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We may now define $|\xi|_h^2$ for $\xi \in L_x$ with $x \in X$ by

$$|\xi|_h^2 = \begin{cases} e^{-\gamma(x)}|\xi/s(x)|^2 & \text{if } x \in X \setminus Y, \\ e^{-\alpha(x)-\epsilon\beta(x)}|\xi|_k^2 & \text{if } x \in V. \end{cases}$$

Then h is a well-defined C^∞ Hermitian metric in L since, for $x \in V \setminus Y$ and $\xi \in L_x$, we have

$$e^{-\gamma(x)}|\xi/s(x)|^2 = e^{-\left(\alpha(x)-\log|s(x)|_k^2+\epsilon\beta(x)\right)}|\xi|_k^2/|s(x)|_k^2 = e^{-\alpha(x)-\epsilon\beta(x)}|\xi|_k^2 .$$

Furthermore, on $X \setminus Y$ we have

$$\mathcal{R}_h = \Delta_g\left(-\log|s|_h^2\right) = \Delta_g\gamma \begin{cases} > 0 & \text{on } X \setminus Y \\ > 1 & \text{on } V \setminus Y \end{cases}$$

By continuity, we also have $\mathcal{R}_h \geq 1 > 0$ at points in Y. Thus $\mathcal{R}_h > 0$ on X. $\square$

For X a Riemann surface, the above proofs become especially simple. For example, the construction of α in the proof of Theorem 2.3 is trivial for $\dim X = 1$ because Y is discrete. For X an open Riemann surface, Theorem 0.1 provides a C^∞ strictly plurisubharmonic exhaustion function and, therefore, by [Gr] and [DG], one gets the theorem of [BS] that an open Riemann surface is Stein. For a compact Riemann surface X, Theorem 2.3 becomes the familiar fact (see, for example, [GriH]) that the holomorphic line bundle associated to a nontrivial effective divisor admits a C^∞ Hermitian metric h with positive curvature Θ_h.

REFERENCES

[BS] BEHNKE, H. und K. STEIN. Konvergente Folgen von Regularitätsbereiche. *Math. Ann. 116* (1938), 204–216.

[D] DEMAILLY, J.-P. Cohomology of q-convex spaces in top degrees. *Math. Z. 204* (1990), 283–295.

[DG] DOCQUIER, F. und H. GRAUERT. Levisches Problem und Rungescher Satz für Teilgebiete Steinscher Mannigfaltigkeiten. *Math. Ann. 140* (1960), 94–123.

[EM] ELIASHBERG, Y. and N. MISHACHEV. *Introduction to the h-Principle.* Graduate Studies in Math. *48*. Amer. Math. Soc., Providence, 2002.

[GoG] GOLUBITSKY, M. and V. GUILLEMIN. *Stable Mappings and their Singularities.* Grad. Texts in Math. *14*. Springer-Verlag, New York–Heidelberg, 1973.

[Gr] GRAUERT, H. On Levi's problem and the imbedding of real-analytic manifolds. *Ann. Math. (2) 68* (1958), 460–472.

[GreW] GREENE, R. and H. WU. Embedding of open Riemannian manifolds by harmonic functions. *Ann. Inst. Fourier (Grenoble) 25* (1975), 215–235.

[GriH] GRIFFITHS, P. and J. HARRIS. *Principles of Algebraic Geometry.* Wiley–Interscience, New York, 1978.

[M] MALGRANGE, B. Existence et approximation des solutions des équations aux dérivées partielles et des équations de convolution. *Ann. Inst. Fourier (Grenoble) 6* (1956), 271–355.

[N] NARASIMHAN, R. *Complex Analysis in One Variable*, second edition. Birkhäuser, Boston, 2001.

[O] OHSAWA, T. Completeness of noncompact analytic spaces. *Publ. Res. Inst. Math. Sci. 20* (1984), 683–692.

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