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52

RESIDUALLY NILPOTENT GROUPS

by Roman MIKHAILOV and Inder Bir S. PASSI

In general, it is difficult to decide whether a given group is residually nilpotent; this is so even for rather simple looking one-relator groups [2]. There is thus need to develop general methods for checking residual nilpotence. Such an investigation will have wide impact in group theory and topology; for example, in the context of Baumslag's parafree conjecture [1], Whitehead's asphericity conjecture [4], etc.

We list here two problems on residual nilpotence (see also Kourovka Notebook 2006, Problem 16.65).

PROBLEM 52.1. If F is a free group with finite basis x_1, \dots, x_n , and r a basic commutator, then is the group $\langle x_1, \dots, x_n \mid r \rangle$ residually nilpotent?

Let G be a residually nilpotent group. We say that G is *absolutely residually nilpotent* if for any k -central extension

$$1 \rightarrow N \rightarrow \tilde{G} \rightarrow G \rightarrow 1,$$

of G , i.e., a central extension satisfying $[N, \underbrace{\tilde{G}, \dots, \tilde{G}}_{k \text{ terms}}] = 1$, the group \tilde{G} is again residually nilpotent.

It would be of interest to investigate such groups; for instance, it can be shown that the following two statements are equivalent:

- (i) finitely-generated parafree groups are absolutely residually nilpotent;
- (ii) Baumslag's parafree conjecture: $H_2(G) = 0$ for a finitely-generated parafree group G .

As a first step, on examining absolute residual nilpotence for one-relator groups, it turns out that *every central extension of a one-relator residually*

nilpotent group is again residually nilpotent [3]. We are thus motivated to raise the following:

PROBLEM 52.2. Is every one-relator residually nilpotent group absolutely residually nilpotent?

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