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Generalisations of Einstein's Theory of Gravitation Considered from the Point of View of Quantum Field Theory

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The following considerations are based on the assumption that the principle of general relativistic invariance is neither limited to the macroscopic aspect, nor in contradiction with the fundamental principles of quantum theory, but is, on the contrary, to be regarded as an important guide in the search for an adequate formulation of quantum field theory. Against this assumption doubts have often been raised founded on the weakness of gravitational forces even at nuclear dimensions especially in connection with the view that quantum field theory needs some further deepgoing revision entailing the introduction of a fundamental length comparable in size with the cut-off distances in meson theories and thus not very much smaller than the range of nuclear forces. On the other hand it is well-known that the need of such revision is comparatively little urgent in quantum electrodynam icsbased on DIRAC's relativistic wave equation for the spinor field and MAXWELL's equations for the electromagnetic field, which also in many other respects is clearly the best founded part of quantum field theory. Already visible in WEISSKOPF's calculation of the electron self energy according to the DIRAC equation [1], the divergence of which proved to be very much weaker than that corresponding to the scalar wave equation, this fact has been strongly emphasized through the success of the renormalisation procedure developed during later years. And it appears still more strikingly through the recent work by PAULI and KÄLLÉN[1]) on the LEE model, suggesting according to PAULI a definite limit of this procedure necessitating a change of the theory at high energies and correspondingly small distances.

The principle of general relativity in combination with the quantum postulate is hardly sufficient, however, for the formulation of adequate field laws. In the first place we have here the principle of invariance

[1]) W. PAULI and G. KÄLLÉN, [2]. I am indebted to professor PAULI and Dr. KÄLLÉN for kindly letting me see their considerations before publication. Added in proofs: See also L. LANDAU [3], whose considerations (mentioned below by PAULI) I learnt about at the Bern meeting.

against so called gauge transformations closely connected with the conservation of electric charge. The importance of this invariance was strongly emphasized by WEYL [4] in the first attempt to extend the frame of gravitational relativity theory so as to comprise the electromagnetic phenomena. In the following we shall take as a starting point the fivedimensional representation of this principle, which, as far as it goes, has given a rather satisfactory solution to the problem posed by WEYL.

The main problem to be solved by a generalised quantum field theory is, however, the adequate formulation of the laws governing nuclear and mesonic phenomena. Here YUKAWA's idea of the connection between nuclear forces and charged and neutral BOSE-EINSTEIN particles with restmass corresponding to the range of the forces has been leading in the great amount of work done to bring order into this new part of physics. And as a further guide the assumption of the charge independence of these phenomena — neglecting electromagnetic forces — first introduced by KEMMER has played an important rôle. In the search for a more rigid basis for the description of these phenomena it seems natural to fix the attention on the appearance of BOSE-EINSTEIN fields with electrically charged quanta as being the essentially new feature of YUKAWA's theory as compared with the EINSTEIN-MAXWELL theory of gravitational and electromagnetic fields. In fact, the appearance of finite restmasses does not in itself demand any new principle of invariance, exhibiting rather the lack of that kind of gauge invariance, which forbids a finite photon restmass.

Now the fivedimensional theory, mentioned above, seemed to demand a generalisation including such charged fields. In order to account for the existence of an elementary electric charge one had hereby to assume a periodic dependence of the field quantities on the extra coordinate x^0, conjugated to the electric charge, the period corresponding to a small length $\sqrt{2\varkappa}\, h\, c/e \approx 0{\cdot}8 \times 10^{-30}$ cm, where $\varkappa$ $(= 8\,\pi\,\gamma/c^4)$ is the EINSTEIN gravitational constant, h PLANCK's quantum of action, c the vacuum velocity of light and e the elementary electric charge.

Now, such a theory, although in a certain sense the most direct generalisation of relativity theory including gauge invariance and charge conservation so as to comprise electrically charged fields, has such strange features that it should hardly be taken litteraly. In the same direction points the similarity of the periodicity condition to a quantum condition in classical disguise. We shall see, however, how the fivedimensional relativity theory with the periodicity assumption may be used as a model or stepping stone towards a theory of more physical aspect, whereby charge invariance appears as part of a natural generalisation of gauge invariance. But in order to have a background on which to consider the somewhat repellent appearance of the small length just mentioned in the generalised quantum field theory we shall first return to the question touched upon above of the natural, unit length. For this purpose we shall regard the quantum field theory built on EINSTEIN'S gravitational theory and DIRACS theory of the electron, but so far only in a general way without entering on the specific difficulties of the quantisation problem.

Let us use the Lagrangian formalism of quantum theory developed by FEYNMAN and SCHWINGER according to which e^{iS}, with $S = \int L\, d^4\, x/\hbar\, c$ (L is the Lagrangian density of the system, the integral is to be taken over the space-time region separating two space-like hypersurfaces and the time coordinate x^4 is taken as $c\,t$), is connected to the transformation matrix relating expectations at one of the hypersurfaces with those at the other hypersurface. Let now L signify the Lagrangian density of a system of spinor particles in a gravitational field. Then the total FEYNMAN-SCHWINGER integral of the spinor field and the gravitational field is

$$S = \frac{1}{\hbar\, c} \int \left(L + \frac{1}{2\varkappa}\, G\right) d^4x \tag{1}$$

where G is the wellknown Lagrangian density of the pure gravitational field, of which we shall for the present only use the property that, with coordinates

$$x^k = 1_0\, \xi^k, \qquad k = 1, 2, 3, 4\,, \tag{2}$$

where 1_0 is a (so far arbitrary) unit length and the ξ^k dimensionless parameters, it is a function G_0 divided by 1_0^2 of the dimensionless EINSTEIN g_{ik} and their derivatives with respect to the ξ^k. Introducing the parameters ξ^k into the Lagrangian density of the spinor particles and replacing the spinor wave function ψ by $\varphi = 1_0^{3/2}\, \psi$, which is again dimensionless, we have

$$L = \frac{\hbar\, c}{1_0^4}\, L_0\,, \qquad G = \frac{1}{1_0^2}\, G_0. \tag{3}$$

where L_0 is obtained from L by replacing the x^k by the ξ^k, the ψ by φ and $\hbar$ and c by unity, L_0 being thus dimensionless. Then we get

$$S = \int d^4\xi \left(L_0 + \frac{1_0^2}{2\varkappa\hbar c}\, G_0\right), \tag{4}$$

from which equation we see that the quantum field theory of the combined spinor and gravitational field will take a particularly simple form, if for the unit length 1_0 we choose the expression

$$1_0 = \sqrt{2\varkappa\hbar c} = \sqrt{\frac{8\,\gamma\,\hbar}{c^3}} \approx 1.1 \times 10^{-32}\,\text{cm}\,, \tag{5}$$

γ being the ordinary gravitational constant. We shall compare the length l_0 with the period characteristic of the fivedimensional theory mentioned above

$$l = \frac{\sqrt{2\varkappa}\, h\, c}{e} \tag{6}$$

It follows

$$l = 2\pi \sqrt{\frac{\hbar c}{e^2}}\; l_0 \tag{7}$$

Now l_0 is the outcome of the ordinary quantisation of gravitational theory, while l comes from the fivedimensional, quasigeometrical interpretation of the elementary quantum of electricity, which we regard as a quantisation in disguise. To have these two processes of quantisation connected is thus the same as to determine the value of $\hbar c/e^2$. A near lying possibility of such a connection is that the relation between l and l_0 is determined by the renormalisation of the electric charge through vacuum polarisation, which in an adequate theory ought to be finite. If thus the basic equations instead of e would contain a quantity e_0 simply connected to $\sqrt{\hbar c}$ (say $\sqrt{\hbar c}$) their form would become very simple, if l_0 is chosen as the unit of length.

Before leaving the question of l_0 we shall regard this quantity from a more elementary point of view [5]. Let us assume that we have to do with a particle described approximately as a quantum belonging to a linear wave equation. Then by superposition we may make a wave package representing the particle confined to a volume of linear dimensions λ. If λ is small compared to the COMPTON wavelength of the particle the wave package will represent an energy $\sim h c/\lambda$ and thus a mass $\sim h/c\,\lambda$. Thus the difference in gravitational potential between the centre and the edge of the wave package will be $\sim \gamma\, h/c\,\lambda^2$ and will mean a negligible change of the metrics only if $\gamma h/c\lambda^2 \ll c^2$, i. e. if $\lambda \gg \sqrt{\gamma\, h/c^3} \sim l_0$. From this consideration it would seem to follow that the linear wave equation for the particle in question would break down when the wave length approaches the length l_0. The condition in question can also be expressed by stating that for λ approaching the length l_0 the gravitational self energy of the particle approaches the kinetic energy corresponding to its volume. It is perhaps not unreasonable to expect that the rigorous consideration of gravitational and perhaps other similar non-linear effects would do away with the remaining divergencies of electron theory. In this connection it is interesting that PAULI's estimate[1]) of the energies of the 'ghost'

[1]) Kindly communicated to me in a letter.

states, those states where the unphysical, indefinite metrics of renormalized electron theory makes itself felt, is of the order of magnitude $\hbar c/l_0$.[1])

The five dimensional representation of the connection between gravitation and electromagnetism is based on the gauge transformation of the electromagnetic potentials

$$A'_k = A_k + \frac{\partial f}{\partial x^k}, \qquad k = 1, 2, 3, 4 \tag{8}$$

and the corresponding transformation of the wave function ψ of an elecrtic particle of charge q

$$\psi' = \psi\, e^{\frac{iq}{\hbar c} f}, \tag{9}$$

where f is an arbitrary function of the space-time coordinates. The essential idea of the five-dimensional representation is now to regard the electric charge (multiplied by a suitable, constant factor to give it the dimension of a momentum) as a fifth component p_0 of the momentum-energy vector and to introduce a parameter x^0 (of the dimension of a length) as its canonically conjugate. Thus a wave function φ of an electric particle of charge q will be written

$$\varphi(x^0, x) = \psi(x)\, e^{\frac{i}{\hbar} p_0 x^0}, \tag{10}$$

where x is shorthand for the four space-time coordinates. With

$$p_0 = \frac{q}{\beta c}, \qquad f_0(x) = -\beta f(x), \tag{11}$$

where β is a constant of the dimension of a reciprocal potential, the transformation

$$x^{0\prime} = x^0 + f_0(x) \tag{12}$$

is seen to leave the fivedimensional wave function $\varphi(x^0, x)$ invariant. Thus we have a simple representation of the phase part of the gauge transformation, which is analogous to the shift of the origin of the space-time coordinates

$$x^{k\prime} = x^k + f_k(x), \qquad k = 1, 2, 3, 4 \tag{13}$$

so closely connected with the conservation of momentum and energy.

While the introduction of the gravitational field in the general theory of relativity could be based on the metric invariant (the square of the fourdimensional line element) of special relativity theory a generalised field theory should not be based on some extended line element, the physical significance of which would be rather obscure. In stead of this we

[1]) see also L. Landau [3], where a similar estimate is made, an its connection with $\frac{\hbar c}{l_0}$ is pointed out.

have, as has often been remarked to chose some fundamental, physical law, the invariance of which under an extended transformation group is plausible. It would seem that the natural choice to make is the DIRAC equation, the generally relativistic form of which has long been known thanks to the work of FOCK, SCHRÖDINGER, BARGMANN and others[1]).

Thus we consider five matrix functions γ^μ ($\mu = 0, 1, 2, 3, 4$) of the coordinates (in the restricted theory of the space-time coordinates alone), which in general coordinate transformations are supposed to behave as the contravariant components of a five-vector. Then the DIRAC equation will take the form

$$\gamma^\mu \left(\frac{\partial}{\partial x^\mu} - \Gamma_\mu\right) \psi + \frac{m c}{\hbar} \psi = 0 . \tag{14}$$

Here the Γ_μ are another set of matrices, which are known to appear in the theory in order to make it invariant with respect to linear transformations of the ψ-components, the coefficients of which may be functions of the coordinates. The γ^μ are supposed to fulfil the following commutation relations

$$[\gamma^\lambda, \{\gamma^\mu, \gamma^\nu\}] = 0 , \qquad \lambda, \mu, \nu = 0, 1, 2, 3, 4 , \tag{15}$$

where as usual $[a, b]$ and $\{a, b\}$ denote the expressions $a b - b a$ and $a b + b a$ respectively. Denoting the symmetric quantities $\frac{1}{2}\{\gamma^\mu, \gamma^\nu\}$, which transform as a tensor, by $\gamma_{\mu\nu}$ we may define the corresponding covariant tensor components by means of

$$\gamma_{\mu\varrho}\, \gamma^{\varrho\nu} = \delta_\mu^\nu , \tag{16}$$

δ_μ^ν being KRONECKER symbols, and the quantities

$$\gamma_\mu = \gamma_{\mu\varrho}\, \gamma^\varrho , \tag{17}$$

from which follows

$$\gamma_{\mu\nu} = \frac{1}{2}\{\gamma_\mu, \gamma_\nu\}, \; \delta_\mu^\nu = \frac{1}{2}\{\gamma_\mu, \gamma^\nu\} . \tag{18}$$

As well known, each of the matrices Γ_μ can (apart from an arbitrary term proportional to the unit matrix) be simply expressed in terms of the γ^μ, γ_μ and their first derivatives with respect to the coordinates. In order to have a complete, generalised quantum field theory based on the equation (14) we may try to define the Lagrangian density of the γ^μ field by means of a procedure connected with the DIRAC equation in question, which leads to the correct result in the purely gravitational case. For this purpose we consider the process of parallel displacement of a spinor ψ first introduced by FOCK [6] and defined by means of the covariant derivatives

[1]) In connection with projective relativity theory it was early used by VEBLEN, PAULI and others.

$$\Delta_\mu \psi = \left(\frac{\partial}{\partial x^\mu} - \Gamma_\mu\right)\psi, \tag{19}$$

which in a linear transformation of the spinor components behave like spinors. Now, the parallel displacement of a spinor is in general non-integrable, the commutators $[\Delta_\mu, \Delta_\nu]$ being linear, homogeneous expressions in the components of the curvature tensor of the RIEMANN space, whose metric tensor is given by the $\gamma_{\mu\nu}$. Through this process the curvature tensor and the corresponding invariant, playing the rôle of Lagrangian density in the EINSTEIN theory of gravitation, may thus be defined by means of processes and quantities directly connected with the DIRAC equation without any recurrence to 'geometry'.

Now, the wellknown result [7] of the restricted, fivedimensional theory is that the Lagrangian obtained in this way corresponds exactly to the EINSTEIN-MAXWELL theory of gravitation and electromagnetism, if the following restrictions are made a) the $\gamma_{\mu\nu}$ depend only on the four space-time coordinates b) γ_{00} is constant — restrictions compatible with the transformations (12) and (13) — and if the following connections are made between the $\gamma_{\mu\nu}$ and the g_{ik} and A_i of the ordinary theory

$$\gamma_{i0} = \gamma_{00}\,\beta\,A_i, \;\; \gamma_{ik} = g_{ik} + \gamma_{00}\,\beta^2 A_i\,A_k, \tag{20}$$

and if, further, the constant β is determined by the relation

$$\varkappa = \frac{1}{2}\,\gamma_{00}\,\beta^2.$$

The restriction of γ_{00} to be constant is certainly not natural and has been the subject of much discussion [5]. The most obvious assumption to make is to leave out this restriction altogether and let γ_{00} be determined by the fifteenth field equation then obtained from the variational principle. In the absence of 'matter' (here the spinor particles) this can easily be carried through and leads to a variation of γ_{00} in the presence of electromagnetic fields, which, however, is extremely weak and probably far outside the reach of experimental investigation. In the presence of matter the corresponding part of the generalised energy-momentum tensor is still uncertain being tied up with the problem of the masses of elementary particles. To me it seems plausible that the solution of this problem, which certainly needs further generalisation of the field theory, would lead to a negligible, average variation of γ_{00} also in the presence of matter, although its variation within regions of the dimension l_0 may be important for the problem just mentioned. Outside of matter and when the variation of γ_{00} may be neglected we may put $\gamma_{00} = 1$ so as to obtain the same scale for x^0 as for the other coordinates in an ordinary coordinate system, where gravitation may be neglected. Then in stead of the above relation we may write

$$\beta = \sqrt{2\varkappa}, \tag{21}$$

which I think ought to be regarded as a relation between two constants, from which follows the validity of the ordinary laws, as soon as the deviations from the restrictions a) and b) may be neglected.

Coming now to the generalisation of the theory we shall still restrict ourselves as far as possible. Thus we shall leave the transformation (13) of the space-time coordinates unchanged, just extending the transformation (12) to

$$x^{0\prime} = x^0 + f_0(x^0, x), \tag{22}$$

where f_0 is supposed to be a periodic function of x^0. Using 1_0 as unit of length we shall assume the period to be 2π, which with $\gamma_{00} \to 1$ in free space contains a physical assumption perhaps to be changed at a later stage. At present it is made for reasons of simplicity. Since according to (13) the γ^k, $k = 1, 2, 3, 4$, transform among themselves we may assume that they are functions of the space-time coordinates alone, while γ^0 will have to contain x^0 as well. Of ψ we shall also assume that it is a periodic function of x^0 corresponding to a superposition of states belonging to particles of charge $0, \pm 1, \pm 2, \ldots$ quanta of electricity. This is equivalent to its expansion according to the set of eigenfunctions

$$U_n(x^0) = \frac{1}{\sqrt{2\pi}}\, e^{inx^0}, \quad n = 0, \pm 1, \pm 2, \ldots, \tag{23}$$

thus

$$\psi(x^0, x) = \sum_n \psi_n(x)\, U_n(x^0). \tag{24}$$

Let now $F(x, x^0)$ be any field function, e.g. γ^0, depending on x^0 as well as on x. Then the introduction of the expansion (24) into the wave equation (14) will lead to a system of wave equations for the $\psi_n(x)$ no longer containing x^0, in which matrices of the kind

$$(n' \mid F(x^0, x) \mid n'') = F_{n'-n''}(x) \tag{25}$$

will appear, $F_n(x)$ being the FOURIER coefficients of the expansion

$$F(x^0, x) = \sum_n F_n(x)\, e^{inx^0}. \tag{26}$$

On the other hand for p_0 itself we obtain the following matrix representation

$$(n' \mid p_0 \mid n'') = n'\, \delta_{n'n''} \tag{27}$$

in conformity with the above statement about the charge belonging to the states U_n.

We shall now find also the matrix representation of the generalised gauge transformation (22), whereby we may limit ourselves to the infinitesimal transformation

$$x^{0\prime} = x^0 + \varepsilon \sum_{s=-\infty}^{+\infty} \xi_s(x)\, e^{i s x^0}, \tag{28}$$

ε being an infinitesimal, constant parameter. Now, to a function $U_n(x^0)$ corresponds a function $\overline{U}_n(x^{0\prime})$ given by

$$\overline{U}_n(x^{0\prime}) = U_n(x^0) \left| \frac{dx^0}{dx^{0\prime}} \right|^{\frac{1}{2}}, \tag{29}$$

where x^0 has to be expressed in terms of $x^{0\prime}$ by means of (28). The $\overline{U}_n(x^{0\prime})$ form again a complete, orthogonal and normalized set of eigenfunctions for the same set of states as the $U_n(x^0)$, every state corresponding to a particle of given charge from the x^0-standpoint. From the $x^{0\prime}$-standpoint such a state is, however, a mixture of states of given charge represented by the functions $U_n(x^{0\prime})$, and we may easily find the expansion of $\overline{U}_n(x^{0\prime})$ in terms of the $U_n(x^{0\prime})$ set, the result being

$$\overline{U}_n(x^{0\prime}) = U_n(x^{0\prime}) - i\,\varepsilon \sum_{n'} \frac{n + n'}{2}\, \xi_{n'-n}\, U_{n'}(x^{0\prime})\,. \tag{30}$$

Now, the state defined by the wave function $\psi(x^0, x)$ of (24) may just as well be represented by a wave function $\overline{\psi}(x^{0\prime}, x)$ given by

$$\overline{\psi}(x^{0\prime}, x) = \sum_n \psi_n(x)\, \overline{U}_n(x^{0\prime})\,, \tag{24a}$$

where the coefficients are the same as in (24). On the other hand we may expand $\overline{\psi}$ in terms of the functions $U_n(x^{0\prime})$

$$\overline{\psi}(x^{0\prime}, x) = \sum_n \psi'_n\, U_n(x^{0\prime})\,. \tag{31}$$

Comparing (31) with (24a) and (30) we get

$$\psi'_n(x) = \sum_{n'} \left(\delta_{nn'} + \varepsilon \sum_{n'} (n\, Q\, n')\right) \psi_{n'}(x) \tag{32}$$

with

$$(n' \mid Q \mid n') = -i\, \frac{n' + n''}{2}\, \xi_{n'-n''}\,. \tag{33}$$

If, as we shall assume, the transformation (28) is real we have

$$\xi_s(x) = \xi^*_{-s}(x)\,, \tag{34}$$

from which follows that $1 + \varepsilon Q$ is a unitary matrix.

Remembering that what we need is a quantum theory comprising charged fields, in which the elementary quantum of electricity has found its adequate place the theory just outlined with its states of multiple charge looks too complicated. It is therefore a hopeful feature that it may be very much simplified without loosing its consistency and essential properties. Thus we can take any number N of consecutive integers to be the eigenvalues of p_0 and cut out the corresponding part of any matrix $(n' \,|\, F \,|\, n'')$ simply by putting all the ψ_n equal to zero, which do not belong to the eigenvalues of p_0. Thus already the case of two row matrices with

$$p_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{35}$$

will give a mathematically possible theory. This case will correspond to spinor particles of positive and negative, unit charge and of zero charge, the negative particles being antiparticles of the positive ones.

The obvious resemblance of this theory to the symmetric meson theory is strengthened when we regard the corresponding Q-matrix, which is seen to be

$$Q = -i \begin{pmatrix} \xi_0\,, & \dfrac{\xi_1}{2} \\ \dfrac{\xi_1^*}{2}\,, & 0 \end{pmatrix}. \tag{36}$$

If for a moment we disregard the dependence of the ξ's on the coordinates and their consequent lack of commutability with the momenta nothing is changed, when to Q in (36) we add the following multiple of the unit matrix

$$i\,\frac{\xi_0}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

In this way we get a new matrix $\overline{Q}$ given by

$$\overline{Q} = -\frac{i}{2} \begin{pmatrix} \xi_0, & \xi_1 \\ \xi_1^*, & -\xi_0 \end{pmatrix}, \tag{37}$$

or, if we introduce the isotopic spin matrices

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{38}$$

$$\overline{Q} = -\frac{i}{2} \left(\frac{\xi_1 + \xi_1^*}{2}\, \tau_1 + i\, \frac{\xi_1 - \xi_1^*}{2}\, \tau_2 + \xi_0\, \tau_3 \right). \tag{39}$$

But this matrix represents an arbitrary, infinitesimal, real rotation in isotopic spin space, defining just that transformation group, which is characteristic of the symmetric meson theory.

Now, the difference between Q and $\overline{Q}$ is probably what should be expected from the neglect of electromagnetic forces in the latter theory. Thus putting $\xi_1 = 0$ and taking the dependence of ξ_1 on x into account Q will just correspond to the gauge transformation of electromagnetism, while $\overline{Q}$ will also change the phases of the ψ-components belonging to neutral particles.

Let us for a moment return to the general case. Here the field is represented by the F-matrices. We may say that $(n' \mid F \mid n'')$ represents the field connected with a transition of the spinor particle from a n'-fold to a n''-fold unit charge corresponding to quanta of $(n' - n'')$-fold unit charge. Thus the diagonal represents neutral fields, while the lines parallel to the diagonal represent fields of charged quanta of higher and higher multiplicity the farther away from the diagonal they are situated. The interpretation just outlined is seen to correspond closely to the commutation relation

$$(n' \mid [p_0, F] \mid n'') = (n' - n'') (n' \mid F \mid n'') . \tag{40}$$

In the case of two row matrices the field is seen to correspond to neutral and to positive and negative quanta of unit charge.

As to the further development of the theory outlined it would probably need much work before any quantitative conclusions, comparable with nuclear and mesonic experiments, could be drawn from it, this being due to its pronounced non-linearity. On the other hand, the non-linearity would seem to justify the hope that the wellknown difficulty of five-dimensional relativity, the appearance of enormous particle mass terms, may be overcome in the way touched upon above, whereby the quantity corresponding to γ_{00} may perhaps be of importance. On the whole, the relation of the theory to the fivedimensional representation of gravitation and electromagnetism on the one hand and to symmetric meson theory on the other hand — through the appearance of the charge invariance group — may perhaps justify the confidence in its essential soundness.

Diskussion – Discussion

W. Pauli: The existence of a finite cut-off momentum in quantized field theories as a boundary of its mathematical consistency was proved by G. Källén and myself [2] only for a particular academic model. In analogy to this I formulated in private communications the conjecture of a finite energy range of consistency in quantum electrodynamics with a cut-off momentum P given by

$$\log \frac{P^2}{m^2 c^2} \sim \frac{1}{\alpha} \sim 137 \tag{1}$$

Here m is the restmass of the electron, c the velocity of light and $\alpha = e^2/\hbar c$ the fine-structure constant.

Independently LANDAU [3] and his collaborators obtained the same order of magnitude, as given by (1), for the maximum cut-off momentum P in quantum electrodynamics by a detailed mathematical analysis of the series which expresses the physical electric charge e in powers of the mathematical charge e_0. Unfortunately the passage from the asymptotic behaviour for large P of the single terms of this power series to the asymptotic behaviour of its sum needs additional mathematical assumptions of uniformity which have not yet been proved rigorously. Nevertheless the still hypothetical cut-off moment in quantum electrodynamics, given by (1), is rather suggestive. For us here it is important that LANDAU pointed to the fact that for a momentum P of this high order of magnitude the gravitational forces between two electrons are becoming of the same magnitude as the Coulomb forces. The relation $\varkappa P^2 \sim 1$ in units $\hbar = c = 1$, which LANDAU derives in this way[1]), gives in KLEIN's notation just the relation mentioned by him

$$P \sim \frac{\hbar}{l_0} \tag{2}$$

with $l_0 = \sqrt{\varkappa \hbar c}$, where $\varkappa$ is EINSTEIN's gravitational constant.

The question whether such a very high limit of mathematical consistency for quantum electrodynamics can have any direct physical meaning at all has been much disputed at the Physics Conference in Pisa in June. In view of the possibility of the occurence of mesons or nucleons in intermediate states the view has been stated, that the limit of the physical validity of quantum electrodynamics will be reached already at energies about corresponding to the mass of the nucleons.

On the other hand, the connection (2) of the mathematical limitation of quantum electrodynamics with gravitation, pointed out by LANDAU and KLEIN, seems to me to hint at the indeterminacy in space-time of the light-cone, which is governed by probability-laws in a quantized field theory, invariant with respect to the wider group of general relativity. It is possible that this new situation so different from quantized theories, invariant with respect to the LORENTZ group only, may help to overcome the divergence difficulties which are so intimately connected with a c-number equation for the light-cone in the latter theories.

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[1]) The argument is not accurate enough to distinguish between 1 and α on the right side of (2).

W. HEITLER: LANDAU's (very high) cut-off represents an upper limit imposed such that quantum electrodynamics should be selfconsistent and not lead to the catastrophes (negative probabilities, etc.) otherwise occuring as a result of charge renormalization. But it may be that the true cut-off lies considerably lower. There are strong arguments for the assumption that the cut-off momentum should lie at the order of magnitude of the proton mass. It is very probable that our present meson theory requires some fundamental physical changes and that not even the theory of the nucleon is in order. Quantum electrodynamics is not independent of all the other particles (mesons and nucleons, etc.). Not even the electrodynamics of π-mesons is free of fundamental difficulties (meson-meson scattering) and there can be little doubt that quantum electrodynamics can only be regarded as correct so long as these particles do not enter in virtual processes. It seems therefore plausible to assume that something goes wrong for virtual momenta not higher than the order of the nucleon mass. On the other hand one can verify that, by introducing such a cut-off, none of the established results of quantum electrodynamics (line shift, magnetic moment, collision cross sections) are changed appreciably, i.e. the changes are beyond the accuracy with which these effects are established.

A. LICHNEROWICZ: Si j'ai bien compris, la signature de la métrique pentadimensionelle introduite est $+----$. J'en suis fort heureux, car l'autre signature parfois introduite: $++---$ conduit, en ce qui concerne les équations du champ, à des problèmes un peu tératologiques.

B. JOUVET: Au sujet de la relation entre la constante de structure fine $e^2/\hbar c$ et la constante de coupure, je voudrais faire la remarque suivante: La construction des particules élémentaires à partir de Fermions plus élémentaires couplés par des couplages de FERMI conduit à exprimer les constantes de couplages des Bosons avec les paires de Fermions en fonction des constantes de coupure des impulsions des Fermions élémentaires. Dans le cas du photon, on obtient le résultat indiqué par le Prof. PAULI, équation (1). De plus cette théorie prévoit l'existence d'une particule de spin 2, qu'on peut interpréter comme étant le graviton. La constante de gravitation que l'on peut alors calculer est une fonction de la constante de coupure et de la constante de FERMI. Inversement, on peut espérer exprimer les constantes de coupure, en fonction de la constante de gravitation.

O. COSTA DE BEAUREGARD: La nécessité logique d'une synthèse entre la théorie des Quanta et la Relativité générale ressort d'un très bel argument relatif à la $4^{\text{ème}}$ relation d'incertitude, élucidé par BOHR et par EINSTEIN au cours de leurs âpres discussions. La loi d'équivalence entre énergie et masse inerte de la Relativité restreinte semble d'abord mettre en défaut la $4^{\text{ème}}$ relation d'incertitude: l'on peut peser la boite munie d'un volet mobile d'où s'échappe une particule quantique avant l'ouverture et après la fermeture du volet. Mais il faut examiner *comment* seront faites les pesées au moyen d'une balance. Il apparait alors que la 4$^{\text{ème}}$ relation d'incertitude est rétablie *exactement*, dès qu'on évoque la loi einsteinienne de variation de l'étalon du temps dans la direction de l'accélération de la pesanteur. Tout l'argument est très proche parent de l'argument d'équivalence entre inertie et gravitation par lequel EINSTEIN établit initialement l'effet DOPPLER de gravitation; il se situe dans le même cadre pré-riemannien que lui. Par là se manifeste l'intimité profonde de la mécanique ondulatoire et de l'optique.

H. BONDI: There is a connection between gravitation and electromagnetism additional to those discussed already.

NEWTON'S achievement can be described as establishing the sun, a *visible* body as causing the planetary motions. His theory therefore links two observations, one dynamical and one electromagnetic. Special relativity preserves this link under transformations.

In general relativity the SCHWARZSCHILD singularity raises a difficulty for if a body of mass m were to have a radius less than $2m$ then such a body would be invisible but would still be observable through its gravitational field. This intolerable possibility has been ruled out on the basis of the properties of materials by considerations due to EDDINGTON and to CURTIS. Would not a more fundamental denial of this possibility be a result of any satisfactory unitary theory?

O. KLEIN: I fear that I have missed Professor BONDI's point. Thus the observability of a given star by a given observer by means of light rays is no invariant and may be arbitrarily poor, e.g. if the observer moves away from the star with sufficient velocity. Further the difficulty of the singularity of the SCHWARZSCHILD solution has, as far as I can see, no more to do with electromagnetism than with particle dynamics, any kind of particle requiring an infinite time to come out from the interior of the star as judged by the outside observer.

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