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On the Two-Body Problem of General Relativity

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Recently GUPTA [1] put forward the view that EINSTEIN's theory of gravitation can be reinterpreted as a theory in flat space with a Lagrangian density consisting of an infinite number of terms. On the basis of this idea, he has evolved a practicable method of quantization of the gravitational field in interaction with the electromagnetic field, which is an extension of ROSENFELD's early work on the quantization of the linearized EINSTEIN equations.

In this note we shall endeavour to present, as an argument for the soundness of GUPTA's approach, a field theoretical derivation of the equations of the two-body problem of general relativity, which were found by EINSTEIN, HOFFMANN and INFELD [2] (the HEI equations).

Consider the Lagrangian

$$L = -\frac{1}{4}\left(\gamma_{\mu\nu,\lambda}\,\gamma_{\mu\nu,\lambda} - \frac{1}{2}\gamma_{,\lambda}\gamma_{,\lambda}\right) + \sum_{i=1}^{2}\left\{L^{(i)} - \frac{1}{2}\varkappa\, h_{\mu\nu}\, T^{(i)}_{\nu\mu}\right\}$$

with $h_{\mu\nu} = \gamma_{\mu\nu} - \gamma/2\;\delta_{\mu\nu}$. Here $\gamma_{\mu\nu}$ and $\gamma = \gamma_{\mu\mu}$ represent the gravitational field, and $L^{(i)}$, $T^{(i)}_{\mu\nu}$ are the Lagrangians and the energy momentum tensors of two fields of scalar particles of mass m_1 and m_2. Terms of higher order in $\varkappa$ have been neglected as they are not necessary for our purpose. The Lagrangian in its entirety would be equivalent to the general theory of the gravitational field co-existing with the two scalar fields.

From the Lagrangian it is easy to deduce the interaction Hamiltonian in the interaction representation, and to calculate, by the FEYNMAN-DYSON method, the matrix element for scattering of a particle m_1 by a particle m_2 due to the exchange of one graviton. The matrix element thus obtained $(\vec{p}_1\,\vec{p}_2\,|\,V\,|\,\vec{p}_1{}'\,\vec{p}_2{}')$, is translated into configuration space by a FOURIER transformation, and defines a velocity dependent potential $(\vec{r}_1\,\vec{r}_2\,|\,V\,|\,\vec{r}_1{}'\,\vec{r}_2{}')$ of the order of $\varkappa^2$, which reduces to the NEWTON potential if the velocity dependent terms are neglected. This potential is then used in the two particle Hamiltonian

$$H = \frac{1}{2\,m_1}\,\vec{p}_1^{\,2} + \frac{1}{2\,m_2}\,\vec{p}_2^{\,2} - \frac{1}{8m_1^3}\,(\vec{p}_1^{\,2})^2 - \frac{1}{8m_2^3}\,(\vec{p}_2^{\,2})^2 + \ldots + V$$

by the help of which the second time derivative of the coordinate r_1^i is evaluated by the formula

$$\ddot{r}_1^i = -[H,\,[H,\,r_1^i]]\,.$$

The calculation has actually been performed in the coordinate representation, and the matrix element $(\vec{r}_1\,\vec{r}_2\,|\,\ddot{r}_1^i\,|\,\vec{r}_1'\,\vec{r}_2')$ has been found.

Take now a wave function $\Psi\,(\vec{r}_1\,\vec{r}_2)$ representing two spatially separated wave packets for the two particles m_1 and m_2. The expectation value

$$\langle\ddot{r}_1^i\rangle = \int \Psi^*\,(\vec{r}_1\,\vec{r}_2)\,(\vec{r}_1\,\vec{r}_2\,|\,\ddot{r}_1^i\,|\,\vec{r}_1'\,\vec{r}_2')\,\Psi\,(\vec{r}_1'\,\vec{r}_2')\,d\vec{r}_1\ldots d\vec{r}_2'$$

is, by a series of partial integrations and by neglecting terms more than quadratic in the velocities, found to reduce to an expression which agrees with the HEI equations.

It is to be remarked that this derivation of the equations of the two-body problem is not necessarily quantum-mechanical. The procedure is of a Hamiltonian type and admits of a classical version.

Diskussion – Discussion

B. Jouvet: Je voudrais faire une remarque sur le sens possible des termes non-linéaires contenus dans le Lagrangian du champ de gravitation d'Einstein, à la lumière des nouvelles théories des particules élémentaires, basées sur la donnée de couplages de Fermi entre paires de Fermions [3] et [4]. Il ressort de ces théories que les particules de spin plus grand que 1, et en particulier les particules de spin 2 (gravitons), n'ont généralement pas d'interaction 'locales' avec les particules qui les émettent (absorbent). Cela provient du fait que le graviton, formé de 4 Fermions, doit pour être émis, faire intervenir nécessairement deux fois le couplage de Fermi qui crée chaque fois une paire de Fermions, en deux instants-points qui n'ont pas de raison d'être confondus. Le champ du graviton est alors un champ non local $\varphi\,(x_1,\,x_2)$. On est conduit à rapprocher cet aspect non local de l'interaction, de la forme non linéaire des équations d'Einstein. Il serait utile de savoir si la solution générale du problème des deux corps en relativité générale, mise sous la forme d'une matrice de diffusion comme l'a fait le DR. CORINALDESI, conduit aussi à une interaction non locale de la forme $\int d_1 d_{1'} \, d_2 d_{2'} \, T_{\mu\nu}(x_1, x_1') \, K_{\mu\nu\sigma\tau}(x_1, x_1', x_2, x_2') \, T_{\sigma\tau}(x_2, x_2')$, les fonctions $T_{\mu\nu}(x, x')$ tendant vers le tenseur d'énergie $T_{\mu\nu}(x_1)$ dans la limite où $x - x' \to 0$.

L. INFELD: Is the Lagrangian arranged this way so as to give the right equations of motion?

E. CORINALDESI: It is obtained by expanding in powers of $\varkappa$ the general relativistic Lagrangian including the scalar fields of mass m_1 and m_2. The present work therefore, claims to be a consistent derivation of the HEI equations.

O. COSTA DE BEAUREGARD: Je voudrais faire une remarque sur la théorie du graviton, particule de spin 2: La déduction de l'approximation quasi-minkowskienne de la théorie de la gravitation à partir de la théorie de la particule de spin 2, et de l'hamiltonien d'interaction indiqué par M. CORINALDESI a été méthodiquement démontrée par Mme TONNELAT [5]. Récemment, nous sommes revenus nous-mêmes sur le sujet [6]. Il est permis de penser que l'actuelle incapacité de la mécanique ondulatoire à dépasser l'approximation quasi-minkowskienne ne prouve pas l'inexistence des gravitons, mais

1° l'inadéquation radicale de la «représentation d'interaction» (méthode de pertubations) en ce domaine,

2° la probabilité nulle ou évanescente d'émission-absorption de gravitons libres, c'est à dire en particulier l'absence d'écrans à la gravitation.

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