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INTRODUCTION: THE PROBLEM

To mind is an activity which always continues, a kind of work in progress aimed to understand everything. In 1450, Nicholas of Cusa writes his dialogue *De idiota*, whose third book, *De mente*, really contains the most of his theory of knowledge; this theory sheds light about the meaning of his scientific research, therefore it leads on an epistemology of mathematics. Now, to show the Cusanus' theory of the mathematical objects' nature (numbers, figures and proportions), I propose a reading of *De mente* (chapters 6 to 10), from a practical question. On the one hand, Nicholas of Cusa continues the famous biblical formula "You have ordered all things in measure, and number, and weight" which bestows a divine origin to the mathematical objects, but on the other hand he shows, in *De mente*, that mind "makes" these objects. What does this production mean?

1. THE PROBLEM OF THE KNOWLEDGE ORIGIN

In *De mente*, chapter two, Cusanus expounds the debate between peripatetic philosophers and academic ones about the origin of forms: are they beings of reason enlightened by intellect thanks to sensitive observation (Aristotle) or are they exemplary models which precede the sensitive things, as humanity itself precedes all human beings (Plato)? According to peripatetic school, forms result of a reason working from the sensitive perception and "in the intellect, there can be nothing that was not first in the senses." According to platonic school, something exists in the mind which does not come from senses nor from reason, as the exemplary forms which are reflecting in things. These forms are independent of the mind. For instance, the form of humanity in itself and by itself has first existed, then human beings have physically existed, and finally, human species has existed in human reason.

But these two doctrines are inadequate. If you are adopting the Aristotelian point of view, then you will have to consider that truth is relative to your knowledge and will therefore constantly change. If you adopt the Platonic point of view, then you must acknowledge that there are as much forms as things, that is to say an innumerable multiplicity of forms. What will be the best point of view?

Nicholas of Cusa settles it with a new answer and calls for the notion of infinity: there is a form of all forms, an infinite form, beyond the separate exemplars or beyond the beings of reason, and this ineffable form is God. The divine mind owns the exact forms for everything and is able to create things. This resort to infinity to resolve the problem of knowledge is the Cusanus' own originality. Here is the revolution the Cusanus brings about, in ancient thought: where infinity was unthinkable, it becomes a condition for thinking. Now, how will the human mind be able to know the divine forms? And more particularly, how will man be able to have mathematical knowledge?

Physical objects can be observed in nature by our senses; on the contrary, mathematical objects are ideal objects which have no material existence; a circle marked on the ground is not a real circle. Mathematical objects are in our mind and, however, they can't be invented just as you like it; their nature necessarily comes from their definition; if a triangle had four angles, it would not be a triangle. Last but not least, even if they have a divine origin, mathematical objects are not separated from each other creatures; they are united by a genealogy and they are constituting families; point generates line, which generates surfaces, which generate bodies; rectilinear figures are divided in trilateral ones, quadrilateral ones, polygons, and so on. However, a geometer have to "make" them. Why the geometer would have to produce what is generated?

2. The Cusanus' lexicon

So, to understand how Nicholas of Cusa saw his own mathematical activity, we have to study his lexicon. First, it's necessary to distinguish production and creation, because man is a producer but is not a creator. So, mathematics is not a human creation. The man produces, only God creates. That's the meaning of the formula *ex nihilo*, from nothing. God created the world only by his spirit, without anything out of him. In *De principio*, Cusanus differentiates God creator from man assimilator. The creator (*conditor*) gives the essence of things while the assimilator (*assimilator*) is the "understander". The creator sees everything in himself and considers himself as exemplar of everyone and everything he creates. According to Cusanus, to understand is to create. On the contrary, "The assimilator-intellect, which is a likeness of the Creator-intellect, sees within itself all things; i.e., it sees itself as the conceptual or befiguring exemplar of all things; and for it to understand is for it to assimilate"[1]. The creator is form of forms;

[1] Nicolai de Cusa: *Opera Omnia, De princ.*, (h. X, n. 21). Hamburg: Felix Meiner Verlag 1988: Assimilator intellectus, qui est conditoris similitudo, in se omnia videt, hoc est se omnium videt notionale sive figurativum exemplar, et eius intelligere est assimilare. (The en-

the assimilator is representation of representations. Since geometer sees the mathematical objects in himself, we can't say that he creates them: these objects are generated, but not created.

In spite of the fact that, in *De mente*, Cusanus speaks a lot about generation, the notion of begetting doesn't appear in his mathematical writings. He uses two verbs: to make and to see. He makes figures and sees what happens on them. More precisely, he uses these two verbs at imperative mood: make and see. What does it mean? We can distinguish two categories of actions: by verbs like *causare*, *effecere*, *oriri*, *exurgere*, and expressions like *per ... esse*, he describes the effects of a motion without a geometer's direct action, as, for example, the emergence of a circumference by the motion of a point at the end of a radius; then, it's possible to say that radius generates circle. The second category concerns the effects of a geometer's direct action, by verbs like *constituere*, *componere*, *manuducere*, *ducere*, and mostly *facere* (more than forty occurrences); the idea of production is mostly expressed by *ducere* and his composite *manuducere*, particularly used for the drawing of a line with a direction. So, the word *production* doesn't mean a creation nor an artificial manufacturing, but precisely the direction of a motion: *producere* is to walk forward. The geometer is at the same time active and watcher: he's active for he draws figures and watcher for he observes what happens. So the mind produces objects that generate each other. Where this theory of the mathematical practice comes from?

3. THE PROCLUSEAN SOURCE

I am making the assumption of a decisive influence of Proclus on Cusanus' theory in *De mente*, and particularly the philosophy of mathematics which Proclus exposes in his *Commentary on the first book of Euclid's Elements*, although there is a little problem of chronology to prove it! Indeed, the Greek text of this work has been printed for the first time in Basel, by Simon Grynaeus in 1533, and it occurs that the latin translation has been made by Francis Barozzi (or Barocius) in Padua in 1560, nearly a century after Cusanus died; it's nevertheless true that handwritten copies of Proclus' work have spread around in the fifteenth century. We know that very well, we also know that Bessarion, with whom Nicholas of Cusa was related, owned many of these copies. Consequently nothing prevented Cusanus from accessing to the *Commentary on the first book of Euclid's Ele-*

glish quotations of Cusanus'work come from HOPKINS, Jasper: *Complete Philosophical and Theological Treatises of Nicholas of Cusa*, translation by Jasper Hopkins, 2 volumes. Minneapolis: The Arthur J. Banning Press 2001).

ments in its Greek version before its translation, either he could have read the text by himself, or somebody else could have translated it for him.

That is the point: in short, this reading paves the way for a third solution between Plato and Aristotle. The mathematical objects are mixed: unlike the physical things, they are eternal and always identical to themselves; unlike the intelligible objects, they are composed with different parts and are divisible; they can exist in several variants (for example, triangle may be isosceles, right-angled, equilateral, scalene). On the one hand, they are devoid of material and, unlike physical things, they don't change, but on the other hand, they have a kind of expanse: numbers are discrete and geometrical figures are divisible. They have a kind of material, not a physical one, but a mathematical material.

In the chapter six of his prologue and in his commentary on the definition fourteen about figures, Proclus defines his theory about projection (*probolè*, in Greek): imagination provides to geometrical objects a *sui generis* material, a kind of white canvas,—or mirror—on which the discursive mind projects mathematical forms. Since mind can't see them in an involved way, it derives these forms from itself and unfolds them in the material of imagination; it gives to them the expanse that allows seeing them. So, thė mind at the same time projects and unfolds the mathematical reasons to show the variety and complexity of mathematical forms it explores.

For instance, the circle in the mind, before being projected on the imagination screen, is only one, without expanse, without center or circumference; then, when it's pictured in imagination, it's extended and may appear in one of its guises in size and position. If a circle would not be so presented, the mind could not explore its components nor their connections. By way of these geometrical figures, mind can catch a glimpse of the universal nature of all imaginary circles and, at the same time, can see them as paradigms for sensible forms. This function of imagination is Proclus' own contribution to the Platonic mathematics theory. In his commentary on Euclid's definition seven, he writes: "And thus we must think of the plane as projected and lying before our eyes and the understanding as writing everything upon it, the imagination becoming something like a plane mirror to which the ideas of the understanding send down impressions of themselves"[2]. We find the same metaphor in Cusanus' *De mente*:

[2] PROCLUS: *A commentary on the first book of Euclid's Elements*, translated with introduction and notes by Glenn R. Morrow. Princeton: University Press 1970, §. 121, 98; *In Procli Diadochi Lycii commentariorum in primum euclidis elementorum, latin translation of Francisco Barocio*, Grazioso Percacino. Padua: 1560: Et hoc pacto planum quidem intelligere oportet, ut pote projectum, et ante oculos constitutum: cuncta vero in hoc cogitationem describentem, phantasia quidem quasi plano aequiparata speculo, rationibus vero, quae in cogitatione sunt suas in illud demittentibus imagines.

God created mind and "He then added mind as a living mirror"[3]. And again in *De venatione sapientiae*: "the intellect finds all things to be within itself as in a mirror that is alive with an intellectual life"[4].

This proclusean theory about the geometrical figures' projection provides a specific insight on the Cusanus' mathematical epistemology. According to him, mathematical objects are generated from the mind stock, but are not invented (like a child who is fathered but not made by his parents). When he goes from definitions to figures, geometer invents nothing at all; he only follows the unfolding of the *logoï*, that is to say of the mathematical reasons. We may say that mathematical objects are made only by the mind in the sense of finished in their unfolding. If Cusanus adopts the proclusean theory, what does it mean to him "to do mathematics"?

4. Some examples

Let us glance through some definitions of elemental mathematical objects. With number, we discover the main mind's activity. Indeed, mind is computation, enumeration; it's the "discretive" power of soul. Every finite thing receives its limit and measure from the mind. Nicholas of Cusa surmises that *mens* comes from *mensurare*; he compares mind to a living compass which would encompass the variety of beings, or to a living number, that is to say a number which enumerates itself. To mind or to compute is to discern one another, is to develop the power of oneness by multiplying the one. Number allows to introduce the distinction where the common properties are confused, and also to gather the diverse properties of things. Number discerns things with multiplicity, and enables order, proportion and harmony.

Let us compare oneness and point. According to Proclus, oneness is more perfect than multiplicity because the one is indivisible. Oneness is simpler than point and precedes point. Unlike point, oneness is without position because it is immaterial, without any size or place, while point has really a position. Nicholas of Cusa also supports the simplicity of oneness: "The one, or the monad, is simpler than the point. Therefore, the indivisibility of a point is a likeness of the indivisibility of the one"[5]. Point is an unfolding of oneness in the field of quantity: "With respect to quantity, which is the unfolding of oneness, oneness is said to be a point. For in quantity only

3 Nicolai de Cusa: *De mente*, c. 5, (h. V, n. 87): cui deinde addidit mentem quasi vivum speculum.

4 Nicolai de Cusa: *De ven. sap.*, c. 17, (h. XII, n. 50): Unde, cum cognitio sit assimilatio, reperit omnia in se ipso ut in speculo vivo vita intellectuali.

5 Nicolai de Cusa: *De beryl.*, (h. XI/I, n. 21): Unum seu monas est simplicius puncto. Puncti igitur indivisibilitas est similitudo indivisibilitatis ipsius unius.

a point is present”[6]. Oneness and point are principles: “I thought that a point is the enfolding of a line as oneness is the enfolding of number. For anywhere in a line there is found nothing but a point, even as in number there is nowhere found anything but oneness”[7].

What about figures? It’s sure that mathematical objects generate one another in mind; for instance, a line can generate a plan area. But geometer has to develop the drawn lines on a paper, using rule and compass (these instruments themselves are made) so that these lines become sensible realities. Geometer is able to see moving figures in the mirror of his imagination, but to show them to his reader, he must make them, that is to say to draw them. Besides, it’s necessary for him to draw figures if he needs time for reasoning about them.

However, even when they are made, figures don’t show all their properties. For example, we can approximately locate a point position, but we can’t see the point itself, because it has no measurable size. So, we have to appeal to the reader’s imagination which, in turn, has to move figures.

To make figures, we have to draw lines. A line is a length without breadth, a magnitude with only one dimension. According to Proclus, in a line, the point procession (*proodos*, in Greek) occurs according to a continuous process which is a flow (*rhusis*, in Greek). This flow takes place in the immaterial expanse of mind[8]. According to Nicholas of Cusa, line is the point unfolding itself; point unfolds itself and develops itself.

> “What do you mean [by saying that] a line is the ‘development’ of a point?—[I mean that it is] the development, i.e., the unfolding, [of the point]—which [is to say] none other than the following: viz., that the point is present in the many atoms in such a way that it is present in each of them qua combined and connected”[9].

Point participates in line. This definition remains very empirical, similar to the gesture of drawing a line.

Let us finish with the circle. Proclus considers that it’s the most simple and the most perfect geometrical figure. A circle surpasses all figures thanks to its similitude and its identity with itself. It corresponds to finite, to oneness, to the best of all arrangements. Its nature is more divine than

6 NICOLAI DE CUSA: *De docta ign.*, II, c. 3, (h. I, n. 105): Ipsa quidem unitas punctus dicitur in respectu quantitatis ipsam unitatem explicantis, quando nihil in quantitate reperitur nisi punctus.

7 NICOLAI DE CUSA: *De mente*, c. 9, (h. V, n. 121): Putabam punctum complicationem lineae sicut unitatem numeri, quia nihil in linea reperitur nisi punctus ubique sicut in numero nihil nisi unitas.

8 cf. PROCLUS: *A commentary…*, § 97, 79.

9 NICOLAI DE CUSA: *De mente*, c. 9, (h. V, n. 119): Quomodo intelligis lineam puncti evolutionem? – Evolutionem id est explicationem, quod non est aliud quam punctum in atomis pluribus ita quod in singulis coniunctis et continuatis esse.

the others. The circle is assigned to heaven, while rectilinear forms are assigned to generation. Cusanus does not exactly follow Proclus about the symbolic of the circle; he considers its simplicity ("A circle is a perfect figure of oneness and simplicity"[10]), but he does not see finite in it; he considers circle as the eternity symbol: "For in a circle—in which there is no beginning or end, since in it there is no point that is a beginning rather than an end—I see the image of eternity"[11]. During Middle Age, the circle shows a change of sign for infinity which passes from negative to positive. However, Proclus and Cusanus are coming across again the symbol for heaven and for corruptible matter; man is a becoming being like a rectilinear figure; if he ascends to God, through the breed of its angles, it will look like more and more the divine circle:

> "Assume that a polygon inscribed in a circle were the human nature and the circle were the divine nature. Then, if the polygon were to be a maximum polygon, than which there cannot be a greater polygon, it would exist not through itself with finite angles but in the circular shape. Thus, it would not have its own shape for existing—[i.e., it would not have a shape which was] even conceivably separable from the circular and eternal shape"[12].

CONCLUSION: OBJECTS KNOWLEDGE AS KNOWLEDGE OF ONESELF

To understand the Cusanus' knowledge theory, we could inadvertently consider the dichotomy of subject and object, which, today, seems obvious to us, for we define knowledge as the representation of an object by a subject. However, this theory is inappropriate: it implies externality of the object to the subject, while, for Cusanus, knowledge is an inner pursuit.

According to Neo-Platonic tradition, mathematical objects are innate: they first exist in soul and to know them is to develop them. Their essence is given with their definition. We are far from modern mathematics which creates new objects by inventing new definitions. For instance Cardan invents the imaginary number as the square root of a negative number; Lobatchevski invents the assumption of an infinite number of parallels[13]. For

[10] NICOLAI DE CUSA: *De docta ign.*, I, c. 21, (h. I, n. 63): Circulus est figura perfecta unitatis et simplicitatis.

[11] NICOLAI DE CUSA: *De ludo.*, I, (h. IX, n. 16): In circulo enim, ubi non est principium nec finis, cum nullus punctus in eo sit, qui potius sit principium quam finis, video imaginem aeternitatis.

[12] NICOLAI DE CUSA: *De docta ign.*, III, c. 4, (h. I, n. 206): Quasi ut si polygonia circulo inscripta natura foret humana, et circulus divina: si ipsa polygonia maxima esse debet, qua maior esse non potest, nequaquam in finitis angulis per se subsisteret, sed in circulari figura, ita ut non haberet propriam subsistendi figuram, etiam intellectualiter ab ipsa circulari et aeterna figura separabilem.

[13] Unlike the physical space that he opens to infinity, Nicholas of Cusa takes the geometric Euclidean space without any change, in his mathematical treatises.

Cusanus, on the contrary, definitions have to be received and that's absolutely out of the question to invent them.

Proclus gives to mathematics an anamnestic function. In chapter fifteen of his prologue, he explains the origin of the word "mathematic" (*mathesis*, in Greek) which means learning, in the Platonic sense of reminiscence (*anamnesis*, in Greek). When the soul "makes" mathematics, its projects its essential reasons in imagination and comes across her innate knowledge again. So, mathematics allows the soul to get rid of its sensible obstacles and to make a *catharsis*. Learning is a reminiscence which is stimulated by appearances, then projected from the inside by discursive mind which returns to it. Brought into geometrical figures, the soul remembers its own essential reasons. Nicholas of Cusa does admit this finality of knowledge: to know is to know oneself.

Abstract

While practicing mathematics, Nicholas of Cusa studied the working of the mind and wrote De mente *in which he outlines his theory about the nature of mathematical objects (numbers, figures and proportions). The debate is between the Platonic theory and the Aristotelian one. While he quotes the famous biblical sentence which gives to mathematical objects a divine origin, Nicholas of Cusa demonstrates that the mind "makes" these objects. What does this production mean? Moreover, the mathematical objects belong to a genealogy. This generation has its own laws but only becomes true if the mind conceives it. Of what does this generation of numbers and figures consist?*