*

Notice that if $c = 0$, then $eH_0e = \mathcal{D}(\mathfrak{h})^W$, so Theorem 3.21 reduces to the Levasseur-Stafford theorem.

REMARK. It is believed that this result holds without the assumption of generic c. Moreover, it is known to be true for all c if W is a Weyl group not of type E and F, since in this case Wallach proved that the corresponding classical statement for Poisson algebras holds true. Nevertheless, the genericity assumption is needed for the proof, because, similarly to the proof of the Levasseur-Stafford theorem, it is based on the simplicity of H_c.

3.9 The action of the Cherednik algebra to quasi-invariants

We now go back to the study of Q_m. Notice that the algebra eH_me acts on $\mathbf{C}[\mathfrak{h}]^W$, since e gives the W-equivariant projection of $\mathbf{C}[\mathfrak{h}]$ onto $\mathbf{C}[\mathfrak{h}]^W$. It is clear that this action is by differential operators. For instance, the subalgebra $\mathbf{C}[\mathfrak{h}]^W \subset eH_me$ acts by multiplication. Also, an element $q \in \mathbf{C}[\mathfrak{h}^*]^W \subset eH_me$ acts via the operator $q(D_{x_1}, \ldots, D_{x_n})$. By definition this operator coincides with L_q on $\mathbf{C}[\mathfrak{h}]^W$.