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Quantum Theory in Real Hilbert Space II (Addenda and Errata)

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(15. II. 1961)

Abstract: A more concise demonstration of the necessity of the thermodynamic signature of the metric $g^{\mu\nu}$ and of the pseudo-chronous character of $\overset{\circ}{J}$ (corresponding to $i = \sqrt{-1}$) is given (§ 4 of I¹). Furthermore, the anti-unitary operator (Annex 3 of I) is wrong. A more concise demonstration in terms of observables is given, showing the perfect correspondence between RHS (Real Hilbert Space) and CHS (Complex Hilbert Space).

A more concise demonstration for the *thermodynamic signature* and the *pseudo-chronous character* of $\overset{\circ}{J}$ is:

§ 4 bis. The Thermodynamic Signature of $\mathfrak{g}^{\mu\nu}$ and the Pseudo-Chronous Character of $\mathbf{J} = \overset{\circ}{\mathbf{J}}$

For a *local scalar observable*, we have the identity ($\alpha \beta \dots = 12 \dots n$):

$$'F('x) = F(x) = F(L^{-1}'x) = O^{-1} F('x) O,$$

$$L \leftarrow O: 'x = Lx = \{ 'x'^{\alpha} = L'^{\alpha}_{\alpha} (x^{\alpha} + L^{\alpha}) \},$$

$$\mathfrak{g}'^{\mu\nu} = L'^{\mu}_{\mu} L'^{\nu}_{\nu} g^{\mu\nu}. \quad (4 \text{ bis. } 1)$$

We form the *bilocal observable*

$$\overset{\circ}{G}(x y) = \overset{\circ}{J} [F(x), F(y)] \quad (4 \text{ bis. } 2)$$

appearing in the UP:

$$\langle \Delta F(x)^2 \rangle_{\Psi} \langle \Delta F(y)^2 \rangle_{\Psi} \geq \frac{1}{4} \langle \overset{\circ}{G}(x y) \rangle_{\Psi}^2. \quad (4 \text{ bis. } 3)$$

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The hypothesis, that an L -invariant vacuum state Ψ^0 exists

$$\langle \check{G}(x y) \rangle_{\Psi^0} = \check{f}(x y) = -\check{f}(y x) \quad (4 \text{ bis. } 4)$$

requires that

$$\check{f}'(x' y) = c(L) \check{f}(x y) = c(L) \check{f}(L^{-1} x L^{-1} y) \quad (4 \text{ bis. } 5)$$

is the same function in every frame, i.e.

$$\check{f}'(x' y) = \check{f}(x' y) \quad (4 \text{ bis. } 6)$$

because *vacuum is a frame-invariant concept*.

$c(L)$ is a (real) *number*, satisfying the representation condition

$$c(L_2) C(L_1) = c(L_2 L_1) . \quad (4 \text{ bis. } 7)$$

From the identity, following from (4 bis. 5) and (4 bis. 6)

$$c(L) \check{f}(L^{-1} x L^{-1} y) = \check{f}'(x' y) = -\check{f}'(y' x) \quad (4 \text{ bis. } 8)$$

follows, that $\check{f}(x y)$ is, except for the factor $c(L)$, an invariant anti-symmetric function. Such a function exists certainly, if we choose a metric

$$\text{signat}(g^{\alpha\beta}) = \pm (11 \dots 1 - 1) . \quad (4 \text{ bis. } 9)$$

We call $\{x^i\} = \vec{x}$ ($ik \dots = 12 \dots d$, $d = n - 1$) *space* and $x^n = t$ *time* and define

$$\check{f}(x y) = \text{sig}(x^n - y^n) \cdot f((x - y)^2) = -\check{f}(y x) \quad (4 \text{ bis. } 10)$$

$$f(z^2) = 0 \quad \text{for } z = x - y = \text{spatial}.$$

We have therefore

$$\begin{aligned} c(L) \check{f}(L^{-1} x L^{-1} y) &= c(L) \text{sig}(L^{-1 n} (x'^n - y'^n)) \cdot f((x - y)^2) \\ &= c(L) \text{sig}(L'^n) \cdot \text{sig}(x'^n - y'^n) \cdot f((x - y)^2) \\ &= c(L) \text{sig}(L'^n) \cdot \check{f}'(x' y) \equiv \check{f}'(x' y), \end{aligned} \quad (4 \text{ bis. } 11)$$

i.e.

$$c(L) = \text{sig}(L'^n) . \quad (4 \text{ bis. } 12)$$

This means, that $\check{f}(x y)$ is a *pseudo-chronous number*, characterising the *vacuum*. Therefore $\check{G}(x y)$ must be a *pseudochronous observable* i.e.

$$\begin{aligned}
 \check{G}('x 'y) &= \text{sig}(L'^n_n) \check{G}(x y) = \text{sig}(L'^n_n) \check{G}(L^{-1} 'x L^{-1} 'y) = \\
 &= O^{-1} \check{G}('x 'y) O = (O^{-1} \check{J} O) [O^{-1} F('x) O, O^{-1} F('y) O] = \\
 &= (O^{-1} \check{J} O) ['F('x), 'F('y)] = (O^{-1} \check{J} O) [F(x), F(y)] = \\
 &= (O^{-1} \check{J} O \check{J}^{-1}) \check{J}[F(x), F(y)] = (O^{-1} \check{J} O \check{J}^{-1}) \check{G}(x y)
 \end{aligned}
 \tag{4 bis. 13}$$

where we have made use of (4 bis. 1). Comparing the second and the last member, we have $O^{-1} \check{J} O \check{J}^{-1} = \text{sig}(L'^n_n).1$

$$O^{-1} \check{J} O = \text{sig}(L'^n_n) \check{J} \equiv ' \check{J}
 \tag{4 bis. 14}$$

which is equation (4.14) of I. \check{J} is thus a *pseudochronous operator*. Writing $L_{(\text{ochr})} \leftarrow O_{(\text{ochr})}$ and $L_{(\text{pchr})} \leftarrow O_{(\text{pchr})}$ for *ortho-chronous* ($\text{sig}(L'^n_n) = +1$) and *pseudo-chronous* ($\text{sig}(L'^n_n) = -1$) *Lorentz-transformations*, we have

$$[O_{(\text{ochr})}, \check{J}] = [O_{(\text{ochr})}, \check{J}]_- = 0,
 \tag{4 bis. 15_-}$$

$$[O_{(\text{pchr})}, \check{J}] = [O_{(\text{pchr})}, \check{J}]_+ = 0.
 \tag{4 bis. 15_+}$$

We are left to show, that (4 bis. 9) are the only two possible signatures of $g^{\mu\nu}$. To show this, let us assume a metric, in which $z = \{z^\alpha\}$ has $d = n - r$ *space components* and $r \leq 1/2 n$ *time components* $\vec{t} = \{t^a\} = \{z^a\}$ ($i k \dots = 12 \dots n - r, a b \dots = (n - r + 1) (n - r + 2) \dots n$).

We have to build a function $f(z) = f(\vec{x}, \vec{t})$ which is *invariant* under *homogeneous continuous transformations* $L_{(\text{cont})}$ (= *proper Lorentz-transformations*) and *changes sign* under *reflections* ($'x = (PT) x = -x$). Thus, we look for a vector $z_0 = \{z_0^\alpha\}$, which changes sign under the transformation $L = PT$ but does not change sign under any continuous transformation. If $r > 1$, it is evident, at least for *space-like* and *time-like vectors* z , that no such vector z_0 exists, because every vector $z = (\vec{t}, \vec{z})$ can be transformed into $-z$ by rotations in \vec{z} -space and \vec{t} -space. For null-vectors ($g_{\alpha\beta} z^\alpha z^\beta = 0$) we use the theorem²⁾, that in n dimensional space time with $r \leq 1/2 n$, there exist exactly r two-by-two orthogonal linearly independent null vectors z . Therefore, for $r > 1$, a rotation of this r -dimensional 'frame', transformes z into $-z$ (for instance z on the 'frame'). Thus only in the case $r = 1$, time-like vectors and null vectors (on the light cone) exist, which change signe under $L = PT$.

The text of I is correct, if we write, on page 748, under b):

'We pose...' instead of 'We try, posing...' and leave out, on p. 749, the text following (A-3.20). (A-3.20) is the correct formula, the text following (A-3.20) is erroneous. However, a more logical deduction, using only observables $F^X = F^{XT} = F^{\alpha\beta\dots}(xy\dots)$ ($X = \{\alpha\beta\dots xy\dots\}$) shall be given below in:

Annex 3 bis. Unitary (\widehat{U}) and Anti-Unitary (\widehat{V}) Operators in CHS

We rewrite the operator identity of (I 3.9) in RHS

$$F'_{a'b}{}^X = L'^X{}_X O'_{aa} F_{ab} O'^T{}_{b'b}; \quad F^X_{ab} = F^X_{(ab)}. \quad (\text{A-3 bis. 1})$$

For:

a) Orthochronous Lorentz Transformations

We decompose the RHS (a -space) in the direct product of a two-dimensional ((r) (i)-space) and a $\omega_C = (1/2) \omega_R$ dimensional p -space, using the two-dimensional matrices (in (r) (i)-space)

$$1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad j = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \quad (\text{A-3 bis. 2})$$

On account of (4 bis. 15₋), we have ($O = O_{(\text{ochr})}$):

$$\begin{aligned} F'_{a'b}{}^X &= 1 \times F'^X_{(r)'p'q} + j \times F'^X_{(i)'p'q} \\ &= L'^X{}_X (1 \times O_{(r)'pp} + j \times O_{(i)'pp}) (1 \times F^X_{(r)pq} + j \times F^X_{(i)pq}) \cdot \\ &\quad \cdot (1 \times O^T_{(r)q'q} - j \times O^T_{(i)q'q}), \end{aligned}$$

$$F^X_{(r)pq} = F^X_{(r)(pq)}; \quad F^X_{(i)pq} = F^X_{(i)[pq]} \quad (\text{A-3 bis. 3})$$

formula, which, may be rewritten in CHS, using the correspondence $\text{RHS} \rightleftharpoons \text{CHS}$, $A \rightleftharpoons \widehat{A}$ given in (I A-2.3) \rightleftharpoons (I A-2.5) in the form

$$\widehat{F}'_{p'q}{}^X = L'^X{}_X \widehat{O}_{p'p} \widehat{F}^X_{pq} \widehat{O}^\dagger_{q'q} = L'^X{}_X \widehat{U}_{p'p} \widehat{F}^X_{pq} \widehat{U}^\dagger_{q'q} \quad (\text{A-3 bis. 4})$$

with:

$$\widehat{F}^\dagger = \widehat{F} = \widehat{F}^{T*} = \widehat{F}^{*T},$$

$$\widehat{U} = \widehat{O}; \quad \widehat{U}^\dagger \widehat{U} = \widehat{U} \widehat{U}^\dagger = 1; \quad \widehat{U}^\dagger = \widehat{U}^{-1} \quad (\text{A-3 bis. 5})$$

where \widehat{A}^* is the *complex conjugate operator* ($(\widehat{A}\widehat{B})^* = \widehat{A}^*\widehat{B}^*$) and $\widehat{A}^\dagger = \widehat{A}^{*T} = \widehat{A}^{T*}$ is the *hermitian conjugate operator* ($(\widehat{A}\widehat{B})^\dagger = \widehat{B}^\dagger\widehat{A}^\dagger$) in CHS. Explicitly written, and omitting the indices $p q \dots = 12 \dots \omega_C$ referring to a frame in ω_C -dimensional CHS, the identity

$$\widehat{F}'^{\alpha'\beta\dots} ('x' 'y \dots) = c(L) L'^{\alpha'}_{\alpha} L'^{\beta}_{\beta} \dots \widehat{U} F^{\alpha\beta\dots} (L^{-1} 'x L^{-1} 'y \dots) \widehat{U}^{-1}$$

holds, with $c(L) = 1, \text{sig}(\det(L'^i_i))$ and $\text{sig}(\det(L'^{\alpha}_{\alpha}))$ for *ortho* (\widehat{F}), *pseudo-chronous* ($\overset{\sim}{\widehat{F}}$), *pseudochorous* ($\overset{\circ}{\widehat{F}}$) and *pseudo-(\widehat{F})* observables. The *transformed operators* are given by

$$\begin{aligned} \widehat{F}'^{\alpha'\beta\dots} ('x' 'y \dots) &= c(L) L'_{(\text{ochr})\alpha}{}^{\alpha'} L'_{(\text{ochr})\beta}{}^{\beta} \dots \widehat{F}^{\alpha\beta\dots} (L^{-1} 'x L^{-1} 'y \dots) \\ &= \widehat{U}^{-1} F'^{\alpha'\beta\dots} ('x' 'y \dots) \widehat{U}. \end{aligned} \tag{A-3 bis. 6}$$

They lead to the identity, for expectation values:

$$\begin{aligned} \langle \widehat{\Psi}, \widehat{F}'^{\alpha\dots} ('x \dots) \widehat{\Psi} \rangle &= c(L) L'^{\alpha}_{\alpha} \dots \langle \widehat{\Psi}, \widehat{F}^{\alpha\dots} (L^{-1} 'x \dots) \widehat{\Psi} \rangle \\ &= \langle \widehat{\Psi}, \widehat{F}'^{\alpha\dots} ('x \dots) \widehat{\Psi} \rangle \end{aligned} \tag{A-3 bis. 7}$$

with

$$\widehat{\Psi} = \widehat{U} \Psi; \quad L_{(\text{ochr})} \leftarrow \widehat{U}_{(\text{ochr})}. \tag{A-3 bis. 8}$$

b) *Pseudo-Chronous Lorentz-Transformations*

We need an operator

$$K = k \times 1; \quad K^T = K = K^{-1}; \quad K^2 = 1 \tag{A-3 bis. 8}$$

which transformes

$$\tilde{j} = K^{-1} \tilde{j} K = -\tilde{j}; \quad \tilde{j} = j \times 1. \tag{A-3 bis. 9)*}$$

Such an operator can be given in terms of the pseudoquaternions (I A-4.8). We chose, for exemple, the two-dimensional matrix in (r) (i)-space

$$k = k^T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad k^2 = 1, k^T = k, (k, j) = 0. \tag{A-3 bis. 10}$$

) In CHS $K \rightleftharpoons \widehat{K} : \widehat{K} i \widehat{K} = -i$ (A-3 bis 9).

The operator $O_{(\text{pchr})} = O$ can now be given in terms of

$$O = O' K; \quad O^T = O^{-1} = K^T O'^T = K O'^T; \quad [O', \overset{\smile}{J}] = 0,$$

$$O'^T O' = O' O'^T = 1 \quad (\text{A-3 bis. 11})$$

and the identity (A-3 bis. 1) can be written, using the decomposition

$$\begin{aligned} F'_{a'b}{}^X &= 1 \times F'_{(r)'p'q}{}^X + j \times F'_{(i)'p'q}{}^X \\ &= L'^X_X (1 \times O'_{(r)'p'p} + j \times O'_{(i)'p'p}) (k \times 1) (1 \times F^X_{(r)pq} + j \times F^X_{(i)pq}) \cdot \\ &\quad \cdot (k \times 1) (1 \times O'^T_{(r)q'q} - j \times O'^T_{(i)q'q}) \\ &= L'^X_X (1 \times O'_{(r)'p'p} + j \times O'_{(i)'p'p}) (1 \times F^X_{(r)pq} - j \times F^X_{(i)pq}) \cdot \\ &\quad \cdot (1 \times O'^T_{(r)q'q} - j \times O'^T_{(i)q'q}). \end{aligned} \quad (\text{A-3 bis. 12})$$

We have the correspondence $\text{RHS} \rightleftharpoons \text{CHS}$.

$$1 \times F^X_{(r)pq} - j \times F^X_{(i)pq} \rightleftharpoons F^X_{pq} - i F^X_{pq} = \widehat{F}^X_{pq}{}^*. \quad (\text{A-3 bis. 13})$$

Now, using again the $\text{RHS} \rightleftharpoons \text{CHS}$ correspondence (I A-2.3) \rightleftharpoons (I A-2.5), we may write (A-3 bis. 12) in CHS, omitting again the p -space indices,

$$\begin{aligned} \widehat{F}'^X &= F'^X + i F'^X_{(i)} = L'^X_X \widehat{O}' \widehat{F}^X{}^* \widehat{O}'^{-1} \\ &= L'^X_X \widehat{U}' \widehat{F}^X{}^* \widehat{U}'^{-1}; \quad \widehat{U}'^{-1} = U'^{\dagger} \end{aligned} \quad (\text{A-3 bis. 14})$$

or, explicitly

$$\widehat{F}'^{\alpha\beta\dots} ('x' y \dots) = c(L) L'^{\alpha}_{\alpha} L'^{\beta}_{\beta} \dots \widehat{U}' \widehat{F}^{\alpha\beta\dots}{}^* (L^{-1} 'x L^{-1} 'y \dots) \widehat{U}'^{-1}. \quad (\text{A-3 bis. 15})$$

The transformed operator is defined by

$$\begin{aligned} \widehat{F}'^{\alpha\dots} ('x \dots) &= c(L_{(\text{pchr})}) L'^{\alpha}_{(\text{pchr})\alpha} \dots \widehat{F}^{\alpha\dots} (L^{-1} 'x \dots) \\ &= \widehat{U}'^{-1*} \widehat{F}^{\alpha\dots}{}^* ('x \dots) \widehat{U}'^*. \end{aligned} \quad (\text{A-3 bis. 16})$$

The identity for expectation values takes the form

$$\begin{aligned} \langle \widehat{\Psi}, \widehat{F}'^{\alpha\dots} ('x \dots) \widehat{\Psi} \rangle &= c(L) L'^{\alpha}_{\alpha\dots} \langle \widehat{\Psi}, \widehat{F}^{\alpha\dots} (L^{-1} 'x \dots) \widehat{\Psi} \rangle \\ &= \langle \widehat{\Psi}, \widehat{F}'^{\alpha\dots} ('x \dots) \widehat{\Psi} \rangle. \end{aligned} \quad (\text{A-3 bis. 17})$$

Defining the anti-unitary operator \widehat{V} by

$$\widehat{\Psi} = \widehat{V} \widehat{\Psi} = \widehat{U} \widehat{\Psi}^* \rightarrow \widehat{\Psi}'_p = U'_{p'p} \Psi_p^*; \quad L_{(\text{pchr})} \leftarrow \widehat{V}, \quad (\text{A-3 bis. 18})^*$$

we have

$$\begin{aligned} \langle \widehat{\Psi}', \widehat{F}'^{\alpha \dots} ('x \dots) \widehat{\Psi}' \rangle &= c(L) L'^{\alpha \dots} \langle \widehat{\Psi}, \widehat{F}^{\alpha \dots} (L^{-1} 'x \dots) \widehat{\Psi} \rangle = \\ &= \widehat{\Psi}'_p \widehat{U}'_{p'p}{}^T{}^* \widehat{F}'_{p'q}{}^{\alpha \dots} ('x) \widehat{U}'_{qq} \Psi_q^* = \widehat{\Psi}'_q \widehat{U}'_{q'q}{}^T{}^* \widehat{F}'_{q'p}{}^{\alpha \dots} T('x) \widehat{U}'_{p'p}{}^* \widehat{\Psi}'_p = \\ &= \langle \widehat{\Psi}', \widehat{U}'{}^\dagger{}^* \widehat{F}'^{\alpha \dots} T('x) \widehat{U}'^* \widehat{\Psi}' \rangle. \end{aligned} \quad (\text{A-3 bis. 19})$$

By definition, $F^{\alpha \dots} (x \dots)$ is an observable. Therefore it follows, from (A-3 bis. 5), that in (A-3 bis. 19), we have

$$\widehat{F}'^{\alpha \dots} T ('x \dots) = \widehat{F}'^{\alpha \dots}{}^* ('x \dots) \quad (\text{A-3 bis. 20})$$

and, from (A-3 bis. 16) ($\widehat{U}'{}^\dagger = U'^{-1}$)

$$\widehat{U}'{}^\dagger{}^* \widehat{F}'^{\alpha \dots} T ('x \dots) \widehat{U}'^* = c(L) L'^{\alpha \dots} \widehat{F}^{\alpha \dots} (L^{-1} 'x \dots). \quad (\text{A-3 bis. 21})$$

Thus, formula (A-3 bis. 19) is identical with the second equation (A-3 bis. 17) and with (A-3 bis. 16).

In order to show the physical significance of (A-3 bis. 16) let us consider the '*real*' scalar free field***) considered as an observable:

$$w(x) = w^T(x); \quad \widehat{w}(x) = \widehat{w}^\dagger(x), \quad (\text{A-3 bis. 22})$$

expanded in plane waves (signat $(g^{\alpha\beta}) = (11 \dots 1 - 1)$).

$$\widehat{w}(x) = (2)^{-1/2} (2\pi)^{-d/2} \int d\sigma(\check{k}) (\widehat{a}(\check{k}) e^{i(\check{k}, x)} + \widehat{a}^\dagger(\check{k}) e^{-i(\check{k}, x)}),$$

$$\check{k}^2 + M^2 = 0; \quad \check{k}^n > |M|,$$

$$d\sigma(\check{k}) = (\check{k}^n)^{-1} d^d \check{k}; \quad d^d \check{k} = \prod_{i=1}^d d\check{k}^i, \quad (\text{A-3 bis. 23})^{***}$$

) $\widehat{\Psi}' = \widehat{U}' \widehat{K} \Psi = \widehat{U}' \widehat{\Psi}^$.

***) '*Field Quantization in Real Hilbert Space*' will be the object of a forthcoming paper in this journal (referred to as III).

***) The surface integral is to be extended over the *positive shell* of the hyperboloid $\check{k}^2 + M^2 = 0$. $\check{k} = \{\check{k}^\alpha\}$ is therefore a *pseudochronous vector* (see III).

where, if the sum over plane waves is made denumerable, $\widehat{a}(\check{k})$ and $\widehat{a}^\dagger(\check{k})$ are the usual annihilation and creation operators of quanta in a state \check{k} . Using (A-3 bis. 16) for the transformation $L = PT$

$$'x = (PT) x = -x; \quad 'x'^\alpha = -x'^\alpha \quad (\text{A-3 bis. 24})$$

we have (A-3 bis. 16):

$$\begin{aligned} \widehat{w}'(x) &= \widehat{w}(x) \equiv \widehat{w}(-'x) \\ &= (2)^{-1/2} (2\pi)^{-d/2} \int d\sigma(\check{k}) (\widehat{a}(\check{k}) e^{-i(\check{k}, 'x)} + \widehat{a}^\dagger(\check{k}) e^{i(\check{k}, 'x)}) \\ &= \widehat{U}'^{-1*} \widehat{w}^*(x) \widehat{U}'^* \\ &= (2)^{-1/2} (2\pi)^{-d/2} \int d\sigma(\check{k}) ((\widehat{U}'^*{}^{-1} \widehat{a}^*(\check{k}) \widehat{U}'^*) e^{-i(\check{k}, 'x)} + \\ &\quad + \widehat{U}'^*{}^{-1} \widehat{a}^{\dagger*}(\check{k}) \widehat{U}'^*) e^{i(\check{k}, 'x)}. \end{aligned} \quad (\text{A-3 bis. 25})$$

Thus, we have

$$\widehat{U}'^{-1*} \widehat{a}^*(\check{k}) \widehat{U}'^* = \widehat{a}(\check{k}), \quad (\text{A-3 bis. 26})$$

$$\widehat{U}'^{-1*} \widehat{a}^{\dagger*}(\check{k}) \widehat{U}'^* = \widehat{a}^\dagger(\check{k}). \quad (\text{A-3 bis. 26})$$

As the relations

$$[\widehat{a}(\check{k}), \widehat{a}^\dagger(\check{k}')] = \delta(\check{k}, \check{k}'); \quad \delta(\check{k}, \check{k}') = \check{k}^n \delta(\vec{k} - \vec{k}'), \quad (\text{A-3 bis. 27})$$

$$[\widehat{a}(\check{k}), \widehat{a}(\check{k}')] = 0, \quad (\text{A-3 bis. 28})$$

are invariant, if we go over to the conjugate complex operators, an unitary matrix \widehat{U}'^* exists always, satisfying (A-3 bis. 26). In particular, if the annihilation and creation operators are chosen real, $\widehat{U}'^* = 1$ is the unit operator.

References

- 1) E. C. G. STUECKELBERG, *Helv. Phys. Acta* 33, 727 (1960) to be referred to as I.
- 2) R. JOST and M. GUENIN, unpublished.