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COHOMOLOGY OF PURE AUTOMORPHISM GROUPS OF FREE PRODUCTS OF FINITE GROUPS

by Craig JENSEN

Recently, John McCammond, John Meier and I verified [2] the Brownstein–Lee conjecture [1]. It concerns the cohomology of the *pure symmetric automorphism group of a free group*, $P\Sigma_n$, namely the group which has to take each of the preferred generators of free group F_n to a conjugate of itself.

THEOREM 41.1 (The Brownstein–Lee Conjecture). *The cohomology ring* $H^*(P\Sigma_n, \mathbf{Z})$ *is generated by one-dimensional classes* α^*_{ij}, *where* $i \neq j$, *subject to the relations*

(1) $\alpha^*_{ij} \wedge \alpha^*_{ij} = 0$

(2) $\alpha^*_{ij} \wedge \alpha^*_{ji} = 0$

(3) $\alpha^*_{kj} \wedge \alpha^*_{ji} = (\alpha^*_{kj} - \alpha^*_{ij}) \wedge \alpha^*_{ki}$

and the Poincaré series is $\mathfrak{p}(z) = (1 + nz)^{n-1}$.

What can we say about the cohomology of similar groups ? A first question might be:

QUESTION 41.2. *What is the cohomology of* $PAut(\mathbf{Z}/p * \cdots * \mathbf{Z}/p)$? *That is, what is the cohomology of the pure (meaning each* $\mathbf{Z}/p$ *in the free product has to be taken to a conjugate of itself) automorphism group of a free product of* n *copies of* $\mathbf{Z}/p$?

After we get this, a more generalized question would be:

QUESTION 41.3. *Let* $G_1, \dots, G_n$ *be finite abelian groups. What is the cohomology of* $PAut(G_1 * \cdots * G_n)$?

or perhaps

QUESTION 41.4. *Let $G_1, \ldots, G_n$ be finite abelian groups or* $\mathbf{Z}$. *What is the cohomology of* $PAut(G_1 * \cdots * G_n)$ *?*

or perhaps even

QUESTION 41.5. *Let $G_1, \ldots, G_n$ be finite groups or* $\mathbf{Z}$. *What is the cohomology of* $PAut(G_1 * \cdots * G_n)$ *?*

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