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Remarks on the problem of the vacuum polarization and the photon-self-energy

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As is well known the formulae for the polarization of the vacuum and the expression for the photon self energy are not well defined mathematical symbols and lead to conflict with the postulate of gauge invariance unless a certain regularizing procedure[1]) is applied. This procedure of regularization means however a change in the fundamentals of field theory, so that the question as to other possibilities of obtaining the desired result (i. e. the gauge invariance of the vacuum polarization current and the vanishing of the ph. s. e.) seems still to be of some interest. Indeed, it has been shown by Umesawa[2]), and independently, by one of us[3]) that by coupling the electromagnetic field to several charged fields of spinor and scalar (or pseudoscalar) types the gauge invariance of the vacuum polarization and the vanishing of the ph. s. e. may be achieved (at least in the second order of approximation e^2) due to some compensations. It would be premature to decide whether such compensations posess any deeper meaning or whether they should be considered merely as accidental. In any case it should be noted that one of the rules for regularization (i. e. the regularization of the whole expressions without factorization) appears as if borrowed from the theory of several mutually compensating fields. On the other hand it should be stressed that there still remains another difficulty i. e. the infinite self charge produced by the external field. This is, of course, immaterial for the photon self energy but plays a role in the problem of the vacuum polarization. In order to obtain a reasonable value for the vacuum polarization effects, the charges of the spinor and scalar fields have to be renormalized.

An external electromagnetic field $A_\mu(x)$ coupled with n fields of a spinor type (with the mass constants m_i) and with N fields of

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a scalar (or pseudoscalar) type (with masses M_i) is given. The charge and current of the vacuum polarization then assumes the form:

$$<j_\mu(x)>_{\text{vac}} = -e^2 \int d^4 x' K_{\mu\nu}(x-x') A_\nu(x') \tag{1}$$

where $K_{\mu\nu}$ (in the e^2 approximation) consists additively of separate terms for different charged fields:

$$K_{\mu\nu} = \sum_{i=1}^{n} K_{\mu\nu}^{(1/2)}(m_i) + \sum_{t=1}^{N} K_{\mu\nu}^{(0)}(M_i) \tag{2}$$

with

$$K_{\mu\nu}^{(1/2)} = 4\left\{\frac{\partial \bar{\Delta}}{\partial x_\mu}\frac{\partial \Delta^{(1)}}{\partial x_\nu} + \frac{\partial \bar{\Delta}}{\partial x_\nu}\frac{\partial \Delta^{(1)}}{\partial x_\mu} - \delta_{\mu\nu}\left(\frac{\partial \bar{\Delta}}{\partial x_\lambda}\frac{\partial \Delta^{(1)}}{\partial x_\lambda} + m^2 \bar{\Delta}\Delta^{(1)}\right)\right\} \tag{2'}$$

and

$$K_{\mu\nu}^{(0)} = \left\{-\frac{\partial \bar{\Delta}}{\partial x_\mu}\frac{\partial \Delta^{(1)}}{\partial x_\nu} - \frac{\partial \bar{\Delta}}{\partial x_\nu}\frac{\partial \Delta^{(1)}}{\partial x_\mu} + \frac{\partial^2 \bar{\Delta}}{\partial x_\mu \partial x_\nu}\Delta^{(1)} + \bar{\Delta}\frac{\partial^2 \Delta^{(1)}}{\partial x_\mu \partial x_\nu}\right.$$
$$\left. + \delta_{\mu\nu}(-\bar{\Delta}\cdot\square\Delta^{(1)} - \Delta^{(1)}\cdot\square\bar{\Delta} + 2M^2\bar{\Delta}\Delta^{(1)})\right\} \tag{2''}$$

where the $\bar{\Delta}$ and $\Delta^{(1)}$ functions are given by J. Schwinger[4]). As the expression for the vacuum polarization current of a spinor field has already been extensively discussed in the literature (compare e. g. footnote [1]), we limit ourselves here to giving only the derivation of $K_{\mu\nu}^{(0)}$:

The Hamiltonian of a scalar (or pseudoscalar) field coupled with an external electromagnetic field is[5]):

$$H_0 + H'$$

where $$H_0 = \tfrac{1}{2}\int d^3 x\, (\pi^*\pi + \operatorname{grad}\varphi^* \operatorname{grad}\varphi + M^2\varphi^*\varphi) \tag{3'}$$

and $$H' = \int d^3 x\, (\varrho A_0 - \sum_{i=1}^{3} s_i A_i + e^2\varphi^*\varphi \sum_{i=1}^{3} A_i A_i) \tag{3''}$$

with $$\varrho = i e(\pi^*\varphi^* - \pi\varphi) \qquad s_i = i e\left(\frac{\partial \varphi^*}{\partial x_i}\varphi - \frac{\partial \varphi}{\partial x_i}\varphi^*\right) \tag{4}$$

The current four vector j_μ has the components:

$$j_4 = i\varrho \quad \text{and} \quad j_i = s_i - 2e^2\varphi^*\varphi A_i \tag{5}$$

The Schrödinger equation is:

$$i\frac{\partial \Psi}{\partial t} = (H_0 + H')\Psi \tag{6}$$

In the interaction representation[6]) this becomes:

$$i \frac{\partial \Psi'}{\partial t} = H'(t) \Psi' \tag{7}$$

$$\text{with } \Psi' = e^{iH_0 t} \Psi \text{ and } H'(t) = e^{iH_0 t} H' e^{-iH_0 t} \tag{8}$$

In general, the time dependence of an operator F is given by

$$F(t) = e^{iH_0 t} F e^{-iH_0 t} \tag{9}$$

In this representation the canonically conjugate momenta become

$$\pi(t) = \frac{\partial \varphi^*(t)}{\partial t} \qquad \pi^*(t) = \frac{\partial \varphi(t)}{\partial t} \tag{10}$$

while the commutation relations remain the same as in the interaction free case

$$[\varphi^*(x), \varphi(x')] = i \Delta(x - x')$$

$$[\varphi(x), \varphi(x')] = [\varphi^*(x), \varphi^*(x')] = 0 \tag{11}$$

where x is written instead of x, y, z, t. With the aid of the notation:

$$s_\nu(t) = i e \left(\frac{\partial \varphi^*}{\partial x_\nu} \varphi - \frac{\partial \varphi}{\partial x_\nu} \varphi^* \right) \tag{12}$$

we obtain for the interaction energy:

$$H'(t) = \int d^3 x \left(-s_\nu A_\nu + e^2 \varphi^* \varphi \sum_{i=1}^{3} A_i A_i\right) \tag{13}$$

while the current j_μ becomes:

$$j_\mu(x) = s_\mu(x) + 2 e^2 \varphi^*(x) \varphi(x) (\delta_{\mu 4} \delta_{\nu 4} - \delta_{\mu\nu}) A_\nu(x) . \tag{14}$$

The vacuum expectation value of the perturbed (due to the existence of the external field) current j_μ is:

$$< j_\mu(x) >_{\text{vac}} = \frac{i}{2} \int d^4 x' \, \varepsilon(x - x') < [s_\mu(x), s_\nu(x')] >_{\text{vac}} A_\nu(x')$$

$$+ e^2 \Delta^{(1)}(0) (\delta_{\mu 4} \delta_{\nu 4} - \delta_{\mu\nu}) A_\nu(x) \tag{15}$$

The commutator in (15) yields

$$\frac{i}{2}\,\varepsilon\,(x-x')\,<[s_\mu, s_\nu']>_{\text{vac}} = e^2\left\{\frac{\partial\bar{\Delta}}{\partial x_\mu}\,\frac{\partial\Delta^{(1)}}{\partial x_\nu} + \frac{\partial\bar{\Delta}}{\partial x_\nu}\,\frac{\partial\Delta^{(1)}}{\partial x_\mu} - \frac{\partial^2\bar{\Delta}}{\partial x_\mu\,\partial x_\nu}\,\Delta^{(1)}\right.$$

$$\left. -\,\bar{\Delta}\,\frac{\partial^2\Delta^{(1)}}{\partial x_\mu\,\partial x_\nu}\right\} - e^2\,\delta^{(4)}\,(x-x')\,\Delta^{(1)}\,(x-x')\,\delta_{\mu 4}\,\delta_{\nu 4} \tag{16}$$

Here use has been made of the identity:

$$\varepsilon\,\frac{\partial^2\Delta}{\partial x_\mu\,\partial x_\nu} = \frac{\partial^2}{\partial x_\mu\,\partial x_\nu}\,\varepsilon\,(x)\,\Delta\,(x) - 2\,\delta^{(4)}\,(x)\,\delta_{\mu 4}\,\delta_{\nu 4}$$

which, together with the definition of the $\bar{\Delta}$ function, easily yields (16)*).

Therefore

$$<j_\mu\,(x)>_{\text{vac}} = e^2\int d^4\,x'\left(\frac{\partial\bar{\Delta}}{\partial x_\mu}\,\frac{\partial\Delta^{(1)}}{\partial x_\nu} + \frac{\partial\bar{\Delta}}{\partial x_\nu}\,\frac{\partial\Delta^{(1)}}{\partial x_\mu} - \frac{\partial^2\bar{\Delta}}{\partial x_\mu\,\partial x_\nu}\,\Delta^{(1)}\right.$$

$$\left. -\,\bar{\Delta}\,\frac{\partial^2\Delta^{(1)}}{\partial x_\mu\,\partial x_\nu}\right)A_\nu\,(x') - e^2\,\Delta^{(1)}\,(0)\,A_\mu\,(x) \tag{17}$$

or

$$<j_\mu\,(x)>_{\text{vac}} = -\,e^2\int d^4\,x'\,K^{(0)}_{\mu\nu}(x-x')\,A_\nu\,(x')$$

with

$$K^{(0)}_{\mu\nu}\,(x) = -\,\frac{\partial\bar{\Delta}(x)}{\partial x_\mu}\,\frac{\partial\Delta^{(1)}(x)}{\partial x_\nu} - \frac{\partial\bar{\Delta}\,(x)}{\partial x_\nu}\,\frac{\partial\Delta^{(1)}\,(x)}{\partial x_\mu} + \frac{\partial^2\bar{\Delta}\,(x)}{\partial x_\mu\,\partial x_\nu}\,\Delta^{(1)}\,(x)$$

$$+\,\bar{\Delta}\,(x)\,\frac{\partial^2\Delta^{(1)}\,(x)}{\partial x_\mu\,\partial x_\nu} + \Delta^{(1)}\,(x)\,\delta^{(4)}\,(x)\,\delta_{\mu\nu}\,. \tag{18}$$

With the aid of the equations:

$$\Box\,\Delta^{(1)} - M^2\,\Delta^{(1)} = 0 \quad \text{and} \quad \Box\,\bar{\Delta} - M^2\,\bar{\Delta} = -\,\delta^4 \tag{19}$$

(18) may be put into the form (2″).

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*) The arguments of $\bar{\Delta}$ and $\Delta^{(1)}$ are in (16): $x - x'$.

The condition for the gauge invariance of the total vacuum polarization current is:

$$\frac{\partial K_{\mu\nu}}{\partial x_\nu} = 0 \tag{20}$$

the separate terms, however, yield:

$$\frac{\partial K^{(1/2)}_{\mu\nu}(m_i)}{\partial x_\nu} = -4\,\delta^{(4)}(x)\,\frac{\partial \Delta^{(1)}(m_i)}{\partial x_\mu}$$

and (21)

$$\frac{\partial K^{(0)}_{\mu\nu}(M_i)}{\partial x_\nu} = 2\,\delta^{('')}(x)\,\frac{\partial \Delta^{(1)}(M_i)}{\partial x_\mu}$$

whose right hand sides are not well defined at the origin[1]). But the total $K_{\mu u}$ yields:

$$\frac{\partial K_{\mu\nu}}{\partial x_\nu}\,2\,\delta^{(4)}(x)\,\frac{\partial}{\partial x_\mu}\left(\sum_{i=1}^{N}\Delta^{(1)}(M_i) - 2\sum_{i=1}^{n}\Delta^{(1)}(m_i)\right) \tag{22}$$

where the right hand side is regular and vanishes identically if two following conditions are satisfied:

$$\mathrm{N} = 2\,n \tag{I}$$

$$\sum_{i=1}^{N} M_i^2 = 2\sum_{i=1}^{n} m_i^2\,. \tag{II}$$

These two conditions show a remarkable analogy with the conditions for regularization and are sufficient for the vanishing of the photon self energy. The mathematically simplest way to satisfy (I) and (II) is to take two scalar fields and to put their masses equal $M_1 = M_2 = m$. More general solutions exist too: we may introduce more than one spinor fields and assume the masses m_i and M_i very large except the mass of the electron m_1 so that the auxiliary charged particles would play a role in the extreme relativistic region only.

Next we proceed to the evaluation of the vacuum polarization current. We may Fourier analyse the tensor $K_{\mu\nu}(x)$:

$$K_{\mu\nu}(x) = \frac{1}{(2\pi)^4}\int d^4 k\, K_{\mu\nu}(k)\, e^{ik_\lambda x_\lambda} \tag{23}$$

with

$$K^{(1/2)}_{\mu\nu}(k) = \frac{4}{(2\pi)^3}\int d^4 p\,[+p_\mu(p_\nu - k_\nu) + p_\nu(p_\mu - k_\mu) - \delta_{\mu\nu}(p_\lambda(p_\lambda - k_\lambda) + m^2)]\,\frac{\delta(p_\lambda p_\lambda + m^2)}{(p_\lambda - k_\lambda)(p_\lambda - k_\lambda) + m^2} \tag{23'}$$

$$K^{(0)}_{\mu\nu}(k) = \frac{1}{(2\pi)^3}\int d^4 p\,[-p_\mu(p_\nu - k_\nu) - p_\nu(p_\mu - k_\mu)$$
$$-p_\mu p_\nu - (p_\mu - k_\mu)(p_\nu - k_\nu) + \delta_{\mu\nu}(p_\lambda p_\lambda + (p_\lambda - k_\lambda)(p_\lambda - k_\lambda)$$
$$+ 2M^2)]\,\frac{\delta(p_\lambda p_\lambda + M^2)}{(p_\lambda - k_\lambda)(p_\lambda - k_\lambda) + M^2} \qquad (23'')$$

Here use has been made of the FOURIER representations of the $\bar{\Delta}$ and $\Delta^{(1)}$ functions[4]). We introduce a new variable:

$$p' = p - \frac{1}{2}k$$

and denote for brevity:

$$p = p' + \frac{1}{2}k = p^{(1)}, \qquad p - k = p' - \frac{1}{2}k = -p^{(2)}$$

Now (23′) and (23″) assume the form:

$$K^{(1/2)}_{\mu\nu}(k) = \frac{4}{(2\pi)^3}\int d^4 p\,\Big(-p^{(1)}_\mu p^{(2)}_\nu - p^{(1)}_\nu p^{(2)}_\mu$$
$$+ \delta_{\mu\nu}\cdot(p^{(1)}_\lambda p^{(2)}_\lambda - m^2)\Big)\,\frac{\delta(p^{(1)}_\lambda p^{(1)}_\lambda + m^2)}{p^{(2)}_\lambda p^{(2)}_\lambda + m^2} \qquad (24')$$

$$K^{(0)}_{\mu\nu}(k) = \frac{1}{(2\pi)^3}\int d^4 p\,\Big(p^{(1)}_\mu p^{(2)}_\nu + p^{(1)}_\nu p^{(2)}_\mu - p^{(1)}_\mu p^{(1)}_\nu - p^{(2)}_\mu p^{(2)}_\nu$$
$$+ \delta_{\mu\nu}\cdot(p^{(1)}_\lambda p^{(1)}_\lambda + p^{(2)}_\lambda p^{(2)}_\lambda + 2M^2)\Big)\,\frac{\delta(p^{(1)}_\lambda p^{(1)}_\lambda + M^2)}{p^{(2)}_\lambda p^{(2)}_\lambda + M^2} \qquad (24'')$$

where p has been written as integration variable instead of p' for convenience sake. The above integrals may be symmetrized by writing under the sign of integration:

$$U = \frac{1}{2}\left(\frac{\delta(p^{(1)}_\lambda p^{(1)}_\lambda + m^2)}{p^{(2)}_\lambda p^{(2)}_\lambda + m^2} + \frac{\delta(p^{(2)}_\lambda p^{(2)}_\lambda + m^2)}{p^{(1)}_\lambda p^{(1)}_\lambda + m^2}\right) \text{ instead of } \frac{\delta(p^{(1)}_\lambda p^{(1)}_\lambda + m^2)}{p^{(2)}_\lambda p^{(2)}_\lambda + m^2}$$

which obviously does not change the values of the integrals in question but has the advantage of permitting the replacement of

$$U = \frac{1}{2}\left(\frac{\delta(p_\lambda p_\lambda + k_\lambda p_\lambda + m^2 + \frac{1}{4}k_\lambda k_\lambda)}{-2k_\lambda p_\lambda} + \frac{\delta(p_\lambda p_\lambda - k_\lambda p_\lambda + m^2 + \frac{1}{4}k_\lambda k_\lambda)}{2k_\lambda p_\lambda}\right)$$

by [7])

$$-\frac{1}{4}\int_{-1}^{1} d u\, \delta'(p_\lambda p_\lambda + m^2 + \frac{1}{4} k_\lambda k_\lambda + k_\lambda p_\lambda u)\,. \qquad (25)$$

We introduce again new variables

$$p' = p - \frac{1}{2} k\, u \qquad (26)$$

whence

$$p^{(1)} = \frac{1}{2} k\,(1+u) + p', \qquad p^{(2)} = \frac{1}{2} k\,(1-u) - p' \qquad (26')$$

which converts the argument of the δ' function in (25) into:

$$\delta'\big(p_\lambda p_\lambda + m^2 + \frac{1}{4} k_\lambda k_\lambda (1-u^2)\big). \qquad (25')$$

Now we introduce (25) and (26) into (24) and split $K_{\mu\nu}$ into two parts:

$$K_{\mu\nu}(k) = \overline{K}_{\mu\nu}(k) + J_{\mu\nu}(k) \qquad (27)$$

with:

$$\overline{K}^{(1/2)}_{\mu\nu}(k) = \frac{1}{2}\,\frac{1}{(2\pi)^3}\,(k_\mu k_\nu - k^2\,\delta_{\mu\nu}) \int d^4 p \int_{-1}^{1} d u\,(1-u^2)\,\delta'\big(p^2 + m^2 + \frac{1}{4} k^2 (1-u^2)\big) \qquad (28')$$

$$J^{(1/2)}_{\mu\nu}(k) = -\frac{2}{(2\pi)^3}\int d^4 p \int_{-1}^{1} d u \{p_\mu p_\nu - \frac{1}{2}\big(p^2 + m^2 + \frac{1}{4} k^2 (1-u^2)\big)\,\delta_{\mu\nu}\}\; \delta'\big(p^2 + m^2 + \frac{1}{4} k^2 (1-u^2)\big)^{*)} \qquad (28'')$$

and

$$\overline{K}^{(0)}_{\mu\nu}(k) = \frac{1}{4\,(2\pi)^3}\,(k_\mu k_\nu - k^2\,\delta_{\mu\nu}) \int d^4 p \int_{-1}^{1} d u\, u^2\,\delta'\big(p^2 + M^2 + \frac{1}{4} k^2 (1-u^2)\big) \qquad (29')$$

$$J^{(0)}_{\mu\nu}(k) = \frac{1}{(2\pi)^3}\int d^4 p \int_{-1}^{1} d u \{p_\mu p_\nu - \tfrac{1}{2}\big(p^2 + M^2 + \frac{1}{4} k^2 (1-u^2)\big)\,\delta_{\mu\nu}\}\; \delta'\big(p^2 + M^2 + \frac{1}{4} k^2 (1-u^2)\big) \qquad (29'')$$

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*) As one sees from what follows, it is *not permissible* to put $[p^2 + m^2 + \tfrac{1}{4} k^2 (1-u^2)]\,\delta'(p^2 + m^2 + \tfrac{1}{4} k^2 (1-u^2)) = -\delta(p^2 + m^2 + \tfrac{1}{4} k^2 (1-u^2))$, which, however, makes the non-gauge-invariant term vanish.

Here k^2 and p^2 have been written instead of $k_\lambda k_\lambda$ and $p_\lambda p_\lambda$ for brevity. We discuss first the part $J_{\mu\nu}$. Denoting

$$A = M^2 + \frac{1}{4} k^2 (1 - u^2)$$

we may write:

$$\int d^4 p \{p_\mu p_\nu - \frac{1}{2}(p^2 + A)\,\delta_{\mu\nu}\}\delta'(p^2 + A) = \frac{i}{2\pi}\int_{-\infty}^{+\infty} dz\, z \int d^4 p \{p_\mu p_\nu - \frac{1}{2}(p^2 + A)\,\delta_{\mu\nu}\} e^{i(p^2 + A)z}.$$

With the aid of the formulae:

$$\int d^4 p\, e^{ip_\lambda p_\lambda z} = \frac{i\pi^2}{z^2}\,\varepsilon(z) \qquad \int d^4 p\, p_\mu p_\nu e^{ip_\lambda p_\lambda z} = -\delta_{\mu\nu}\frac{\pi^2}{2z^3}\,\varepsilon(z) \quad (30)$$

the above integral yields

$$-\frac{i\,\delta_{\mu\nu}\pi}{4}\int_{-\infty}^{\infty} dz\,\varepsilon(z)\left(\frac{1}{z^2} - \frac{iA}{z}\right)e^{iAz} = -\frac{i\,\delta_{\mu\nu}}{4}\pi\int_{-\infty}^{\infty} dz\,\varepsilon(z)\,\frac{d}{dz}\frac{e^{iAz}}{z}$$

$$= \frac{i\pi}{2}\,\delta_{\mu\nu}\,\frac{e^{iAz}}{z}\Big|_{[z=0]}$$

whence

$$J_{\mu\nu}(k) = \frac{i\pi}{2(2\pi)^3}\int_{-1}^{1} du \left\{\frac{e^{i k^2/_4 (1-u^2) z}}{z}\left(\sum_{i=1}^{N} e^{iM_i^2 z} - 2\sum_{i=1}^{n} e^{im_i^2 z}\right)\right\}_{[z=0]}$$

It is immediately seen from (I) and (II) that this integral vanishes identically*). Only the gauge invariant part of $\overline{K}_{\mu\nu}(k)$ from (28) and (29) remains. We integrate $\overline{K}_{\mu\nu}^{(0)}(k)$ by parts:

$$\int_{-1}^{1} du\, u^2\,\delta'\left(p^2 + M^2 + \frac{1}{4}k^2(1-u^2)\right) = \frac{u^3}{3}\,\delta'\left(p^2 + M^2 + \frac{1}{4}k^2(1-u^2)\right)\Big|_{-1}^{+1}$$

$$+ \frac{k^2}{6}\int_{-1}^{1} du\, u^4\,\delta''\left(p^2 + M^2 + \frac{1}{4}k^2(1-u^2)\right)$$

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*) Elementary integration of:

$$I_{\mu\nu} = \int d^4 p \left\{p_\mu p_\nu - \tfrac{1}{2}(p^2 + A)\,\delta_{\mu\nu}\right\}\delta'(p^2 + A)$$

by performing the integration over p_0 first and with a large radius R gives (for $A > 0$):

$$I_{00} = 0,\ I_{11} = I_{22} = I_{33} = \frac{R^3}{\sqrt{R^2 + A}} = R^2 - \tfrac{1}{2}A + \dots$$

which leads to the same conditions for vanishing of $J_{\mu\nu}$.

whence

$$\overline{K}^{(0)}_{\mu\nu}(k) = \frac{1}{4(2\pi)^3}(k_\mu k_\nu - k^2\delta_{\mu\nu})\int d^4p\left\{\frac{2}{3}\delta'(p^2+M^2) + \frac{1}{6}k^2\int_{-1}^{1} du\, u^4\,\delta''\left(p^2+M^2+\frac{1}{4}k^2(1-u^2)\right)\right\}.$$

The first term yields from (30):

$$-\frac{1}{(2\pi)^3}\frac{\pi}{12}(k_\mu k_\nu - k^2\delta_{\mu\nu})\int_{-\infty}^{+\infty} dz\,\frac{\varepsilon(z)}{z}\,e^{iM^2z} \tag{31'}$$

while the second term may be integrated with the aid of the formula[1]):

$$\int d^4p\,\delta''(p_\lambda p_\lambda + A) = \pi/A$$

giving:

$$\frac{1}{(2\pi)^3}\frac{\pi}{24}k^2(k_\mu k_\nu - k^2\delta_{\mu\nu})\,P\int_{-1}^{1} du\,\frac{u^4}{M^2+\frac{1}{4}k^2(1-u^2)} = K^{(0)}_{\mu\nu}(k)_{\text{(regular)}} \tag{31''}$$

In the same way $\overline{K}^{(1/2)}_{\mu\nu}$ yields:

$$-\frac{1}{(2\pi)^3}\frac{\pi}{3}(k_\mu k_\nu - k^2\delta_{\mu\nu})\int_{-\infty}^{+\infty} dz\,\frac{\varepsilon(z)}{z}\,e^{im^2z} \tag{32'}$$

and

$$\frac{1}{(2\pi)^3}\frac{\pi}{4}k^2(k_\mu k_\nu - k^2\delta_{\mu\nu})\,P\int_{-1}^{+1} du\,\frac{u^2\left(1-\frac{u^2}{3}\right)}{m^2+\frac{1}{4}k^2(1-u^2)} = K^{(1/2)}_{\mu\nu}(k)_{\text{(regular)}} \tag{32''}$$

The parts (31′) and (32′) are logarithmically divergent and cannot compensate each other as the signs in both cases are equal. These terms are unphysical and have to be subtracted by means of a charge renormalization. The remaining terms (31″) and 32″) are finite and just responsible for the vacuum polarization effects.

In conclusions we state that the introduction of additional charged fields is sufficient to secure the gauge invariance of the vacuum polarization current and to yield a value zero for the photon self energy. On the other hand there remains the difficulty with the charge renormalization in an external field. This result contributes to the notion that the problem of the self charge is quite different from the problem of the self energy, and the analogy between the

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procedures of mass and charge renormalizations may be more or less superficial.

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