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The Production and Absorption of Neutrinos in Beta-Decay Theory

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Summary. The implications of non-conservation of parity and of lepton charge for the processes (1) $(A, Z-1) \rightarrow (A, Z) + e^- + \nu$ and (2) $p+\nu \rightarrow n+e^+$ are studied. The neutrinos from (1) are described by a density matrix which, in a form applicable to pile neutrinos, is used, in particular, to calculate the cross section $\bar{\sigma}$ for reaction (2). From the general dependence of $\bar{\sigma}$ on the coupling constants the result of LEE and YANG, that $\bar{\sigma}$ is twice as big in the two-component theory as in the conventional theory, is recognized as a special case. In addition experiments are discussed to determine the spectrum and polarization of pile neutrinos. Attention is also given to Pauli's invariance principle.

1. Introduction

Now that enormous neutrino fluxes are available from uranium piles, the neutrino has become a particle of the experimentalists as is seen from the work of COWAN and REINES[1]) and of DAVIS[2]). On the other hand, since the breakdown of parity was discovered, the neutrino has also become a particle of great interest to the theoreticians as is examplified by the proposals of a two-component theory[3]). We believe that in the present situation a formal description of neutrinos emerging from a pile is neither entirely trivial nor is it without interest. In fact, if one adopts the point of view of a phenomenological beta-decay interaction with non-conservation of parity and of lepton charge as proposed by PAULI[4]), one finds that free neutrinos have to be described by a *density matrix*. The reason for this is, that unless lepton charge is conserved, there will be interference between the two states of different neutrino "charge" $l = 1, 2$ and given longitudinal spin $s = +, -$. This is a consequence of Pauli's canonical transformations of the neutrino field[4]) which mix "particle" and "antiparticle" states, but do not mix states of different longitudinal spin for a given neutrino momentum. Thus, the density matrix is diagonal in s as is shown explicitely in section 4.

On the other hand, the interest of such a description of free neutrinos lies in the fact that the discussion in terms of the coupling constants of the cross section for any neutrino experiment is only possible if due account is taken of the production process. The example which has been most discussed in the last months is the cross section $\bar{\sigma}$ for the Cowan and Reines experiment. The reason is that from this experiment, if accurate enough, conclusions for or against the two-component theory can be drawn. Indeed, in their paper quoted in ref. 3 LEE and YANG indicated that in the two-component theory $\bar{\sigma}$ is twice as big as $\bar{\sigma}^c$, the value of the conventional theory. This statement was based on a purely qualitative detailed balance argument. In section 2 we shall reproduce this argument (as we understand it) in a more quantitative form. The statement will reappear in section 5 as particular value of the expression $\bar{\sigma}/\bar{\sigma}^c$ which exhibits a continuous variation with the coupling constants. It will be seen from the results of section 5 that for the calculation of $\bar{\sigma}$ the neutrino spectrum and its polarization in function of energy is needed (and still a third function if lepton charge is not conserved). The possibility of a measurement of these functions is discussed in section 6.

It should be mentioned also that the density matrix concept is not the only means of arriving at $\bar{\sigma}$. Indeed, production and re-absorption could also be considered as a double process analogous to those studied in an earlier note[5]). The connection between the two methods, which are entirely equivalent, will be briefly discussed in section 3. Of course the density matrix is the more physical concept and has the further advantage that it can be used to describe any experiment with free neutrinos.

We should like to add a remark about Pauli's *invariance principle* since the formulae of the present paper are typical examples of its applicability. A general formulation of this principle was given in ref. 4. In a special case it was also applied by PURSEY[6]) and reformulated by KAHANA and PURSEY[7]). An important contribution is due to LÜDERS[8]). It is contained in the following statement: If O is any invariant operator and c_{ls}^*, c_{ls} are the creation and destruction operators for a neutrino in a state of "charge" $l = 1,2$ and longitudinal spin $s = +, -$, then the projection operator $\sum_l c_{ls}^* O c_{ls}$ into the (two dimensional) subspace of given spin component s is invariant[9]).

Based on Lüders' result we propose the following formulation of the invariance principle, which is well adapted to the examples occurring in the following sections:

A sum of products of S-matrix elements is invariant with respect to the Pauli group if the state vectors with 0, 1, 2, ... *neutrinos,*

$$| U \rangle, c_{ls}^* | U \rangle, c_{ls}^* c_{l's'}^* | U \rangle, \ldots,$$

form projection operators into the subspaces of given neutrino spin component

$$| U\rangle \langle V |, \sum_l c_{ls}^* | U\rangle \langle V | c_{ls}, \sum_{l,l'} c_{ls}^* c_{l's'}^* | U\rangle \langle V | c_{l's'} c_{ls}, \ldots$$

This statement becomes evident if one considers as elements of the Pauli group the *combined* transformation of the neutrino field and of the coupling constants, such that the interaction Hamiltonian stays invariant. Since, for zero neutrino mass, this group also leaves the free neutrino Hamiltonian and the commutation relations invariant, the invariance of state vectors is guaranteed, except for transformations within the two dimensional subspace of given neutrino spin component.

2. Preliminary Discussion

A "theory" shall be defined by the coupling parameters in Pauli's interaction Hamiltonian[4])

$$H_{\text{int}}(x) = \sum_i (\bar{\psi}_n O_i \psi_p)\{(\bar{\psi} Q_{1i} \psi_e) + (\bar{\psi}_\nu^c Q_{2i} \psi_e)\} + \text{herm. conj.} \quad (2.1)$$

Here we have introduced the abbreviations

$$\left.\begin{aligned} Q_{1i} &= (F_{1i} \tfrac{1}{2}(1+\gamma_5) + G_{1i} \tfrac{1}{2}(1-\gamma_5))\, O_i \\ Q_{2i} &= (-F_{2i} \tfrac{1}{2}(1+\gamma_5) + G_{2i} \tfrac{1}{2}(1-\gamma_5))\, O_i \end{aligned}\right\} \quad (2.2)$$

and

$$O_i = 1, \gamma_\mu, i\gamma_\mu\gamma_\nu\ (\mu < \nu), i\gamma_5\gamma_\mu, \gamma_5 \quad (2.3)$$

so that $O_i^* = O_i$, $O_i^2 = 1$. As in ref. 4 and 5, ψ contains the emission of particles and the absorption of antiparticles (see footnote 12). The charge conjugate ψ^c is defined as in ref. 4. We also use the assumption $m_\nu = 0$.

The conventional (parity and lepton charge conserving) theory is characterized by $F_{1i} = G_{1i}$, $F_{2i} = G_{2i} = 0$, the (lepton charge conserving) two-component theory by $G_{1i} = F_{2i} = G_{2i} = 0$.

The S-matrix element for neutrino absorption from a charge and spin state (l, s) in the reaction of Cowan and Reines, $p + \nu \rightarrow n + e^+$, is of the form

$$\langle n e^+ | S | p \nu \rangle = -i(2\pi)^{-2} \delta^4 (p_n + p_e - p_p - p_\nu) \cdot T_{ls}^{\text{abs}}. \quad (2.4)$$

For the absorption cross section $\bar{\sigma}$ an average over the initial states (l, s) has to be formed which in the case of lepton conservation can be written ($l = 1$ for antineutrino)

$$\sum_{ls} a_{ls} |T_{ls}^{\text{abs}}|^2; \ \sum_{ls} a_{ls} = 1,\ a_{2s} = 0$$

(A general expression for $\bar{\sigma}$ will be derived in section 3.) In the conventional theory the antineutrinos are not polarized,

$$a_{1+} = a_{1-} = \tfrac{1}{2} \quad (2.5)$$

whereas in the 2-component theory they are completely polarized backward (or forward)

$$a_{1+} = 0,\ a_{1-} = 1 \quad (\text{or } a_{1+} = 1,\ a_{1-} = 0). \tag{2.6}$$

Now for all states (l, s) for which a_{ls} vanishes T_{ls}^{abs} vanishes too. This is a sort of detailed balance argument since the a_{ls} are determined by matrix elements of the type describing neutrino emission in the decay of the neutron

$$\langle p e^- \nu \mid S \mid n \rangle = -i\,(2\pi)^{-2}\,\delta^4\,(p_p + p_e + p_\nu - p_n) \cdot T_{ls}^{\text{em}}.$$

But if

$$T_{ls}^{\text{abs}} = 0, \text{ all } (l, s) \text{ such that } a_{ls} = 0 \tag{2.7}$$

we can write, using (2.5) and (2.6)

$$\sum_{ls} a_{ls} \mid T_{ls}^{\text{abs}} \mid^2 = \frac{1}{z} \sum_{ls} \mid T_{ls}^{\text{abs}} \mid^2 \tag{2.8}$$

with $z = 2$ for the conventional and $z = 1$ for the 2-component theory. The expression $\sum_{ls} \mid T_{ls}^{\text{abs}} \mid^2$ (summed over spin and angle variables of n, p, e^+) is an example of Pauli's invariance principle and, as we shall see, it depends on the same invariant expressions of the coupling constants as the lifetimes of allowed transitions do. Now since lifetimes are empirical quantities these invariants have to be constants with respect to a change of the theory (coupling parameters). Likewise the neutrino spectrum is an empirical quantity[11]) and thus is a constant in this sense. Then the only quantity in $\bar{\sigma}$ which is different for the two theories in consideration is the number z in (2.8) and it follows

$$\bar{\sigma}^{\text{“2”}} / \bar{\sigma}^{c} = 2 \tag{2.9}$$

("2" for 2-component, c for conventional) which is the result of LEE and YANG.

To be more specific we shall write down explicit expressions for $\langle \mid T_{ls}^{\text{abs}} \mid^2 \rangle$ and $\langle \mid T_{ls}^{\text{em}} \mid^2 \rangle$, where $\langle\rangle$ means an average over the spins of n, p, $e^{\pm}$ and over the direction of $e^{\pm}$. In first order of perturbation theory it follows[12])

$$T_{l\pm}^{\text{abs}} = \sum_i (\bar{u}_n O_i u_p)^* (\bar{v}_{\pm} Q_{li}\, v_e)^* \tag{2.10}$$

$$T_{l\pm}^{\text{em}} = \sum_i (\bar{u}_n O_i u_p)(\bar{v}_{\pm} Q_{li}\, u_e) \tag{2.11}$$

As a consequence of $m_\nu = 0$ the neutrino spinors have the property

$$\left.\begin{aligned} \bar{v}_+ Q_{li} &= G_{li}\, \bar{v}_+ O_i \\ \bar{v}_- Q_{li} &= \pm F_{li}\, \bar{v}_- O_i\,; \quad l = \left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right. \end{aligned}\right\} \tag{2.12}$$

For non-relativistic nucleons it then follows

$$\left.\begin{aligned} 8\langle|T^{\text{em}}_{\substack{\text{abs}\\ l+}}|^2\rangle &= (|G_{ls}|^2+|G_{lv}|^2)+3\,(|G_{lT}|^2+|G_{lA}|^2)\pm\\ &\pm\frac{m}{W}\,[(G^*_{lS}G_{lV}+G^*_{lV}G_{lS})+3\,(G^*_{lT}G_{lA}+G^*_{lA}G_{lT})] \end{aligned}\right\} \qquad (2.13)$$

$$\left.\begin{aligned} 8\langle|T^{\text{em}}_{\substack{\text{abs}\\ l-}}|^2\rangle &= (|F_{lS}|^2+|F_{lV}|^2)+3\,(|F_{lT}|^2+|F_{lA}|^2)\pm\\ &\pm\frac{m}{W}\,[(F^*_{lS}F_{lV}+F^*_{lV}F_{lS})+3\,(F^*_{lT}F_{lA}+F^*_{lA}F_{lT})] \end{aligned}\right\} \qquad (2.13')$$

where m and W are the mass and energy of the $e^{\pm}$. It can immediately be seen that a detailed balancing between the reactions $n\to p+e^-+\nu$ and $p+\nu\to n+e^+$ (which are not inverse) of the form

$$\langle|T^{\text{em}}_{ls}|^2\rangle=\langle|T|^{\text{abs}}_{ls}|^2\rangle \qquad (l,s)\ \text{fixed}$$

is only true for vanishing Fierz-terms but that (2.7) is a true special case of such a relation.

After summation over the states (l, s) one gets

$$\left.\begin{aligned} 4\sum_{ls}\langle|T^{\text{em}}_{\substack{\text{abs}\\ ls}}|^2\rangle &= (K_{SS}+K_{VV})+3\,(K_{TT}+K_{AA}\pm\\ &\pm\frac{m}{W}\,[(K_{SV}+K_{VS})+3\,(K_{TA}+K_{AT})] \end{aligned}\right\} \qquad (2.14)$$

where K_{ij} is a Pursey-Pauli invariant, defined by

$$K_{ij}=\frac{1}{2}\,(A^+_{ij}+A^-_{ij})=K^*_{ij} \qquad (2.15)$$

with

$$\left.\begin{aligned} A^-_{ij}&=F^*_{1i}F_{1j}+F^*_{2i}F_{2j}=A^{-*}_{ji}\\ A^+_{ij}&=G^*_{1i}G_{1j}+G^*_{2i}G_{2j}=A^{+*}_{ji}. \end{aligned}\right\} \qquad (2.16)$$

The only other type of invariants occurring in this paper is

$$L_{ij}=\frac{1}{2}\,(A^+_{ij}-A^-_{ij})=L^*_{ii}. \qquad (2.17)$$

These invariants L_{ij} always occur in connection with pseudoscalar quantities (see for instance the table in ref. 8). If for instance the difference between $s=+$ and $s=-$ in the expressions (2.13, 13′) is taken one arrives at the right side of (2.14) with the K_{ij} replaced by the L_{ij}. This is a measure for the neutrino polarization (see (4.18)).

3. General Form of Density Matrix and Cross Section

Let us consider the nuclear beta-decay

$$N(A, Z-1) \rightarrow N(A, Z) + e^- + \nu. \tag{3.1}$$

The S-matrix element for this reaction is

$$\langle F e^- \nu | S | I \rangle = -i\,(2\pi)^{-1/2}\,\delta(W_N - W - \omega) \cdot T^N_{ls,\vec{q}} \tag{3.2}$$

where

$$T^N_{l\pm,\vec{q}} = \sum_i \int d^3x\, \Phi_i(\vec{x})\, e^{-i\vec{q}\vec{x}} \left(\bar{v}_\pm(\vec{q})\, Q_{li}\, \chi(\vec{x})\right) \tag{3.3}$$

and

$$\Phi_i(\vec{x}) = \int d\tau'\, (U_F^*\, O_i\, \varepsilon U_I). \tag{3.4}$$

Here U_I and U_F are the properly anti-symmetrized wave functions of the initial and final state nucleus, $\vec{x}$ being a neutron and a proton coordinate respectively. $\int d\tau'$ means summation over all nucleon coordinates except $\vec{x}$. The operator ε exchanges the neutron with coordinate $\vec{x}$ in U_I against a proton. $\chi_{p,k,m}(Z, \vec{x})$ is the Coulomb wave function of the electron in a state of positive energy $W = \sqrt{m^2 + p^2}$ with radial functions f_k, g_k $(k = \pm 1, \pm 2, \ldots)$. W_N is the spectrum limit, $\vec{q}$ and $\omega = |\vec{q}|$ the neutrino momentum and energy respectively.

To facilitate trace formation in the density matrix we temporarily introduce discrete energy-momentum variables by a finite normalization volume V for the neutrino field and a finite transition time T. The prescription is

$$\left.\begin{aligned} \delta^3(\vec{q} - \vec{q}\,') &\rightarrow V(2\pi)^{-3}\,\delta^3_{\vec{q},\vec{q}\,'} \\ \delta\,(\omega - \omega') &\rightarrow T(2\pi)^{-1}\,\delta_{\omega,\omega'} \end{aligned}\right\} \tag{3.5}$$

with Kronecker instead of Dirac δ's. Then we can write instead of (3.2)

$$S^N_{ls,\vec{q}} = -iTV^{-1/2}\,\delta_{W_N - W,\,\omega} \cdot T^N_{ls,\vec{q}} \tag{3.2'}$$

For the density matrix of a convergent neutrino beam in the direction $\vec{e} = \vec{q}/\omega = \vec{q}\,'/\omega'$ we need the expression

$$\sum_{\text{nucl}} \sum_{\text{km}} \int dp\, S^N_{ls,\vec{q}}\, S^{N*}_{l's',\vec{q}\,'} = \frac{T^2}{V}\,\delta_{\omega,\omega'} \int dp\, \frac{2\pi}{T}\,\delta(W_N - W - \omega)$$

$$\cdot \sum_{\text{nucl}} \sum_{\text{km}} T^N_{ls,\vec{q}}\, T^{N*}_{l's',\vec{q}} = \frac{T}{V}\, 2\pi\,\delta_{\omega,\omega'}\, R^N_{ls,l's'}(\omega)$$

where

$$R^N_{ls,l's'}(\omega) = \left[\frac{W}{p} \sum_{\text{nucl}} \sum_{\text{km}} T^N_{ls\vec{q}}\;\; T^{N*}_{l's'\vec{q}}\right]_{W = W_N - \omega} \tag{3.6}$$

and $\sum_{\text{nucl}}$ means an averaging over initial and summation over final nuclear states. Then the density matrix is

$$\mathrm{P}^N_{ls\vec{q},\, l's'\vec{q}'} = \delta_{\omega,\omega'} \cdot R^N_{ls,\, l's'} / \sum_{ls} \sum_{\vec{q}}{}' R^N_{ls,\, ls} \,. \tag{3.7}$$

This is normalized to give

$$Tr\ \mathbf{P}^N = \sum_{ls} \sum_{\vec{q}}{}' \mathrm{P}^N_{ls\vec{q},\, ls\vec{q}} = 1 \,. \tag{3.8}$$

Here $\sum_{\vec{q}}{}'$ means summation within a small solid angle in the direction $\vec{e}$[13]).

To give a description of a neutrino beam from a pile we have to take into account all possible processes (3.1)[14]). Let us call λ_N the relative rate of formation in the pile of the initial state nucleus $N(A, Z-1)$. Then the correct averaging procedure is

$$\mathrm{P}_{ls\vec{q},\, l's'\vec{q}'} = \delta_{\omega,\,\omega'} \cdot \langle R_{ls,\, l's'} \rangle / \sum_{ls} \sum_{\vec{q}}{}' \langle R_{ls,\, ls} \rangle \tag{3.7'}$$

where

$$\langle R_{ls,\, l's'}(\omega) \rangle = \sum_N \lambda_N \cdot \Theta(W_N - m - \omega) \cdot R^N_{ls,\, l's'}(\omega)\,; \ \sum_N \lambda_N = 1 \tag{3.9}$$

The step function Θ ensures that the condition $W = W_N - \omega \geq m$ in (3.6) is fulfilled.

This density matrix has to be used to get the cross section for the Cowan Reines experiment. First we write the matrix element (2.4) in the V, T-normalization (discrete energy-momentum)

$$S^{\text{abs}}_{ls,\,\vec{q}} = -i\,\frac{T}{V}\,\delta^4_{p_n+p_e+p_p,\,p_\nu} \cdot T^{\text{abs}}_{ls,\,\vec{q}}$$

where the energy-momentum vectors are (target proton at rest)

$$p_n = (\vec{p}_n,\, E_n = \sqrt{m_n^2 + \vec{p}_n^{\,2}},\ p_p = (0, m_p)$$

$$p_e = (\vec{p}\,',\, W' = \sqrt{m^2 + \vec{p}\,'^2},\ p_\nu = (\vec{q},\, \omega = |\vec{q}|)$$

then

$$\bar{\sigma} = \frac{V}{T} \sum_{\vec{p}_n} \sum_{\vec{p}'} \frac{1}{2} \sum_{pne} \sum_{ls} \sum_{\vec{q}}{}' \sum_{l's'} \sum_{\vec{q}'}{}' S^{\text{abs}}_{ls\vec{q}}\, \mathrm{P}_{ls\vec{q},\, l's'\vec{q}'}\, S^{\text{abs}*}_{l's'\vec{q}'}$$

where $\sum_{pne}$ means the spin sums of p, n and e.

Going back to continuous variables ($V \to \infty$, $T \to \infty$) replacing $\sum_{\vec{q}}{}'$ by $\int d\omega \cdot \omega^2$ and Kronecker by Dirac δ's, one easily finds

$$\bar{\sigma} = \int d\omega \cdot \omega^2 \sum_{ls} \sum_{l's'} \left\{ \omega^2 \langle R_{ls,\, l's'} \rangle / \sum_{ls} \int d\omega \cdot \omega^2 \langle R_{ls,\, ls} \rangle \right\} \cdot$$

$$\cdot (2\pi)^{-2} \int d^3 p'\, \delta(E_n - m_p + W' - \omega)\, \frac{1}{2} \sum_{pne} T^{\text{abs}*}_{l's'\vec{q}}\, T^{\text{abs}}_{ls\vec{q}} \,.$$

This formula takes a simple form if one introduces a reduced density matrix

$$\varrho_{ls,\,l's'}(\omega) = \omega^2 \langle R_{ls,\,l's'}(\omega)\rangle \,/\, \sum_{ls} \int_0^\infty d\omega \cdot \omega^2 \langle R_{ls,\,ls}\rangle \tag{3.10}$$

and a cross section matrix

$$\left.\begin{aligned} \sigma_{ls,\,l's'}(\omega) &= (2\pi)^{-2} \int_{\omega_0}^{\infty} d^3 p'\,\delta\,(E_n - m_p + W' - \omega)\cdot \\ &\cdot \frac{1}{2} \sum_{pns} T^{\mathrm{abs}*}_{ls\vec{q}}\, T^{\mathrm{abs}}_{l's'\vec{q}}\,. \end{aligned}\right\} \tag{3.11}$$

It follows that

$$\overline{\sigma} = \sum_{ls} \sum_{l's'} \int_{\omega_0}^{\infty} d\omega \cdot \omega^2\, \varrho_{ls,\,l's'}(\omega)\, \sigma_{l's',\,ls}(\omega) = \int_{\omega_0}^{\infty} d\omega \cdot \omega^2 \cdot Tr\,(\boldsymbol{\varrho}\cdot\boldsymbol{\sigma})\,. \tag{3.12}$$

Here

$$\omega_0 = m_n - m_p + m \cong 3{,}53 \cdot m \tag{3.13}$$

is the threshold energy for neutrino absorption.

It is clear that

$$\varrho(\omega) = Tr\,\boldsymbol{\varrho} = \omega^2\, Tr\,\langle \boldsymbol{R}\rangle \,/ \int_0^\infty d\omega\cdot\omega^2\, Tr\,\langle \boldsymbol{R}\rangle \tag{3.14}$$

is the neutrino spectrum which as a consequence of (3.10) has the property

$$\int_0^\infty d\omega\cdot\varrho(\omega) = 1\,. \tag{3.15}$$

It is also useful to introduce the matrix

$$\boldsymbol{a}(\omega) = \boldsymbol{\varrho}/\varrho = \langle \boldsymbol{R}\rangle \,/\, Tr\,\langle \boldsymbol{R}\rangle \tag{3.16}$$

for which

$$Tr\,\boldsymbol{a} = 1\,. \tag{3.17}$$

With this matrix the cross section for a given neutrino energy ω is

$$\sigma(\omega) = Tr(\boldsymbol{a}\cdot\boldsymbol{\sigma}) \tag{3.18}$$

so that according to (3.12)

$$\overline{\sigma} = \int_{\omega_0}^{\infty} d\omega\cdot\omega^2\cdot\varrho(\omega)\cdot\sigma(\omega)\,. \tag{3.12'}$$

The important quantity in this formulation obviously is $\boldsymbol{R}^N$.

It is also possible to express lifetime τ^N and β^--spectrum $N^N(W)$ in terms of $\boldsymbol{R}^N$,

$$\left.\begin{aligned} 1/\tau^N &= \frac{1}{T} \sum_{ls} \sum_{\vec{q}} \sum_{km} \int dp\, |S^N_{ls\vec{q}}|^2 = \frac{1}{\pi} \int_0^\infty d\,\omega \cdot \omega^2\, Tr\, \boldsymbol{R}^N \\ &= \int_0^\infty d\omega\, N^N (W_N - \omega)\,. \end{aligned}\right\} \tag{3.19}$$

$$N^N(W) = \frac{1}{\pi} (W_N - W)^2\, Tr\, \boldsymbol{R}\, (W_N - W)\,. \tag{3.20}$$

An other quantity of interest is the polarization vector for the pile neutrinos.

We can write

$$\sum_l a_{ls,\, ls'} = \frac{1}{2} (1 + \vec{P}\vec{\sigma})_{ss'} \tag{3.21}$$

where σ_1, σ_2, σ_3 are the Pauli spin matrices and the coordinate system is such that $\vec{q} = (0, 0, \omega)$. Then from (3.21)

$$\frac{1}{2} (P_1 \pm i\, P_2) = \sum_l a_{l\pm,\, l\mp}\,; \quad P_3 = \sum_l (a_{l+,\, l+} - a_{l-,\, l-})\,. \tag{3.22}$$

Similarly a polarization $\vec{P}^N$ for neutrinos from the decay (3.1) can be defined with the matrix

$$\boldsymbol{a}^N = \boldsymbol{R}^N / Tr \cdot \boldsymbol{R}^N \tag{3.16'}$$

instead of (3.16).

It should be mentioned that there is complete equivalence between this formulation and the method of double-processes described in ref. 5. Indeed the matrix element for the double-process is (in VT-normalization)

$$S_2 = \sum_{ls} \sum_{\vec{q}} S^N_{ls\vec{q}}\, S^{\mathrm{abs}}_{ls\vec{q}}$$

and the cross section is given by the probability for the double-process divided by the probability for neutrino emission:

$$\bar{\sigma} = \frac{V}{T} \Sigma\, |S_2|^2 / \Sigma\, |S^N|^2$$

(Σ means summation over all final state variable and averaging over all initial state variables). Since the summation over the intermediate state variables $ls\vec{q}$ is carried out with the amplitudes S and not with the probabilities $|S|^2$ (coherent summation), interference phenomena are possible and are exhibited in non-diagonal terms of the density matrix.

4. Formal evaluation of Density Matrix

We shall first calculate $\boldsymbol{R}^N$ for an allowed decay. The modifications to get the general case will then be apparent. For allowed decays only the S-wave part $\chi^0_{k,m}$ for $k = \pm 1$ of the electron wave function $\chi_{k,m}$ is important.

In the representation

$$\gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \ \gamma_5 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \ \vec{\gamma} = i \begin{pmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \tag{4.1}$$

we have

$$\left.\begin{aligned} \chi^0_{+1,+\frac{1}{2}} &= -f_{+1}\begin{pmatrix} \alpha \\ 0 \end{pmatrix}, \ \chi^0_{+1,-\frac{1}{2}} = +f_{+1}\begin{pmatrix} \beta \\ 0 \end{pmatrix} \\ \chi^0_{-1,+\frac{1}{2}} &= -ig_{-1}\begin{pmatrix} 0 \\ \alpha \end{pmatrix}, \ \chi^0_{-1,-\frac{1}{2}} = +ig_{-1}\begin{pmatrix} 0 \\ \beta \end{pmatrix} \end{aligned}\right\} \tag{4.2}$$

where $\alpha = \binom{1}{0}$, $\beta = \binom{0}{1}$.

For the neutrino we may without loss of generality write $\vec{q} = (0, 0, \omega)$. It then follows, apart from a phase factor,

$$\overline{v}_+ = 2^{-\frac{1}{2}}(\beta, \beta), \ \overline{v}_- = 2^{-\frac{1}{2}}(\alpha, -\alpha). \tag{4.3}$$

Furthermore, the connection between the relevant matrix elements (3.4) and the usual Fermi and Gamow-Teller matrix elements may be written

$$\left.\begin{aligned} M_F &= \int d^3 x\, \Phi_s = \int d^3 x\, \Phi_V \\ \vec{j}\, M_{GT} &= \int d^3 x\, \vec{\Phi}_T = \int d^3 x\, \vec{\Phi}_A\,. \end{aligned}\right\} \tag{4.4}$$

Here $\vec{j}$ is a unit vector with the property

$$\langle \vec{j} \rangle_I = 0, \qquad \langle j_k\, j_{k'} \rangle_I = \frac{1}{3}\, \delta_{kk}$$

where $\langle\,\rangle_I$ means averaging over initial nuclear states. Finally, the radial functions f_{+1}, g_{-1} in (4.2) taken, as usual, at the nuclear radius give rise to the expressions

$$\left.\begin{aligned} |f_{+1}|^2 + |g_{-1}|^2 &= \frac{p^2}{2\pi^2} \cdot F_0(Z, W) \cdot \frac{1+\gamma}{2} \\ \frac{|f_{+1}|^2 - |g_{-1}|^2}{|f_{+1}|^2 + |g_{+1}|^2} &= \gamma \cdot \frac{m}{W}\ ; \ \gamma \equiv \sqrt{1 - \alpha^2 Z^2} \end{aligned}\right\} \tag{4.5}$$

where $F_0(Z, W)$ is the usual Fermi function and α is the fine structure constant. With the help of these formulae a straight forward calculation of (3.6) with (3.3, 4) yields

$$\left.\begin{aligned} \boldsymbol{R}^N(\omega) = (2\pi)^{-2}\, W p F_0(Z, W) \frac{1+\gamma}{2} \Big\{ & (\boldsymbol{C}^{SS} + \boldsymbol{C}^{VV})\, |M_F|^2 + \\ + (\boldsymbol{C}^{TT} + \boldsymbol{C}^{AA})\, |M^{GT}|^2 + \gamma \frac{m}{W} \big[& (\boldsymbol{C}^{SV} + \boldsymbol{C}^{VS})\, |M_F|^2 + \\ + (\boldsymbol{C}^{TA} + \boldsymbol{C}^{AT})\, |M_{GT}|^2 \big] \Big\}\ ; & \ W = W_N - \omega \end{aligned}\right\} \tag{4.6}$$

where the matrix C^{ij} is defined by the elements

$$C^{ij}_{l+,\,l'+} \equiv G_{li} G^*_{l'j}\,, \quad C^{ij}_{l-,\,l'-} \equiv \pm F_{li} F^*_{l'j} \begin{cases} l = l' \\ l \neq l' \end{cases} \tag{4.7}$$

$$C^{ij}_{l+,\,l'-} = C^{ij}_{l-,\,l'+} = 0\,. \tag{4.7'}$$

Remembering the invariants defined in (2.15, 16) we see that

$$\sum_l C^{ij}_{l+\,l+} = A^+_{ji}\,, \quad \sum_l C^{ij}_{l-,\,l-} = A^-_{ji}\,, \qquad Tr\,\mathbf{C}^{ij} = 2\,K_{ji}\,. \tag{4.8}$$

Thus we conclude from (3.19, 20) that β-spectra and lifetimes for allowed transitions depend on the coupling constants through the invariants $K_{SS} + K_{VV}$, $K_{TT} + K_{AA}$, $K_{SV} + K_{VS}$, $K_{TA} + K_{AT}$.

From (4.7') it is seen that states of different spin do not interfere. We shall now show that this is still the case using the general expression (3.3) of the matrix element. Indeed, (3.6) may be written in the form

$$R^N_{ls,\,l's'} = (\overline{v}_s(\vec{q})\, X_{ls,\,l's'}(\vec{q})\, \overline{v}^*_{s'}(\vec{q})) \tag{4.9}$$

where $X_{ls,\,l's'}$ can be expressed as a linear combination of the 16 Dirac matrices

$$X_{ls,\,l's'} = \sum_{A=1}^{16} x^{(A)}_{ls,\,l's'}\, \gamma^{(A)}. \tag{4.10}$$

Now in $R^N_{ls,\,l's'}$ all directional and spin variables have been averaged out except $\vec{q}$, s. Therefore $R^N_{ls,\,l's'}$ has to be a scalar with respect to 3-dimensional rotations of the coordinate system.

It follows that $\boldsymbol{X}$ is of the form

$$\boldsymbol{X} = \boldsymbol{x}^{(1)} \cdot 1 + \boldsymbol{x}^{(2)} \cdot \gamma_4 + \boldsymbol{x}^{(3)} \gamma_5 + \boldsymbol{x}^{(4)} \gamma_4 \gamma_5 \tag{4.10'}$$

where the $\boldsymbol{x}^{(A)}$ depend on ω only. But with this expression it is easy to see from (4.1, 3) that (4.9) vanishes unless $s = s'$,

$$R^N_{ls,\,l's'} = \delta_{ss'}\, R^N_{is,\,l's} \tag{4.11}$$

which proves our statement above. Furthermore, from (3.3) and (2.12) it is seen that $R^N_{ls,\,l's}$ contains the coupling constants in the form (4.7). Thus we can write

$$R^N_{l\pm,\,l'\pm}(\omega) = \sum_{ij} f^N_{ij\,\pm}(\omega)\, C^{ij}_{l\pm,\,l'\pm}$$

where the f's are independent of the coupling constants and consequently independent of l, l'. Next we want to show that the f's are also independent of $s = \pm$. For this we consider a space reflexion,

$$\chi'(\vec{x}) = i\,\gamma_4\,\chi(-\vec{x}) \qquad \overline{v}'_\pm(\vec{q}) = -i\,\overline{v}_\pm(-\vec{q})\,\gamma_4\,.$$

(Here the phase is fixed because we have used $u = v^c$, see footnote 12.) Then from (3.3) we have, remembering (2.12)

$$T^{N'}_{l+,\vec{q}} = \sum_i G_{li} \int d^3 x \, \Phi'_i(\vec{x}) e^{-i\vec{q}\vec{x}} (\overline{v}_+(-\vec{q}) \gamma_4 O_i \gamma_4 \chi(-\vec{x}))$$

and similarly for $T^{N'}_{l-,\vec{q}}$. But

$$\Phi'_i(\vec{x}) \gamma_4 O_i \gamma_4 = \eta \Phi_i(-\vec{x}) O_i; \qquad |\eta| = 1; \text{ all } i$$

so that

$$T^{N'}_{ls,\vec{q}} = \eta \cdot T^{N}_{ls,\vec{q}}.$$

Now according to (3.6) $R^N_{ls,ls}$ is a summation over $| T^N_{ls\vec{q}}|^2$ hence

$$R^{N'}_{ls,ls} = R^N_{ls,ls}.$$

But then the coefficients $x^{(3)}_{ls,ls}$, $x^{(4)}_{ls,ls}$ in (4.10′) have to be zero. It follows from (4.13) that

$$(\overline{v}_+ X_{l+,l+} \overline{v}^*_+) = (\overline{v}_- X_{l+,l+} \overline{v}^*_-) = x^{(1)}_{l+,l+}$$

Consequently

$$\sum_{ij} f^N_{ij+} C^{ij}_{l+,l+} = \sum_{ij} f^N_{ij-} C^{ij}_{l+,l+}$$

and similarly with $C^{ij}_{l-,l-}$. Therefore we can introduce

$$f^N_{ij} = \frac{1}{2} (f^N_{ij+} + f^N_{ij-})$$

so that

$$\boldsymbol{R}^N(\omega) = \sum_{ij} f_{ij}(\omega) \cdot \boldsymbol{C}^{ij} \tag{4.12}$$

where (4.11) is fulfilled due to (4.7′).

It is seen from (4.8, 12) that spectrum and lifetime for any β^--decay depends on the coupling constants only through the invariants K_{ij}. Indeed from (3.19, 20) we have

$$1/\tau^N = \frac{2}{\pi} \int_0^\infty d\omega \cdot \omega^2 \sum_{ij} f^N_{ij}(\omega) \cdot K_{ji} \tag{4.13}$$

$$N^N(W) = \frac{2}{\pi} (W_N - W)^2 \sum_{ij} f^N_{ij}(W_N - W) \, K_{ji}. \tag{4.14}$$

Since (4.13) is real it follows that

$$f^{N*}_{ij}(\omega) = f^N_{ji}(\omega). \tag{4.15}$$

With the definition (3.9) of the averaging procedure characteristic of the pile it follows from (4.12)

$$\langle \boldsymbol{R}(\omega) \rangle = \sum_{ij} \langle f_{ij}(\omega) \rangle \, \boldsymbol{C}^{ij}. \tag{4.12'}$$

Then the neutrino spectrum (3.14) is

$$\varrho(\omega) = \omega^2 \sum_{ij} \langle f_{ij}(\omega)\rangle K_{ji} \Big/ \int_0^\infty d\omega \cdot \omega^2 \langle f_{ij}(\omega)\rangle K_{ji}. \tag{4.16}$$

It again depends on the coupling constants only through the K_{ij}. The expression for the matrix (3.16) is

$$\boldsymbol{a}(\omega) = \sum_{ij} \langle f_{ij}(\omega)\rangle \mathbf{C}^{ij} \Big/ 2 \sum_{ij} \langle f_{ij}(\omega)\rangle K_{ji}. \tag{4.17}$$

For the conventional and the two-component theory the nonvanishing elements of $\boldsymbol{a}$ take the values given in (2.5, 6). Furthermore, from the property (4.11), that interference between the states of different spin s is absent, it follows that the neutrino polarization is always longitudinal. Indeed, with (3.22), (4.17) and (4.8), (2.17) one gets

$$P_1 = P_2 = 0, \quad P_3 = P(\omega) = \sum_{ij} \langle f_{ij}(\omega)\rangle L_{ji} \Big/ \sum_{ij} \langle f_{ij}(\omega)\rangle K_{ji}. \tag{4.18}$$

Finally, we mention that from (4.13) an average lifetime for the pile in equilibrium is defined as

$$\left\langle \frac{1}{\tau} \right\rangle = \frac{2}{\pi} \int_0^\infty d\omega \cdot \omega^2 \sum_{ij} \langle f_{ij}(\omega)\rangle K_{ji}. \tag{4.13'}$$

5. Evaluation of cross section

For not too high values of ω, say $\omega \leq 100 \cdot m$, the expression (3.11) for the cross section matrix simplifies because energy-momentum conservation (target proton at rest) gives

$$|\vec{p}_n| \leq 2\omega - \omega_0 + m \ll m_n \tag{5.1}$$

where ω_0 is the threshold defined in (3.13). From the experiment of MUEHLHAUSE and OLEKSA[10]) we take the information that the neutrino spectrum $p(\omega)$ falls off at energies $\omega \simeq 13$ MeV $\simeq 26 \cdot m$ so that (5.1) is largely fulfilled and we can replace E_n by m, in the δ-function of (3.11). Thus

$$\sigma_{ls,\,l's'}(\omega) = \left[\frac{p' W'}{2\pi} \sum_{pne} T^{\mathrm{abs}*}_{ls\vec{q}}\, T^{\mathrm{abs}}_{l's'\vec{q}} \right]_{W' = m + \omega - \omega_0}. \tag{5.2}$$

The further evaluation with (2.10) is very similar to that one of the density matrix for an allowed decay given in the preceeding section. In fact the sum in (3.6) changes into the spin sum in the above expression by the formal substitutions

$$|M_F|^2 \to 1, \quad |M_{GT}|^2 \to 3, \quad |f_{+1}|^2 \to \frac{W' - m}{W'}, \quad |g_{-1}|^2 \to \frac{W' + m}{W'}.$$

*

Thus one reads from (4.6)

$$\left.\begin{array}{l}\boldsymbol{\sigma}(\omega)=\dfrac{p'W'}{2\pi}\Big\{(\boldsymbol{C}^{SS}+\boldsymbol{C}^{VV})+3\,(\boldsymbol{C}^{TT}+\boldsymbol{C}^{AA})-\\ -\dfrac{m}{W'}\,[(\boldsymbol{C}^{SV}+\boldsymbol{C}^{VS})+3\,(\boldsymbol{C}^{TA}+\boldsymbol{C}^{AT})]\Big\}\,;\;W'=m+\omega-\omega_0.\end{array}\right\}\quad(5.3)$$

Using this expression together with (4.17) we see that $\sigma(\omega)$, (3.18), is a linear combination of terms $Tr\,\boldsymbol{C}^{ij}\,\boldsymbol{C}^{i'j'}$. From the invariance principle we know that not only these terms but even $\sum_{l'} C^{ij}_{ls,\,l's}\,C^{i'j'}_{l's,\,ls}$, for both signs of s, are invariant. Indeed, according to (2.16) and (4.8),

$$\sum_{ll'} C^{ij}_{l+,\,l'+}\,C^{i'j'}_{l'+,\,l+}=A^{+*}_{ij'}\,A^{+}_{ji'}\,;\quad \sum_{ll'} C^{ij}_{l-,\,l'-}\,C^{i'j'}_{l'-,\,l-}=A^{-*}_{ij'}\,A^{-}_{ji'}\quad(5.4)$$

and with (2.15, 17)

$$\frac{1}{2}\,Tr\,\boldsymbol{C}^{ij}\,\boldsymbol{C}^{i'j'}=K^{*}_{ij'}\,K_{ji'}+L^{*}_{ij'}\,L_{ji'}.\quad(5.5)$$

To correct for the permutation of indices on the right side we introduce the bilinear invariants

$$\Lambda_{ij\,|\,j'i'}\equiv K^{*}_{ij}\,K_{j'i'}-K^{*}_{ij'}\,K_{ji'}+L^{*}_{ij}\,L_{j'i'}-L^{*}_{ij'}\,L_{ji'}=-\Lambda_{ij'\,|\,ji'}\quad(5.6)$$

and remark that the vanishing of the Λ's is a necessary invariant condition for lepton charge conservation[15]). Thus

$$\frac{1}{2}\,Tr\,\boldsymbol{C}^{ij}\,\boldsymbol{C}^{i'j'}=K_{ji}\,K_{j'i'}+L_{ji}\,L_{j'i'}-\Lambda_{ij\,|\,j'i'}\quad(5.5')$$

and according to (3.18), (4.17, 18), (5.3)

$$\sigma(\omega)=\sigma(\omega;K)+P(\omega)\cdot\sigma(\omega;L)-\Delta(\omega;\Lambda)\,/\,\sum_{ij}\langle f_{ij}\rangle\,K_{ji}\quad(5.7)$$

where

$$\left.\begin{array}{l}\sigma(\omega;K)=\dfrac{p'W'}{2\pi}\Big\{(K_{SS}+K_{VV})+3\,(K_{TT}+K_{AA})-\\ -\dfrac{m}{W'}\,[(K_{SV}+K_{VS})+3\,(K_{TA}+K_{AT})]\Big\}\,;\;W'=m+\omega-\omega_0\end{array}\right\}\quad(5.7')$$

$$\left.\begin{array}{c}\Delta(\omega,\Lambda)=\dfrac{p'W'}{2\pi}\displaystyle\sum_{ij}\langle f_{ij}\rangle\,\Big\{(\Lambda_{ij/SS}+\Lambda_{ij/VV})+3\\ (\Lambda_{ij/TT}+\Lambda_{ij/AA})-\dfrac{m}{W'}\,[(\Lambda_{ij/SV}+\Lambda_{ij/VS})+3\,(\Lambda_{ij/TA}+\Lambda_{ij/AT})]\Big\}\\ W'=m+\omega-\omega_0\end{array}\right\}\quad(5.7'')$$

and $\sigma(\omega; L)$ is (5.7′) with the L_{ij} replacing the K_{ij}. The total cross section is according to (3.12′)

$$\bar{\sigma} = \{\Sigma(K) + \Sigma(L) - \bar{\Delta}(\Lambda)\} / \left\langle \frac{1}{\tau} \right\rangle \tag{5.8}$$

where

$$\left.\begin{aligned} \Sigma(K) &= \frac{2}{\pi} \int_{\omega_0}^{\infty} d\omega \cdot \omega^4 \sum_{ij} \langle f_{ij} \rangle K_{ji} \sigma(\omega; K) \\ \bar{\Delta}(\Lambda) &= \frac{2}{\pi} \int_{\omega_0}^{\infty} d\omega \cdot \omega^4 \Delta(\omega; \Lambda) \end{aligned}\right\} \tag{5.8′}$$

and $\langle 1/\tau \rangle$ is given by (4.13′).

To get an idea of the dependence of (5.8) on the invariants we take the conventional theory as a reference theory. Its invariant characterization is[15])

$$L^c_{ij} = 0; \quad \Lambda^c_{ij/j'i'} = 0 \tag{5.9}$$

(c for conventional). Taking all β-spectra $N^N(W)$, lifetimes τ^N and the neutrino spectrum $\varrho(\omega)$ as given empirically, we conclude from (4.13, 14, 16) that all K_{ij} are "measurable" and, therefore, constant with respect to any change of the theory (i.e. its coupling parameters),

$$K_{ij} = K^c_{ij}, \qquad \text{all } ij \;^{16)}. \tag{5.10}$$

Thus

$$\sigma^c(\omega) = \sigma(\omega; K) \;^{17)} \tag{5.11}$$

and

$$\bar{\sigma} / \bar{\sigma}^c = 1 + \{\Sigma(L) + \bar{\Delta}(\Lambda)\} / \Sigma(K). \tag{5.12}$$

Since the invariant characterization of the two-component theory is[15]) $L_{ij} = -K_{ij}$ (or $L_{ij} = +K_{ij}$), $\Lambda_{ij/j'i'} = 0$ we find the previous result (2.9) as a special case. It does not, however, follow in general that the value 2 is an upper limit to (5.12).

6. Discussion of experiments

The principal result (3.12′) and (5.7) shows that for the calculation of the theoretical total cross section $\bar{\sigma}$ three invariant functions of ω are needed in the general case: the spectrum $\varrho(\omega)$, the polarization $P(\omega)$ which is a measure of parity non-conservation and the function

$$L(\omega) = \Delta(\omega; \Lambda) / \sum_{ij} \langle f_{ij}(\omega) \rangle K_{ji} \tag{6.1}$$

which is a measure of lepton charge non-conservation. It is important to note that the information from parity experiments available at the moment is not sufficient to fix $P(\omega)$ and $L(\omega)$; and consequently also the value of the relative cross section $\bar{\sigma}/\sigma^c$ is not fixed. The parity experiments tell us that[18])

$$A^+_{SS} = A^+_{TT} = A^-_{VV} = A^-_{AA} = 0 \tag{6.2}$$

or equivalently

$$\left.\begin{aligned} L_{SS} &= -K_{SS}\,, & L_{TT} &= -K_{TT} \\ L_{VV} &= +K_{VV}\,, & L_{AA} &= +K_{AA}\,. \end{aligned}\right\} \tag{6.2'}$$

But this is not all that can be said since from the inequalities[8])

$$|A^+_{ij}|^2 \leq A^+_{ii}\, A^+_{jj}\,, \quad |A^-_{ij}|^2 \leq A^-_{ii}\, A^-_{jj} \tag{6.3}$$

it follows that

$$A^+_{Si} = A^+_{Ti} = A^-_{Vi} = A^-_{Ai} = 0, \text{ all } i \tag{6.4}$$

or also

$$\left.\begin{aligned} K_{SV} = K_{SA} = K_{TV} = K_{TA} = 0 \\ L_{SV} = L_{SA} = L_{TV} = L_{TA} = 0 \end{aligned}\right\} \tag{6.4'}$$

which is much more than the vanishing of the Fierz terms

$$(\mathrm{Re}\, K_{SV} = \mathrm{Re}\, K_{TA} = 0)$$

Unless recoil experiments will show that the coupling is pure (STP) or pure (VAP) the indeterminacy of $P(\omega)$ and $L(\omega)$ even persists if all neutrinos came from allowed decays, as is seen from (4.6). Therefore, all three functions $\varrho(\omega)$, $P(\omega)$ and $L(\omega)$ should in principle be determined from experiment.

The Muehlhause and Oleksa experiment[11]) does not measure $\varrho(\omega)$ directly, but the averaged β^--spectrum (see (3.9, 20))

$$\langle N(W)\rangle = \sum_N \lambda_N\, N^N(W)$$

whereas according to (3.14, 20) $\varrho(\omega)$ may be written as

$$\varrho(\omega) = \sum_N \lambda_N\, \Theta(W_N - \omega - m)\, N^N(W_N - \omega) \Big/ \left\langle \frac{1}{\tau} \right\rangle.$$

The connection between these two spectra is rather involved unless the symmetry property

$$N^N(W) = N^N(W_N - W) = \Theta(W - m)\cdot N^N(W)$$

is used which, however, is certainly violated at the endpoints $W = m$ and $W = W_N$. Though in ref. 11 account is taken of this asymmetry the resulting neutrino spectrum is probably not very accurate.

In the hope to get a direct determination of $\varrho(\omega)$ REINES has recently made an absorption experiment in which the positron energy W' is discriminated[19]). The corresponding cross section follows from (3.11, 18, 12′) by suppressing the integration over W' in (3.11),

$$\sigma_e(W') = \omega^2 \varrho(\omega) \cdot \sigma(\omega); \quad W' = m + \omega - \omega_0. \tag{6.5}$$

From (5.7) it is seen, however, that $\varrho(\omega)$ is determined only if

$$P(\omega) = P = \text{const.}, \ L(\omega) = 0.$$

According to (4.18) this condition leads to a theory with

$$L_{ij} = P \cdot K_{ij} \quad {}^{20)} \tag{6.6}$$

which reduces to the two-component theory if the experimental information (6.2′) is taken into account.

Inspite of the lack of knowledge about $P(\omega)$ and $L(\omega)$ it follows that a measurement of $\sigma_e(W')$ gives a good test of the two-component theory. Indeed, if first $\varrho(\omega)$ is determined with the assumption $P = -1$ (or $P = +1$), $L(\omega) = 0$ and then is used to calculate $\bar{\sigma}$, an agreement of this value with $\int dW' \sigma_e(W')$ strongly supports the two-component theory. A discrepancy, on the other hand, means that $P(\omega)$ (and even $L(\omega)$) should be determined, too.

We would like to point out that assuming lepton conservation the measurement of $P(\omega)$ is, in principle, possible in an absorption experiment which discriminates not only W' but also the asymmetry in the angle Θ between the neutrino and positron momenta $\vec{q}$ and $\vec{p}'$. (Of course the discrimination of the longitudinal spin of the positron could also be used, but such an experiment would be even more difficult.) We give here the result of a calculation of the corresponding cross section $\sigma_e(W', \Theta)$ which follows from (3.11, 12) by suppressing the integration over W' and Θ. Without any assumptions about the Λ's we get instead of (6.5), using (5.5′)

$$\left.\begin{aligned} \sigma_e(W', \Theta) \sin\Theta \, d\Theta &= \frac{1}{2} \omega^2 \varrho(\omega) \sigma(\omega) \cdot (1 + \alpha(\omega) \cos\Theta) \sin\Theta \, d\Theta \\ W' &= m + \omega - \omega_0 ; \end{aligned}\right\} \tag{6.7}$$

where the asymmetry parameter α is given by

$$\left.\begin{aligned} \alpha(\omega) = \frac{p'^2}{2\pi\sigma(\omega)} \{ & (K_{TT} - K_{AA} - K_{SS} + K_{VV}) + \\ & + P(\omega)(L_{TT} - L_{AA} - L_{SS} + L_{VV}) - \sum_{ij} \langle f_{ij} \rangle \cdot \\ & \cdot (\Lambda_{ij/TT} - \Lambda_{ij/AA} - \Lambda_{ij/SS} + \Lambda_{ij/VV}) / \sum_{ij} \langle f_{ij} \rangle K_{ji} \}. \end{aligned}\right\} \tag{6.8}$$

If in this expression we put all $\Lambda = 0$ (lepton conservation) and use the consequences (6.2′, 4′) of the empirical facts, we arrive at the formula

$$\left.\begin{aligned}\alpha(\omega) = \frac{p'}{W'} \{(K_{TT} - K_{AA} - K_{SS} + K_{VV}) + \\ + P(\omega)\,(K_{TT} + K_{AA} - K_{SS} - K_{VV})\}/\{(K_{SS} + K_{VV}) + \\ + 3(K_{TT} + K_{AA}) + P(\omega)[(K_{VV} - K_{SS}) + 3(K_{AA} - K_{TT})]\}.\end{aligned}\right\} \quad (6.8')$$

from which the neutrino polarization $P(\omega)$ may be deduced. Note that in the two-component theory $\alpha(\omega) = 0$.

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[9]) This is easily verified from the transformation property (see footnote 12)

$$(c'_{1s}, c'_{2s}) = (c_{1s}, c_{2s}) \cdot U_s; \quad \begin{pmatrix} c'^{*}_{1s} \\ c'^{*}_{2s} \end{pmatrix} = U_s^{-1} \cdot \begin{pmatrix} c^{*}_{1s} \\ c^{*}_{2s} \end{pmatrix}$$

where, in the notation of ref. 4, U_s is the unitary matrix

$$U_{\pm} = e^{\pm i\alpha} \begin{pmatrix} a^{*}, & \pm b \\ \mp b^{*}, & a \end{pmatrix}; \quad |a|^2 + |b|^2 = 1\,.$$

[10]) In connection with parity violation the terms "right" and "left helicity" have been introduced to design the projection of the neutrino field on the two spin states

$$\psi_\nu^R = \frac{1}{2}(1-\gamma_5)\,\psi_\nu; \quad \psi_\nu^L = \frac{1}{2}(1+\gamma_5)\,\psi_\nu$$

and similarly for ψ_ν^c. The corresponding coupling constants are simply related to the ones used in this paper,

$$C_{Ri} = F_{1i}^*, \quad C_{Li} = G_{1i}^*, \quad D_{Ri} = F_{2i}^*, \quad D_{Li} = -G_{2i}^*.$$

The use of the F's and G's is somewhat better adapted for our purpose.

[11]) An attempt to measure the neutrino spectrum from uranium fission products is reported by C. O. MUEHLHAUSE and S. OLEKSA, Phys. Rev. *105*, 1332 (1957).

[12]) We use

$$\psi = (2\pi)^{-3/2} \int d^3 p \sum_s \left(c_{1s}^* (\vec{p})\, u_s (\vec{p})\, e^{-i(px)} + c_{2s} (\vec{p})\, v_s (\vec{p})\, e^{+i(px)}\right); \quad \bar{\psi} \equiv \psi^* \gamma_4$$

and define the spinors u, v by the equations

$$(i(\gamma p) + m)\, u_s (\vec{p}) = 0; \; (i(\gamma p) - m)\, v_s (\vec{p}) = 0$$

$$(\mp i (\vec{\Sigma}\vec{p}) + |\vec{p}|)\, u_\pm(\vec{p}) = 0; \; (\pm i (\vec{\Sigma}\vec{p}) + |\vec{p}|)\, v_\pm(\vec{p}) = 0$$

$$\vec{\Sigma} \equiv i\gamma_4 \gamma_5 \vec{\gamma}; \; (u_s^* u_s) = (v_s^* v_s) = 1; \; s = \pm.$$

Furthermore, we fix relative phases by putting

$$v_s = u_s^c \equiv C^{-1} \bar{u}_s; \quad u_s = v_s^c \equiv C^{-1} \bar{v}_s.$$

[13]) It would be easy and perhaps more adapted to experiments to introduce a geometry factor $\gamma(\vec{q})$ which falls off with $|(\vec{q}/\omega) - \vec{e}|$. Then $\sum'_{\vec{p}}$ would have to be replaced by $\sum_{\vec{q}} \gamma(\vec{q})$, summed over all $\vec{q}$. However, this only would complicate the formulae without giving any improvement.

[14]) Since according to the liquid drop model fission products lie on the neutron rich side of the $(A—Z, Z)$ — curve for stable nuclei, β^+-decay and K-capture is practically absent in the pile. Moreover for lepton conservation the neutrinos from β^+-decay are in the wrong charge state for the Cowan Reines experiment. Mesonic and hyperonic neutrino production is of course negligible.

[15]) For an invariant characterization of particular properties of H_{int} see ref. 8.

[16]) If among the functions $f_{ij}^N(\omega)$ there exist linear relations, independent of N, only certain linear combinations of the K_{ij} are "measurable" and (5.10) is correspondingly weakened. This however does not affect the result (5.12). One example are the reality conditions

$$f_{ij}^N(\omega) = f_{ij}^{N*}(\omega), \text{ all } ij \text{ and } N$$

combined with (4.15). In this case the measurable expressions are Re K_{ij} and (5.10) is replaced by

$$Re\, K_{ij} = Re\, K_{ij}^c.$$

As is seen from (4.6) such reality conditions hold for allowed decays. They are also fulfilled for first forbidden decays as may be seen from the expressions $b_0(L, L)$ in the paper by K. ALDER, B. STECH and A. WINTHER, Phys. Rev. *107*, 728 (1957). We believe that they can be proved for the general case using time reversal, similarly to the space reflexion argument given in section 4.

[17]) With only S- and T-coupling (5.7′, 11) reduces to formula (2) of ref. 11 in which, however, there is misprint ($W' = \omega - \omega_0$ instead of $W' = m + \omega - \omega_0$).

[18]) See for instance the tables prepared by C. S. WU in "Proceedings of the International Conference on Nuclear Structure at the Weizmann Institute, Rehovoth, Israel, September 1957". (North-Holland Publishing Co., in press.)

[19]) I am very much indebted to Dr. REINES for information on this and related experiments.

[20]) If there are linear dependences among the functions $\langle f_{ij}(\omega)\rangle$ the conditions (6.6) are weakened. As an example we mention the reality conditions discussed in footnote 16, in which case (6.6) is replaced by

$$Re\ L_{ij} = P \cdot Re\ K_{ij}.$$

The conclusion is, however, the same.

Note added in proof: In an ingenious experiment GOLDHABER, GRODZINS and SUNYAR (to be published in Phys. Rev.) have succeeded in a direct determination of the neutrino polarization P in an allowed GT β^+-decay. They find $P = -1$ which, in accord with recent recoil experiments by HERRMANNSFELDT, STÄHELIN and ALLEN (Bulletin of the American Phys. Soc. *3*, 52 (1958)) with β^+-emitters, rules out an ST-coupling. The interest in an independ determination of P for β^--decays of course persists.