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NOTES [2]). Unfortunately, this part of the proof requires some modest calculations with power series [3]) since we have been unable to stick (as in the case of differential equations) to Cauchy's viewpoint on holomorphic maps and Hadamard's strong maxim: "The shortest way between two truths in the real domain passes through the complex domain."

Cartan's version of differential calculus *à la* Fréchet first appeared in Dieudonné's famous book [3], whose exposition of analytic functions of several variables, followed in 1971 by a proof of the Cauchy-Kowalevski theorem [4], did not venture into infinite dimensions.

Robbin's celebrated proof [14] of Cauchy's theorem on ordinary differential equations (in the usual differentiable setting) is a wonderful application of infinite-dimensional differential calculus, slightly distorted by Lang in an otherwise very good book [11] — and by the author in [2].

Before Douady's thesis [6], the theory of analytic functions between Banach spaces had been developed by Max Zorn in the mid-forties (see the last chapter of [7], which provides many references).

Hans Lewy [12, 4] showed that the existence part of the Cauchy-Kowalevski theorem is false in the smooth category without further hyperbolicity hypotheses. The uniqueness part is much strengthened by Holmgren's theorem [8, 10, 9, 5], of which no infinite-dimensional version seems to be known.

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[2]) The author is grateful to the referee for pointing out many relevant references and facts.

[3]) A more traditional proof involving majorant series can easily be cooked up with the same ingredients...

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