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TABLE 1
Normal surface in the figure eight knot complement

solution	$\nu(\mu)$	$\nu(\lambda)$	slope
(2, 0, 0, 0, 0, 1)	1	4	-4
(0, 2, 0, 0, 0, 1)	-1	4	4
(0, 0, 1, 2, 0, 0)	-1	-4	-4
(0, 0, 1, 0, 2, 0)	1	-4	4

quadrilateral types in M' by r, r', r'' , where $r^{(i)}$ lifts to $p^{(i)}$. The Q -matching equation is $r + r' - 2r'' = 0$. It can be worked out from the triangulation or by observing that the induced involution on quadrilateral types in M is $(p\ q')(p'\ q)(p''\ q'')$. Thus, $\dim PQ(\mathcal{T}') = 1$ and $PF(\mathcal{T}') = \emptyset$.

The boundary curve map is defined via the induced triangulation of the double cover of the Klein bottle; using the generators from the above section, one has: $\nu(\lambda) = -2r - 2r' + 4r'' = 0$ and $\nu(\mu) = -2r' + 2r''$. Generators λ', μ' can be chosen for $H_1(B_v) \cong \mathbf{Z} \oplus \mathbf{Z}_2$ such that the map $H_1(\tilde{B}_v) \rightarrow H_1(B_v)$ is given by $\lambda \rightarrow \lambda'$ and $\mu \rightarrow (\mu')^2 = 0$. The composition

$$Q(\mathcal{T}') \rightarrow \mathbf{Z}^2 \rightarrow H_1(\tilde{B}_v; \mathbf{R}) \rightarrow H_1(B_v; \mathbf{R})$$

is then

$$N \rightarrow (-\nu_N(\lambda), \nu_N(\mu)) = (0, \nu_N(\mu)) \rightarrow \nu_N(\mu)\lambda \rightarrow \nu_N(\mu)\lambda'$$

Since $\nu(\mu) = -2r' + 2r''$, it follows that the map $\partial: Q(\mathcal{T}') \rightarrow H_1(B_v; \mathbf{R})$ is surjective. Its restriction to integral points in $Q(\mathcal{T}')$ has image of index two in $H_1(B_v; \mathbf{Z})$, which gives a subgroup of index four in $H_1(B_v)$.

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