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**Autor:** Putman, Andrew

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any correct expression will work. Now, using an argument of Humphries [9], for  $1 \leq i \leq g-2$  we can express  $T_{\delta_{i+2}}$  as a complicated product of elements in

$$\{T_{\alpha_i}, T_{\alpha_{i+1}}, T_{\alpha_{i+2}}, T_{\beta_i}, T_{\beta_{i+1}}, T_{\delta_i}, T_{\delta_{i+1}}\}^{\pm 1}.$$

This allows us eliminate  $T_{\delta_i}$  from  $S$  for  $i \geq 3$  by adding relations which do not involve both  $T_{\beta_1}^{\pm 1}$  and  $T_{\beta_{g-1}}^{\pm 1}$ . Our final relation is  $[h, T_{\delta_g}] = 1$ ; since this does not involve either  $T_{\beta_1}$  or  $T_{\beta_{g-1}}$ , we are done.  $\square$

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Andrew Putman

Department of Mathematics; MIT, 2-306  
77 Massachusetts Avenue  
Cambridge, MA 02139-4307  
USA  
*e-mail*: andyp@math.mit.edu