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where

$$\widetilde{\Omega}_n = \begin{pmatrix} 1 & & & 0 & \dots & 0 \\ & \ddots & & & & \\ 1-t & & 1 & & & \vdots \\ \vdots & & & \ddots & & 0 \\ & & & & 1 & \\ 1-t & \dots & 1-t & & & 1 \end{pmatrix}$$

REMARK. Squier [Sq] gives an "hermitian" matrix M_n such that

$$\overline{Bu(\sigma)}^t \times M_n \times Bu(\sigma) = M_n\,,$$

but our matrix $\widetilde{\Omega n}$ is much simpler for two reasons:

(a) $\widetilde{\Omega}_n \in GL_n(\mathbf{Z}[t,t^{-1}])$, whereas $M_n \in GL_n(\mathbf{Z}[t^{\pm 1/2}])$;

(b) $\widetilde{\Omega}_n$ is triangular.

The fact that $\widetilde{\Omega}_n$ is triangular imposes more constraints on a matrix to be a Burau matrix, than that of Squier. This will help to understand the group of Burau matrices (recall that we know that the Burau representation is not faithful for $n \geq 5$ by [Moo], [L; P], [Bg]).

COROLLARY 5.4. *Corollary 5.2 is true, if Gassner matrices are replaced by Burau matrices.*

Added in proof. After this paper had been written, the author was informed (in June 2005) that Theorem 0.1 and Lemma 1.2 were obtained previously by V. Turaev in a paper "Intersection loops in two-dimensional manifolds", which appeared in *Mathematics of the USSR Sbornik 35* (1979).

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