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NOTES²). Unfortunately, this part of the proof requires some modest calculations with power series³) since we have been unable to stick (as in the case of differential equations) to Cauchy's viewpoint on holomorphic maps and Hadamard's strong maxim: "The shortest way between two truths in the real domain passes through the complex domain."

Cartan's version of differential calculus *à la* Fréchet first appeared in Dieudonné's famous book [3], whose exposition of analytic functions of several variables, followed in 1971 by a proof of the Cauchy-Kowalevski theorem [4], did not venture into infinite dimensions.

Robbin's celebrated proof [14] of Cauchy's theorem on ordinary differential equations (in the usual differentiable setting) is a wonderful application of infinite-dimensional differential calculus, slightly distorted by Lang in an otherwise very good book [11] — and by the author in [2].

Before Douady's thesis [6], the theory of analytic functions between Banach spaces had been developed by Max Zorn in the mid-forties (see the last chapter of [7], which provides many references).

Hans Lewy [12, 4] showed that the existence part of the Cauchy-Kowalevski theorem is false in the smooth category without further hyperbolicity hypotheses. The uniqueness part is much strengthened by Holmgren's theorem [8, 10, 9, 5], of which no infinite-dimensional version seems to be known.

REFERENCES

- [1] CARTAN, H. *Calcul différentiel*. Hermann, Paris, 1967.
- [2] CHAPERON, M. *Calcul différentiel et calcul intégral*, deuxième édition. Dunod, Paris, 2008.
- [3] DIEUDONNÉ, J. *Foundations of Modern Analysis*. Pure and Applied Mathematics Vol. X. Academic Press, New York-London, 1960.
- [4] ——— *Éléments d'analyse*, tome 4. Gauthier-Villars, Paris, 1971.
- [5] ——— *Éléments d'analyse*, tome 8. Gauthier-Villars, Paris, 1978.
- [6] DOUADY, A. Le problème des modules pour les sous-espaces analytiques compacts d'un espace analytique donné. *Ann. Inst. Fourier (Grenoble)* 16 (1966), 1–95.
- [7] HILLE, E. and R. S. PHILLIPS. *Functional Analysis and Semigroups*. Colloquium Publications 31. Amer. Math. Soc., Providence, RI, 1957.
- [8] HOLMGREN, E. Über Systeme von linearen partiellen Differentialgleichungen. *Öfversigt af Kongl. Vetenskaps-Akad. Förh.* 58 (1901), 91–103.

²) The author is grateful to the referee for pointing out many relevant references and facts.

³) A more traditional proof involving majorant series can easily be cooked up with the same ingredients. . .

- [9] HÖRMANDER, L. *The Analysis of Linear Partial Differential Operators I. Distribution Theory and Fourier Analysis*. Grundlehren der mathematischen Wissenschaften 256. Springer-Verlag, Berlin, 1983.
- [10] JOHN, F. On linear partial differential equations with analytic coefficients. Unique continuation of data. *Comm. Pure Appl. Math.* 2 (1949), 209–253.
- [11] LANG, S. *Analysis II*. Addison-Wesley, New York, 1968, renamed *Real Analysis* in later editions.
- [12] LEWY, H. An example of a smooth linear partial differential equation without solution. *Ann. of Math. (2)* 66 (1957), 155–158.
- [13] MALGRANGE, B. *Systèmes différentiels involutifs*. Panoramas et Synthèses 19. Société mathématique de France, 2005.
- [14] ROBBIN, J. W. On the existence theorem for differential equations. *Proc. Amer. Math. Soc.* 19 (1968), 1005–1006.

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