On quantum theory radiation

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On Quantum theory of Radiation

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This paper is the exact content of an unpublished lecture, given at the Institute for theoretical physics in Copenhagen. It is the wish of many of my colleagues to have it once printed, as its second part has been the suggestion of the final explanation of the last unsolved problem of low energy quantum electrodynamics. The title and the talk were:

Fourth Order Magnetic Moment of the Mu-Meson and Related Problems in Atomic Levels Shifts*)

For the mu-meson, assuming it has a spin 1/2, the magnetic moment is the same, in 4^{th} order as that of the electron, already computed, except for the fact that one has to consider one more term due to the vacuum polarization by electrons during the virtual photon propagation. Its contribution to the magnetic moment is given, in units of $e\,\hbar/2\,m\cdot c$, by the integral

$$\mu_{(Pol)} = \frac{\alpha^2}{\pi^2} \int_0^1 du \int_0^1 dv \, \frac{u^2\,(1-u)\,v^2\left(1-\frac{v^2}{3}\right)}{u^2\,(1-v^2)+\lambda\,(1-u)}\,, \tag{1}$$

where $\lambda = 4\,m_e^2/m_\mu^2$, with m_e and m_μ as the masses of the electron and the mu-meson respectively.

The above integral becomes singular when $\lambda \to 0$ and one can split it in two parts, i.e., one being regular in this limit, the other not. This is written as

$$\mu_{(Pol)} = \frac{\alpha^2}{2\,\pi^2}\,(R+S)\,. \tag{2}$$

The integral R is computed with $\lambda = 0$ since here $\lambda \simeq 10^{-4}$, the error being at most of the order $\lambda^{1/2} \simeq 10^{-2}$, and is trivial.

The integral S is more delicate and is proportional to

$$J = 2\int_0^1 du \int_0^1 dv \, \frac{u^2\,(1-u)\,v^4}{u^2\,(1-v^2)+\lambda\,(1-u)}\,.$$

*) See, in the Annexe C of CERN annual report 1957, the list of the talks of the theoretical division in Copenhagen, section B: Quantum Electrodynamics: A. Peterman (25th February). Fourth order magnetic moment of the mu-meson and related problems in atomic levels shifts.

After the change of variable $r = 1 - v^2$, it is splitted in several parts: the u-integral path: from 0 to $\sqrt{2\lambda}$ and from $\sqrt{2\lambda}$ to 1, then the r-integral path from 0 to 1/2 and from 1/2 to 1 and finally, in the 0 to 1/2 integral, according to the powers of $(1-r)^{3/2}$ after $(1-r)^{3/2}$ has been developped. These various steps are not explicitly given, being straightforward once made. The terms proportional to $Ln\,\lambda$ and the constant terms only are kept; every term proportional to $\lambda^{1/2}$ or to an integer power of it neglected. An estimation (upper bound) of the coefficient of $\lambda^{1/2}$ has been performed with the result that it does not exceed 3 numerically and probably is of order unity.

The J integral comes out with the value

$$J = \frac{1}{2}\,Ln\,\frac{1}{\lambda} + Ln\,2 - \frac{31}{12} + 0(\lambda^{1/2})\,. \tag{3}$$

Finally, after the evaluation of R, $\mu_{(pol)}$, formula (2), gives

$$\mu_{(Pol)} = \frac{\alpha^2}{2\,\pi^2}\,(R+S) = \frac{\alpha^2}{\pi^2}\left[\frac{1}{6}\,Ln\,\frac{1}{\lambda} + \frac{1}{3}\,Ln\,2 - \frac{25}{36} + 0(\lambda^{1/2})\right]$$

and with $\lambda \cong 10^{-4}$, $\mu_{(Pol)}$, formula (1) is

$$\mu_{(Pol)} = \frac{\alpha^2}{\pi^2}\,[1{\cdot}08 + 0(\lambda^{1/2})]\,\frac{e\,\hbar}{2\,m_\mu\,c}\,. \tag{4}$$

If only quantum electrodynamics governs the anomalous magnetic moment of the mu-meson, it yields

$$\mu_{(mu)} = \left[1 + \frac{\alpha}{2\,\pi} - \frac{\alpha^2}{\pi^2}\,1{\cdot}89\right]\frac{e\,\hbar}{2\,m_\mu\,c}\,. \tag{5}$$

The number 1·89 in (5) has been obtained by assuming that the other contributions to $\mu_{(mu)}$, i.e., the numerical value of the electron moment in 4^{th} order (equal to $-\,2{,}97\;\alpha^2/\pi^2$)[1]) is correct. And this introduces the second part of our talk, namely the correctness of the theoretical coefficient $-\,2{,}97$.

Recent measurements at Stanford University have given the value[2])

$$\frac{\mu_{(el)}}{\mu_0} = 1 + \frac{\alpha}{2\,\pi} + (0{,}7 \pm 2{,}0)\,\frac{\alpha^2}{\pi^2} \qquad \text{(limit of error)} \tag{6}$$

for the electron (μ_0 = Bohr magneton), to be compared with the theoretical prediction

$$\frac{\mu_{(el)}}{\mu_0} = 1 + \frac{\alpha}{2\,\pi} - 2{,}97\,\frac{\alpha^2}{\pi^2}\,. \tag{7}$$

This casts a serious doubt concerning the α^2 coefficient in (7), specially if one fits a normal distribution to the result (6) and finds, for the probability for (7) to be correct, the value 0·7‰.

Moreover, the magnetic moment of the electron gives a contribution to the Lamb-shift (energy shift of the $2\,s_{1/2} - 2\,p_{1/2}$ separation) which reads, in units of the Lamb constant $L\,Z^4\,(1 + m_{el}/M_{prot})^{-3}$ ($L = \alpha^3\,R\,y\,d_\infty \cdot c/3\,\pi = 135{\cdot}634$ Mc/sec)

$$4^{th}\ \text{order magn. mom. contribution} = \alpha^2\,(\alpha\,Z)^4\,\frac{\alpha}{\pi}\,(-2{\cdot}97)\,. \tag{8}$$

*) The modification of α due to a change in the anomalous magnetic moment of the order of the difference between (6) and (7) is too small to modify the numerical value of (8).

If, instead of the coefficient $-2,97$, one introduces a modified 4^{th} order coefficient: $(-2,97 + \eta)$, we have got, in collaboration with G. KÄLLEN, the following table

η	δ (Lamb shift) in Mc/sec		Coeff. of α^2/π^2 in the electron moment	
3,0	+0,9	Changes in the Lambshift, according to the new measurements	0,0	New measurements (limit of error)
2,7	+0,8		−0,3	
2,3	+0,7		−0,7	
2,0	+0,6		−1,0	
1,7	+0,5		−1,3	
1,3	+0,4		−1,7	
1,0	+0,3		−2,0	

But, it is well known that the observed Lamb shift and the theoretical value for this shift differ by about $+ 0,6 \pm 0,2$ Mc/sec., both in H and D, and that a explanation of this discrepancy by higher order effects is not possible because it would very much destroy the agreement for the ionized He, problem unsolved for many years, despite many attempts or suggestions. Whereas, according to (8), the agreement is within the errrors for H and D with a modified α^2 coefficient and explicable for He^+ through higher order effects, such as the 3rd order potential interaction.

Now one has to conclude. Without having yet a definite proof that the coefficient $- 2,97$ is incorrect (it will come soon!), we suggest strongly that it is so. The reason for this suggestion is that a change in *one single* theoretical coefficient (never checked) can achieve agreement for *two* effects (namely the magnetic moment and the Lamb shift), between theory and experiment.

As to-day, in 1963, there is no need of comments. The numerics themselves are eloquent enough:

i) the recomputation of the α^2/π^2 coefficient has given the result $-0,3285$ instead of $- 2,973$.

ii) accordingly the electron magnetic moment is theoretical: 1,001159612; experimental[3]) $1,001159622 \pm 0,000000027$

iii) the situation in the Lamb shift is [4]).

Lambshift	H	D	He^+
Theory	$1057,73 \pm 0,22$	$1058,97 \pm 0,22$	$14046,0 \pm 7,0$
Experiment	$1057,77 \pm 0,10$	$1059,00 \pm 0,10$	$14040,2 \pm 4,5$

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