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Absolute Selection Rules for Meson Decay

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Introduction.

The object of the following discussion is to determine a certain class of selection rules on the spontaneous decay of a meson, or a general system of any sort, into several end products. There are two classes of such rules: i) those depending on the explicit type of interaction assumed and on the order of the matrix element involved[1] [2] [3] [4]; ii) those depending only on the transformation properties of the wave functions for the initial and final states of the system, regardless of the intermediate states and the type of interaction[5]. The first class contains only "relative" selection rules, which can be removed by going to the next higher order in a perturbation theory calculation. In meson theory, where the perturbation approach is by no means certainly justified or rapidly convergent, these selection rules may have a very weak effect. The second class contains exclusively "absolute" selection rules, the validity of which does not depend on the strength of the coupling constant or any other details of the calculation. The important difference between relative and absolute selection rules has not been made clear in the literature, and they have often been treated as if they were implicitly equivalent. Therefore the object of the present paper is to explore the complete set of selection rules of class ii), so that any given selection rule may at once be classified as relative or absolute and accorded the appropriate amount of respect.

The derivation is similar to that of reference 5 but is expressed in a simpler and more pictorial form, which can readily be generalized to all possible cases. It turns out that selection rules exist only in the very simplest cases, as might be expected: when the final state is complex, it is generally possible to find a component

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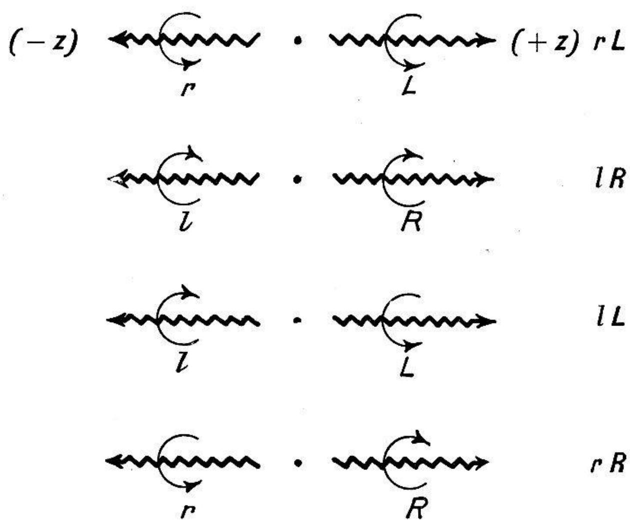
corresponding to any possible symmetry of the initial state. The derivation attempts to be plausible rather than rigorous, but follows exactly the same lines as could be used in a precise treatment. The outline followed is to discuss as examples some special cases, which happen to include most of the selection rules; with this as background, it is then easy to generalize and see that there are few additional restrictions.

1. Two-photon decay.

As a first example, suppose that the final state consists of two photons. In the rest system of the original particle, they must have equal and opposite momenta, the direction of which can be defined as the z-axis. For convenience, consider all photons to be circularly



polarized; then the final, two-photon wave function can be of 4 types, illustrated in the diagram, where l and r signify left and right circularly polarized, and the capital letters L and R refer to the photon traveling in the $+\ z$ direction.

Now photons are emitted in states such that their component of angular momentum j_z along the direction of propagation is always $j_z = \pm\ 1$, regardless of the total angular momentum j carried by the photon; this j_z component is associated with the circular polarization of the quanta. The total z-component for both photons is thus $J_z = \pm\ 2$ for rL and lR, as seen from the diagram, where the spins are parallel; and $J_z = 0$ for rR and lL, where the spins of the two photons are opposed. Since the total angular momentum J of the system must remain a constant during the decay process, we obtain a first lemma: (i) photon functions rL and lR occur only for $J \geqslant 2$, where J is the spin of the initial system.

For further lemmas, it is desirable to combine the photon wave functions into symmetrical and antisymmetrical forms:

$$0^+ = rR + lL \qquad 2^+ = rL + lR$$
$$0^- = rR - lL \qquad 2^- = rL - lR \qquad (1)$$

where the numbers 0, 2 denote the lowest value of J for which the combination occurs.

Now exchange the $+z$ and $-z$ directions by rotating about 180^0, so that all angles Θ measured from the z-axis are replaced by $(\pi - \Theta)$. Under this transformation $l \longleftrightarrow L$, $r \longleftrightarrow R$, as may be seen by turning the previous diagram upside down. Then 0^+, 0^-, 2^+ $\longleftrightarrow 0^+$, 0^-, 2^+, respectively; but $2^- \longleftrightarrow -2^-$. Furthermore, since both J and J_z are conserved throughout the reaction, the dependence of the original system on the angle Θ is $y_J^{J_z}(\Theta) = y_J^{0, \pm 2}(\Theta)$ where Y is a spherical harmonic; and the spherical harmonics have the property that $y_J^{0, \pm 2}(\pi - \Theta) = (-1)^J \, y_J^{0, \pm 2}(\Theta)$. Thus we obtain a second lemma: (ii) only the 2^- photon function can occur for odd J; and only 0^+, 0^-, 2^+ for even J. From (i) and (ii) the first selection rule immediately follows:

$J = 1$ is completely forbidden for two-photon decay. (I)

That is, two-photon decay is forbidden for both vector and pseudovector mesons.

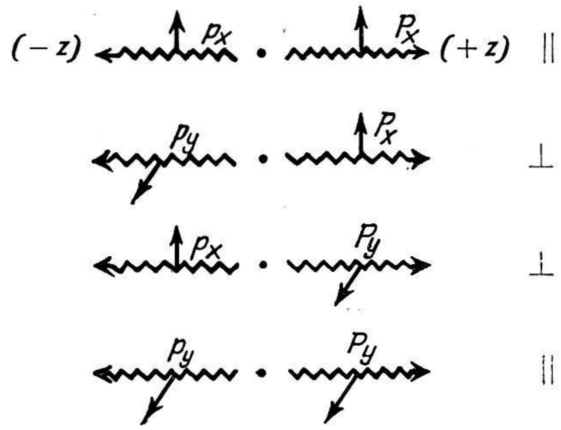
A third lemma can be obtained from consideration of parity, which is also conserved from the intial to the final state. Under the parity transformation $(x, y, z \rightarrow -z, -y, -z)$ the wave function of the original system is multiplied by P, where $P = +1$ or -1 according as the parity is even or odd; and the photon wave functions transform as $l \longleftrightarrow -R$, $r \longleftrightarrow -L$. The third lemma is therefore (iii) for systems of even parity, only the 0^+, 2^+ and 2^- photon functions can occur; for systems of odd parity, only the 0^- photon functions can occur. From (ii) and (iii) comes a second selection rule:

odd J, odd parity is forbidden for two-photon decay. (II)

Rules (I) and (II) overlap only for $J = 1$, odd parity (vector meson).

We may also consider the final photons as plane polarized, the correspondence to the circular polarization being given by $L = P_x + i\,P_y = R^*$, $l = p_x - i\,p_y = r^*$, where the p, P are all real and may be symbolized as in the accompanying diagrams.

These photon wave functions are designated as $||$ or $\perp$ to indicate the relative orientation of the planes of polarization. In terms of



the previous photon wave functions, we have

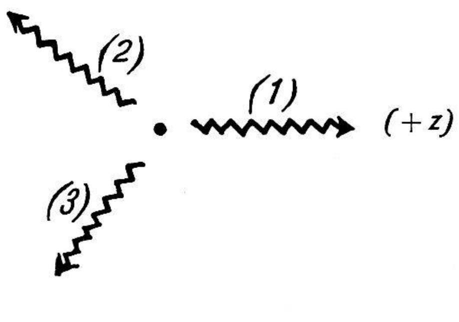
$$
\begin{aligned}
0^+ &= lL + rR = 2\, Re\,\{lL\} = 2\,(p_x P_x + p_y P_y) = || \\
2^+ &= 2\, R_l\ \{rL\} = 2\,(p_x\, P_x - p_y\, P_y) = || \\
0^- &= 2\, I_m\ \{l\, L\} = 2\,(p_x\, P_y - p_y\, P_x) = \perp \\
0^+ &= 2\, I_m\ \{rL\} = 2\,(p_x\, P_y + p_y\, P_x) = \perp
\end{aligned}
\tag{2}
$$

combining this with lemmas (ii) and (iii), we have a third rule

for 2-photon decay, the photons are $||$ for even J, even parity; they are $\perp$ for even J, odd parity or odd J, even parity. (III)

2. Three or more particle decay.

It is next of interest to see how many photons must be present in the final state to permit decay of the systems forbidden for two-photon decay. One can immediately guess that there will be



no restrictions for more than two photons, from the following argument: when only two photons are present, a very special symmetry obtains in the final system because the photons must have equal and opposite momenta; when three or more photons are present, the necessity for this symmetry vanishes, and with it all the selection rules. For example, consider three-photon decay in the center of mass system of the original particle; the z-axis of quantization is taken as the propagation direction of the first photon. Then this photon can have only $j_z = \pm 1$ for its angular momentum components; the other two, however, can have $j_z = 0, \pm 1, \ldots . \pm j_2, \pm j_3$ if they are of total angular momenta j_2 and j_3. Thus there is no lemma corresponding to (i) above.

Furthermore, since the total $J_z = j_{2z} + j_{3z} \pm 1$ is conserved throughout the reaction, the wave function of the original system can have the angular dependence $y_J^M(\Theta)$ where $|M| \leqslant J$. But on reversal of the z-axis, $y_J^M(\pi - \Theta) = (-1)^{M+J} y_J^M(\Theta)$, so that both signs are possible for the original system under this rotation, and no selection rule exists corresponding to lemma (ii) above.

The same is true for considerations of parity: the final photon state has a parity dependence given by $y_{j_1}^{\pm 1} y_{j_2}^{m_2} y_{j_3}^{m}$ but when $(x, y, z) \rightarrow (-x, -y, -z)$ we have $y_j^m \rightarrow (-1)^J y_j^m$, regardless of m. Therefore for any system of original spin J, it is only necessary to be able to write the vector sum $\vec{J} = \vec{j}_1 + \vec{j}_2 + \vec{j}_3$ so that $j_1 + j_2 + j_3 = \left\{ \begin{smallmatrix} \text{even} \\ \text{odd} \end{smallmatrix} \right\}$ to allow decay with $\left\{ \begin{smallmatrix} \text{even} \\ \text{odd} \end{smallmatrix} \right\}$ parity. Such vector sums are always possible for any J, so that no parity selection rule (iii) exists, either.

The consideration for three photons can obviously be repeated for any larger number; furthermore, they are also true if one or more of the photons in the final state are replaced by any other types of particle; thus we have the general conclusion

> the symmetry of the final state imposes no selection rules for the decay of a system into three or more particles of any type, including photons or other massless particles. There is only the general conservation rule that the total number of half-odd spins in the initial and final states together must be even. (IV)

Therefore the statement in reference 2 that three photon decay is forbidden for the pseudovector meson must be a "relative" selection rule of class i) and not an absolute rule of class ii). One expects that three-photon decay is actually possible for the pseudovector meson, but that the first non-vanishing matrix element contains the nuclear coupling constant as g^4 rather than as g^2.

3. Decay into two scalar particles.

There remains only the question what selection rules, if any, arise when the final state consists of two particles, not both photons.

In the center of gravity system they have equal and opposite momenta, taken to define the z-axis. Suppose first that the particles have intrinsic spin 0 and denote their wave functions by a and B, where the different letters indicate distinguishable types of wave functions, and the letters are capital or lower-case according to the direction along the z-axis, as before.

The analysis for photons can be copied exactly. Both a and B have $j_z = 0$ (since the expansion of their wave functions in spherical harmonics contains only y_l^0 terms), so that for the whole system $J \geqslant 0$ and there is no restriction like lemma (i). We form the symmetric and antisymmetric combinations

$$\varphi_+ = aB + bA, \qquad \varphi_- = aB - bA$$

On rotation of the z-axis by 180^0, a $\longleftrightarrow A$, $b \longleftrightarrow B$, so that thus (iv) even J decay by φ_+, odd J by φ_-. For the parity transformation, $a \longleftrightarrow AP_a$, $b \longleftrightarrow BP_b$, where $P_a = P_A$ and $P_b = P_B$ are the internal parities of the respective particles, and can have the values ± 1. Thus $\varphi_\pm \longleftrightarrow \pm P_a P_b \varphi_\pm$, and if the parity of the original system is P, the corresponding lemma is (v) the system decays by $\varphi_\pm$ according as $PP_aP_b = \pm 1$. Since both (iv) and (v) must be compatible, they combine to yield the selection rule,

$$\text{decay into two spin 0 particles is allowed only if } P(-1)^J = P_a P_b \qquad \text{(V)}$$

This rule has direct application in nuclear physics; consider for instance the photo-disintegration of a light nucleus of type $A = 4n$ into an α-particle and $A = 4n - 4$. Such nuclei have ground states of spin 0, even parity; thus P and J of the excited nucleus before emission of the α-particle will be determined by that of the γ-ray. Now for electric multipole transitions $P(-1)^J = +1$ for magnetic multipole $P(-1)^J = -1$; and since the internal parity of an α-particle is $P_\alpha = +1$, rule (V) implies that if the residual nucleus has spin 0 and $P_b = 1$ (which is likely), the γ-ray transition must be electric multipole, and if the residual nucleus has spin 0 and $P_b = -1$ (unlikely among low levels) the transition is magnetic. Thus the relative probability of magnetic multipole disintegration is especially low for these reactions.

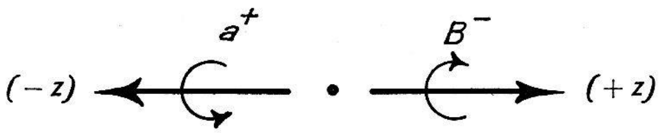
Rule (V) can be further specialized if both particles are identical, a physically rather probable situation. In this case $a = b$, $A = B$, so that $\varphi_- \equiv 0$ and we must have $J =$ even according to (iv). Furthermore, $P_a = P_b$, so that $P_a P_b = P_a{}^2 = 1$, and thus $P = 1$ according to (v). Thus for this case

decay into two identical spin 0 particles requires $P = (-1)^J = +1$ (Va)

This last is the well-known selection rule for the decay of Be^8 into two α-particles.

4. General two-particle decay.

In general the possibility must be allowed that two of the particles (either the two decay particles or the original system and one decay particle) have half-odd spins. For the present considerations, the transformation properties of such wave functions are sufficiently given by non-relativistic approximation in which the Pauli spin functions or their analogues for higher spins are employed. The two half-odd spins can be combined to give expressions with only integral indices, as before. The particle momentum is taken along the z-axis, and the large and small letters are used for wave functions as in section 3. Now, however, each particle can have z-components of total angular momentum $|j_z| \leqslant S$, where S is its intrinsic spin;



this comes from the y_l^0 of its orbital motion compounded with the intrinsic spin, which may have any component in the z-direction. Choose wave functions corresponding to a particular value $m_a = m_A$ for $|j_z|$, and distinguish the two possibilities of $j_z = \pm m_a$ as a^+ and a^-. Then from two essentially different wave functions a, b, with m_a, $m_b \neq 0$, form the 8 independent final wave functions

$$\psi_{\pm}^{pq} = a^p B^q \pm A^p b^q \tag{2}$$

where p, $q = \pm$. Now corresponding to lemma (i), we have for the original system

$$\begin{aligned} J_z &= \pm \mid m_a + m_b \mid \quad pq = + \\ J_z &= \pm \mid m_a - m_b \mid \quad pq = - \end{aligned} \tag{3}$$

For rotation of the z-axis, note that $a^{\pm} \longleftrightarrow A^{\mp}$, $b^{\pm} \longleftrightarrow B^{\mp}$ as is seen by rotating the diagram; thus $\psi^{pq}_{\pm} \longleftrightarrow \pm \psi_{\pm}^{-p-q}$. To form invariant functions under this rotation, take

$$\begin{aligned} \chi_{z+} &= (\psi^{pq}_{\pm} + \psi_{+}^{-p-q}) \text{ and } (\psi^{pq}_{-} - \psi_{-}^{-p-q}) \\ \chi_{z-} &= (\psi^{pq}_{-} + \psi_{-}^{-p-q}) \text{ and } (\psi^{pq}_{+} - \psi_{+}^{-p-q}) \end{aligned} \tag{4}$$

where $\chi_{z\pm} \longleftrightarrow \pm \chi_{z\pm}$ under the transformation. The wave function of the original system transforms as $y_J^{J_z}(\Theta) \longleftrightarrow (-1)^{J+J_z} y_J^{J_z}(\pi - \Theta)$. Thus $\chi_{z\pm}$ is associated with even and odd values of $(J + J_z)$, respectively. This is a very weak restriction, since for decay particles with non-zero spin J_z can generally have several values; thus any value J can be associated with χ_{z+} or χ_{z-} by choosing the appropriate J_z.

Under parity transformation, $a^{\pm} \longleftrightarrow P_a A^{\pm}$, $b^{\pm} \longleftrightarrow P_b B^{\pm}$ so that $\varphi^{pq}_{\pm} \longleftrightarrow \pm P_a P_b \psi^{pq}_{\pm}$. Using these three transformation properties, we can classify the wave functions (2) individually:

$J_z = \lvert m_a + m_b \rvert = J_z^+$	$J_z = \lvert m_a - m_b \rvert = J_z^-$
${}^{+}\chi^{P+}_{z\,+} = (\psi_{+}^{++} + \psi_{+}^{--})$	${}^{-}\chi^{P+}_{z\,+} = (\psi_{+}^{+-} + \psi_{+}^{-+})$
${}^{+}\chi^{P+}_{z\,-} = (\psi_{+}^{++} - \psi_{+}^{--})$	${}^{-}\chi^{P+}_{z\,-} = (\psi_{+}^{+-} - \psi_{+}^{-+})$
${}^{+}\chi^{P-}_{z\,+} = (\psi_{-}^{++} - \psi_{-}^{--})$	${}^{-}\chi^{P-}_{z\,+} = (\psi_{-}^{+-} - \chi_{-}^{-+})$
${}^{+}\chi^{P-}_{z\,-} = (\psi_{-}^{++} + \psi_{+}^{--})$	${}^{-}\chi^{P-}_{z\,-} = (\psi_{-}^{+-} + \psi_{-}^{-+})$ (5)

where ${}^{+}\chi$ and ${}^{-}\chi$ are associated with the large (J_z^+) and small (J_z^-) values of J_z, respectively, and under parity transformation $\chi^{P\pm} \longleftrightarrow \pm P_a P_b \chi^{P\pm}$.

Consider the wave functions (5): there is in general no restriction on ${}^{-}\chi$ since it is always possible to choose $\lvert m_a - m_b \rvert$ equal to the minimum J that could be possessed by the decaying system (0 or $\frac{1}{2}$), regardless of whether half-odd spins are present or not. The only exception occurs when the decay products are a photon and a spin 0 particle. The photon is a transverse spin 1 particle, which has the special feature that $m_a = 1$, but $m_a = 0$ is excluded*). The scalar particle has only $m_b = 0$ so that for this special case $J_z^- = 1$, and ${}^{-}\chi$ is associated only with $J \geqslant 1$ for the original system. The same is obviously true of ${}^{+}\chi$, which leads to the trivial rule that

for $J = 0$, decay into a photon and a particle of spin 0 is forbidden (VI)

*) It is true in general that no massless particle can have more than two independent spin components (cf. M. Fierz, Helv. Phys. Acta **12**, 3 (1939), but these components are selected only by a specific rule for each case. For the photon, this rule is transversality; but in general it must be left indeterminate, so that massless and massive particles are treated here in the same way.

There are a few more restrictions on ${}^{+}\chi$: (vi) it is forbidden for two photon decay if $J < 2$, for a photon and a half-odd particle if $J < 3/2$, for a photon and an integral spin particle if $J < 1$, for two half-odd particles if $J < 1$. Another restriction comes from the fact that if $J_z^+ = 0$, then $m_a = m_b = 0$, so that $a^+ = a^- = a^0$, etc. From (2) and (4) it follows that (vii) for $J_z = 0$, ${}^{+}\chi^{P+}_{z\,-} = {}^{+}\chi^{P-}_{z\,+} = 0$.

None of these restrictions is sufficient to provide a new selection rule, since there is always one of the 8 wave functions that will be allowed for any given initial state. A new selection rule would be obtained only if the number of independent χ were reduced. The most general way of achieving this is to assume both particles identical; then the initial system must, of course, have integral spin. Then we have $a = b$, $A = B$, and

$$\psi^{++}_{-} = \psi^{--}_{-} = 0, \quad \psi^{+-}_{\pm} = \pm \psi^{-+}_{\pm} \tag{6}$$

as may be seen by insertion in (2); and hence from (5)

$$ {}^{+}\chi^{P-}_{z\,+} = {}^{+}\chi^{P-}_{z\,-} = {}^{-}\chi^{P+}_{z\,-} = {}^{-}\chi^{P-}_{z\,-} = 0 \tag{7}$$

where P is now the parity of the original system, since $P_a P_b = P_a^z = 1$. From (7) we have the immediate lemma: (viii) decay into two identical particles is permitted for odd parity only with ${}^{-}\chi^{P-}_{z+}$, for odd $(J + J_z)$ only with ${}^{+}\chi^{P+}_{z-}$. Even with the reduction to 4 independent χ no selection rule is obtained from (vii) and (viii). Since this is the most restricted case outside of the special ones treated in the preceding sections, we have shown that

no other selection rules besides I—VI exist. (VII)

Of course the results for two-photon decay follow immediately from the general treatment of this section: one has only to notice that $J_z = 0, \pm 2$, and lemmas (vi) and (viii) become equivalent to (I) and (II). In the notation of the present section, the wave functions of section 1 are

$$2^{\pm} = {}^{+}\chi^{P+}_{z\,\pm}\,, \quad 0^{\pm} = {}^{-}\chi^{P+}_{z\,+} \tag{8}$$

To obtain the results of section 3 for spin 0 particles, we have $a^+ = a^- = a^0$, etc., and hence $\psi^{++} = \psi^{+-} = \psi^{-+} = \psi^{--}$ or hence

$$ {}^{\pm}\chi^{P+}_{z\,+} = \varphi_+\,, \quad {}^{\pm}\chi^{P-}_{J\,-} = \varphi_- \tag{9}$$

and all other χ's vanish.

The author wishes to thank Professor P. Scherrer for the hospitality of the Physikalisches Institut, and for the colloquium discussions which instigated the present study.

5. Summary.

The spontaneous decay of a system has been examined to determine the complete set of selection rules that do not depend on specific assumptions concerning the matrix elements involved. These rules are independent of the charge, mass, and intrinsic spin and parity of the particles involved, except as otherwise stated. They apply, of course, to any intermediate state as well as the final state of the system. In particular, the rules are

I) Two-photon decay is prohibited if $J = 1$.
II) Two-photon decay is prohibited if $J =$ odd, $P =$ odd.
III) Two-photon decay is $\|$ or $\perp$ as $P(-1)^J = \pm 1$.
IV) Decay into two spin 0 particles requires $P(-1)^J = P_a P_b$; if they are identical, $P = (-1)^J = +1$.
V) Decay into a photon and a spin 0 particle is prohibited for $J = 0$.
VI) All other decay processes are allowed, provided that there is an even total number of half-odd spins in the initial and final states.

Any other selection rules are relative, not absolute, and depend on detailed assumptions concerning the unobserved intermediate states.

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