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Phonon-Helicon Interaction in Piezoelectric Semiconductors*)

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Abstract. The interaction between elastic waves propagating in a piezoelectric semiconductor and the cyclotron mode waves (helicons) on drifting carriers in the material is considered. The coupling between the waves has been calculated for cases where the acoustic propagation is parallel to an applied constant magnetic field and in a crystalline direction such that a transverse electric field is generated. In this case a strong interaction takes place when the phase velocities of the two waves are comparable. Coupled mode analysis has been used to calculate expressions for the exponential rate of growth or decay. The magnitude of the attenuation constants found are smaller than for the case of acoustic interaction with the longitudinal space charge waves because the transverse piezoelectric fields are much weaker than the longitudinal ones, but the effect should still be observable. Because the phase velocity of the fast cyclotron wave can be much greater than the carrier drift velocity, smaller *D C* electric drift fields are required than in the longitudinal case. Wave propagation along the *c*-axis of a hexanonal crystal is treated as a particular example, and possible applications for phonon amplification are considered.

I. Introduction

The discovery by Hutson, McFee, and White[1]) of large amplification of ultrasonic waves by means of a *DC* current in a piezoelectric crystal has given rise to widespread interest in techniques for phonon amplification. In the work of Hutson et al. the interaction between the longitudinal space charge waves and the electric field associated with the acoustic wave provided the mechanism for amplification. The theory of the effect was developed by White[2]), and Quate[3]) has given an illuminating coupled mode analysis of the problem. An interaction with other phonon modes is also feasible, and Woodruff[4]) has suggested that conditions for growing waves can be found in the coupled longitudinal space charge waves on drifting carriers and optical phonons in a compound semiconductor. In the present discussion the interaction of elastic waves with a different normal mode on carriers in a solid, the cyclotron or helicon waves, is considered and the nature of the coupling prividing gain or loss is investigated. The problem is formulated in terms of coupled traveling waves[5]), and it is found that conditions for active coupling, resulting in exponentially growing or decaying waves, can be obtained. The effect is much smaller than in the case of the longitudinal space charge waves because the transverse piezoelectric fields are significantly weaker than the longitudinal ones.

Consider the propagation of elastic waves through a piezoelectric crystal in a direction such that a transverse electric field accompanies the wave. For simplicity

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the calculation is carried out for only one type of carrier, uniformly distributed over a large sample. The background medium dielectric properties are represented by an isotropic static dielectric constant, ε. We restrict the treatment to cases where the space charge density is sufficiently low so that magnetic transverse forces are large compared to space charge forces. Note that non-zero space charge arises only from departures of fluctuations from the equilibrium charge distribution.

In order to linearize the equations we assume that any quantity $\Phi_t\,(\boldsymbol{r}, t)$ may be written

$$\Phi_t\,(\boldsymbol{r}, t) = \Phi_0(\boldsymbol{r}) + \Phi(\boldsymbol{r})\, e^{i\omega t}\,. \tag{1}$$

Here $\Phi_0(\boldsymbol{r})$ is the time average of $\Phi_t\,(\boldsymbol{r}, t)$, and we assume $\Phi \ll \Phi_0$; thus the following is a small signal analysis. (We treat the case of a plane wave traveling along the z-axis, so that $\partial/\partial x = \partial/\partial y = 0$.) We first calculate the normal modes on the drifting carriers and for the elastic waves, and then consider the coupled system.

II. Interaction Between Cyclotron Modes and Acoustic Waves

A. *Cyclotron Mode Waves on Drifting Carriers*

In describing the carriers, which are assumed to be giving rise to a DC current, we shall replace the combined effect of the applied electric field and external scattering processes by a constant drift velocity. This approximation is justifiable if the scattering time, τ, is long compared to the characteristic time scales of the oscillatory phenomena to be investigated. We shall further be interested in the region $\omega\,\tau \gg 1$. We shall carry out the calculation for the simplest case, a cold, collision-less plasma, and consider the effects of collisions and thermal velocities in section III.

The classical equation of motion for carriers of charge q and effective mass m is

$$\frac{d\boldsymbol{v}_t}{dt} = \frac{\partial \boldsymbol{v}_t}{\partial t} + (\boldsymbol{v}_t\, \boldsymbol{\nabla})\, \boldsymbol{v}_t = \frac{q}{m}\,(\boldsymbol{E}_t + \boldsymbol{v}_t \times \boldsymbol{B}_t)\,. \tag{2}$$

If we assume a constant drift velocity v_0 in the z-direction and let $\boldsymbol{E}_t = \boldsymbol{E}_0 + \boldsymbol{E}\, e^{i\omega t}$, $\boldsymbol{B}_t = \boldsymbol{B}_0 + \boldsymbol{B}\, e^{i\omega t}$ Equation (2) yields

$$\left(i\,\omega\,\boldsymbol{v} + \boldsymbol{v}_0 \cdot \frac{d\boldsymbol{v}}{dz}\right) e^{i\omega t} = \frac{q}{m}\,(\boldsymbol{E}_0 + \boldsymbol{v}_0 \times \boldsymbol{B}_0) + \frac{q}{m}\,(\boldsymbol{E} + \boldsymbol{v}_0 \times \boldsymbol{B} + \boldsymbol{v} \times \boldsymbol{B}_0)\, e^{i\omega t} + \frac{q}{m}\,(\boldsymbol{v} \times \boldsymbol{B})\, e^{i2\omega t}. \tag{3}$$

Since the only magnetic field present is applied parallel to $\boldsymbol{v}_0$, and since the carriers are drifting (equivalent to $E_0 = 0$) the time independent term in Equation (3) vanishes. The term in $e^{i2\omega t}$ is of second order in the small AC amplitudes and is neglected. It is convenient to write (3) in terms of the circularly polarized amplitudes $a_\pm = A_\pm\,(v_x \pm i\,v_y)$. Combining the components of (3) then yields

$$\left[\frac{d}{dz} + i\,(\beta_e \pm \beta_c)\right] a_\pm = \frac{A_\pm\, q}{m\, v_0}\,(E_\pm \pm i\, v_0\, B_\pm). \tag{4}$$

Here $E_\pm = E_x \pm i\,E_y$, $B_\pm = B_x \pm i\,B_y$, $\beta_e = (\omega/v_0)$, $\beta_c = \omega_c/v_0 = (q\,B_0/m\,v_0)$, and $A_\pm$ is a normalizing constant. Note that the cyclotron angular frequency, ω_c, is negative for electrons and positive for holes. $E_\pm$ and $B_\pm$ are 'applied' fields which in the case of interest here are generated by elastic waves via the piezoelectric effect. For $E_\pm = B_\pm = 0$ Equation (4) is the normal mode equation for the cyclotron modes. AIGRAIN[6]) has proposed the term 'helicons' for these excitations which have been observed by BOWERS[7]) et al. in metals. This is also the mode of 'whistler' propagation through the ionosphere[8]). The two normal mode solutions $a_\pm$ have opposite circular polarizations and travel with phase velocities

$$v_p^\mp = \frac{v_0}{1 \pm \omega_c/\omega} . \tag{5}$$

Thus one mode propagates faster than the drift velocity, and the other mode slower than the drift velocity.

The terms on the right hand side of (4) provide the coupling with the elastic waves.

B. *Acoustic Wave Propagation in a Piezoelectric Medium*

The equation of motion for an element of volume $dx\ dy\ dz$ and density ϱ is

$$(\varrho\ dx\ dy\ dz)\ \frac{d^2 h_i}{dt^2} = F_i = \frac{\partial T_{ij}}{\partial x_j}\ dx\ dy\ dz\ . \tag{6}$$

Here h_i is the particle displacement along the i-th axis and T_{ij} is an element of the stress tensor. In a piezoelectric material the stress is given by

$$T_{ij} = c_{ijkl}\, S_{kl} - e_{mij}\, E_m\ , \tag{7}$$

where c_{ijkl} and e_{mij} are elements of the elastic and piezoelectric tensors respectively. The strain S is defined as

$$S_{ij} = \frac{1}{2}\left(\frac{\partial h_i}{\partial x_j} + \frac{\partial h_j}{\partial x_i}\right).$$

Equations (6) and (7) may be expressed in terms of the particle velocities, $u_i = \partial h_i/\partial t$, as follows:

$$\varrho\ \frac{du_i}{dt} = \frac{\partial T_{ij}}{\partial x_i}\ , \tag{8}$$

$$\frac{\partial T_{ij}}{\partial t} = \frac{1}{2}\ c_{ijkl}\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right) - e_{mij}\ \frac{\partial E_m}{\partial t}\ . \tag{9}$$

Equations (9) and (10) must be combined in order to express the problem in normal mode form. Rather than continue with a general treatment it is more illustrative to restrict the discussion at this point to a particular case.

Consider a circularly polarized plane shear wave propagating along the axis of symmetry of a hexagonal crystal (such as cadmium sulfide) with symmetry 6 mm. For this class of crystal the non-zero elements of the elastic tensor c and the piezoelectric tensor, e, are, using Voigt notation, $c_{11} = c_{22}$, c_{33}, $c_{44} = c_{55}$, $c_{66} = {}^1/_2\ (c_{11} - c_{12})$, $c_{12} = c_{21}$, $c_{13} = c_{31} = c_{23} = c_{32}$ and $e_{15} = e_{24}$, $e_{31} = e_{32}$, e_{33}.

For a plane shear wave propagated in the z-direction $\partial/\partial x = \partial/\partial y = 0$ and $u_z = 0$. In this case Equations (8) and (9) take on a simple form. In the absence of the piezoelectric effect the normal modes will be linear combinations of $u_\pm = u_x \pm i\, u_y$ and $T_\pm = T_5 \pm i\, T_4$. The appropriate combinations are

$$f_\pm = F_\pm \left(T_\pm - \sqrt{c_{44}\, \varrho}\, u_\pm \right), \tag{10}$$

$$b_\pm = B_\pm \left(T_\pm + \sqrt{c_{44}\, \varrho}\, u_\pm \right). \tag{11}$$

$F_\pm$ and $B_\pm$ are normalizing constants. In terms of $f_\pm$ and $b_\pm$ Equations (8) and (9) become

$$\left(\frac{d}{dz} + i\, \beta_t \right) f_\pm = - i\, F_\pm\, e_{15}\, \beta_t\, E_\pm , \tag{12}$$

$$\left(\frac{d}{dz} - i\, \beta_t \right) b_\pm = - i\, B_\pm\, e_{15}\, \beta_t\, E_\pm , \tag{13}$$

when $e_{15} = 0$ (no piezoelectric effect) the solutions $f_\pm$ and $b_\pm$ correspond to circularly polarized traveling waves traveling in the forward (positive z) and backward directions respectively. The phase velocity of these transverse waves is u_t, and the propagation constant is β_t, where

$$\beta_t \equiv \frac{\omega}{\sqrt{c_{44}/\varrho}} = \frac{\omega}{u_t} . \tag{14}$$

C. *Interaction Between Cyclotron Mode Waves on Drifting Carriers and Elastic Waves*

The terms in $E_\pm$, $B_\pm$ in Equation (4) and in $E_\pm$ in Equations (12) and (13) provide the coupling between the drifting carriers and the lattice waves. In order to formulate the coupled mode theory of the interaction each of these terms must be expressed in terms of the uncoupled normal mode amplitudes $a_\pm$, $f_\pm$, and $b_\pm$. Thus the effect of the coupling between the carriers and the lattice may be viewed as a perturbation on the motion of the two isolated systems considered separately. For the present consider only the forward traveling acoustic wave $f_\pm$ and its interaction with the cyclotron modes $a_\pm$. Since only E_+ or B_+ appear in the equations for f_+ and a_+, and only E_- or B_- appear in equations for f_- and a_-, it is clear that a_+ couples to f_+ and a_- couples to f_-, as one would expect intuitively.

Consider first Equation (4). We must calculate $E_\pm$ and $B_\pm$, which are generated by an elastic wave in a piezoelectric crystal. From MAXWELL's equation, $\boldsymbol{\nabla} \times \boldsymbol{E} = - \partial \boldsymbol{B}/\partial t$ we obtain

$$B_\pm = \mp \frac{1}{\omega} \frac{\partial E_\pm}{\partial z} . \tag{15}$$

This relation can be used to eliminate $B_\pm$ from Equation (4). However, it is necessary to express $E_\pm$ in terms of $f_\pm$, the uncoupled acoustic wave amplitude. To do this use can be made of the piezoelectric equation of state, Equation (7). Strictly speaking this equation is valid only for an insulator, but for low conductivities it may be used as an approximation. The electric displacement D is given by

$$D_n = e_{nij}\, S_{ij} + \varepsilon_{mn}\, E_n . \tag{16}$$

Combining MAXWELL's equations $\boldsymbol{\nabla} \times E = - \partial B/\partial t$ and $\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0\, (\partial \boldsymbol{D}/\partial t)$ neglecting conduction currents, gives

$$- \frac{\partial^2 E_\pm}{\partial z^2} = \mu_0\, \omega^2\, D_\pm . \tag{17}$$

For the case of symmetry 6 mm Equation (16) becomes

$$D_{\pm} = e_{15}\, S_{\pm} + \varepsilon\, E_{\pm}\,. \tag{18}$$

Here ε is the transverse dielectric constant. $S_{\pm}$ may be eliminated from this equation by use of (7), which takes the form

$$T_{\pm} = c_{44}\, S_{\pm} - e_{15}\, E_{\pm}\,, \tag{19}$$

where $S_{\pm} = S_5 \pm i\, S_4$. Combining Equations (17), (18), and (19) gives the relationship between $T_{\pm}$ and $E_{\pm}$:

$$-\frac{\partial^2 E_{\pm}}{\partial z^2} = \frac{\mu_0\,\omega^2\, e_{15}}{c_{44}}\,(T_{\pm} + e_{15}\, E_{\pm}) + \mu_0\,\omega^2\,\varepsilon\, E_{\pm}\,. \tag{20}$$

If we anticipate a traveling wave solution, $e^{-i\gamma z}$, for $E_{\pm}$ Equation (20) can be simplified, and one obtains

$$E_{\pm} = \frac{\dfrac{\mu_0\,\omega^2\, e_{15}}{c_{44}}}{\gamma^2 - \mu_0\,\omega^2\,\varepsilon\left(1 + \dfrac{e_{15}{}^2}{\varepsilon\, c_{44}}\right)}\, T_{\pm}\,. \tag{21}$$

Equations (15) and (21) can be used to write Equation (4) in terms of $f_{\pm}$ by requiring, as an initial condition, that the backward elastic wave, $b_{\pm}$, is not excited.

Then

$$b_{\pm} = B_{\pm}\left(T_{\pm} + \sqrt{c_{44}\,\varrho}\; u_{\pm}\right) = 0 \tag{22}$$

and

$$f_{\pm} = F_{\pm}\left(T_{\pm} - \sqrt{c_{44}\,\varrho}\; u_{\pm}\right) = 2\, F_{\pm}\, T_{\pm}\,. \tag{23}$$

Equations (15), (21), and (23) can now be substituted in (4) to give the desired form, an equation in only $a_{\pm}$ and $f_{\pm}$:

$$\left[\frac{d}{dz} + i\,(\beta_e \pm \beta_c)\right] a_{\pm} = \frac{A_{\pm}}{F_{\pm}}\,\frac{q}{m\, v_0}\,\frac{k^2}{e_{15}}\,\frac{1 - \dfrac{v_0}{\omega}\,\gamma}{\left(\dfrac{\gamma}{\gamma_0}\right)^2 - (1 + 2\, k^2)}\,, \tag{24}$$

where

$$2\, k^2 = \frac{e_{15}{}^2}{\varepsilon\, c_{44}}\,,\quad \gamma^2 = \mu_0\,\varepsilon\,\omega^2\,.$$

Next it is necessary to express $E_{\pm}$ in Equation (12) in terms of $a_{\pm}$. This electric field is that acting on the elastic medium due to the carriers. If $E'_{\pm}$ is the field that would give rise to the current density $J_{\pm}$, the field exerted on the lattice is $E_{\pm} = -\, E'_{\pm}$. From MAXWELL's equations, $\boldsymbol{\nabla}\times\boldsymbol{H} = \boldsymbol{J} + \partial\boldsymbol{D}/\partial t$ and $\boldsymbol{\nabla}\times\boldsymbol{E} = -\,\mu_0\,\partial\boldsymbol{H}/\partial t$ we obtain a relation between $J_{\pm}$ and $E'_{\pm}$. In these equations the conduction current is retained, even though small, since it is the quantity in terms of which $E_{\pm}$ must be expressed. Combining MAXWELL's equations gives

$$\frac{\partial^2 E'_{\pm}}{\partial z^2} = i\,\omega\,\mu_0\, J_{\pm} - \omega^2\,\mu_0\,\varepsilon\, E'_{\pm}\,. \tag{25}$$

Again anticipating a traveling wave solution, $e^{-i\gamma z}$ for $E_\pm$, yields

$$E_\pm = \frac{i\,\omega\,\mu_0}{\omega^2\,\mu_0\,\varepsilon - \gamma^2}\,J_\pm = \frac{i}{\left[\left(\frac{\gamma}{\gamma_0}\right)^2 - 1\right]}\,\frac{\varrho_0}{\omega\,\varepsilon\,A_\pm}\,a_\pm\,. \tag{26}$$

Since $J_\pm = \varrho_0\,u_\pm = \varrho_0\,a_\pm/A_\pm$ where ϱ_0 is the carrier charge density. Upon substitution of (26) into (12) now one obtains

$$\left(\frac{d}{dz} + i\,\beta_t\right) f_\pm = \frac{F_\pm}{A_\pm}\,\frac{e_{15}\,\varrho_0\,\beta_t}{\omega\,\varepsilon\left[\left(\frac{\gamma}{\gamma_0}\right)^2 - 1\right]}\,a_\pm\,. \tag{27}$$

Equations (15) and (30) are now in the desired coupled mode form:

$$\left[\frac{d}{dz} + i\,(\beta_e \pm \beta_c)\right] a_\pm = \lambda_1\,f_\pm\,, \tag{28}$$

$$\left[\frac{d}{dz} + i\,\beta_t\right] f_\pm = \lambda_2\,a_\pm\,, \tag{29}$$

where

$$\lambda_1 = \frac{A_\pm\,q\,k^2}{F_\pm\,m\,v_0\,e_{15}}\left[\frac{1 - \frac{v_0}{\omega}\,\gamma}{\left(\frac{\gamma}{\gamma_0}\right)^2 - (1 + 2\,k^2)}\right]$$

and

$$\lambda_2 = \frac{F_\pm\,e_{15}\,\varrho_0\,\beta_t}{A_\pm\,\omega\,\varepsilon\left[\left(\frac{\gamma}{\gamma_0}\right)^2 - 1\right]}\,.$$

We look for solutions of (28) and (29) of the form $e^{-i\gamma z}$. Since all of the amplitude functions are multiplied by a time factor $e^{i\omega t}$ the solutions are waves traveling in the positive z-direction. An equation for γ is obtained from (28) and (29)

$$[\gamma - (\beta_e \pm \beta_c)]\,[\gamma - \beta_t] = -\,\lambda_1(\gamma)\,\lambda_2(\gamma)\,. \tag{30}$$

In general γ will be complex, $\gamma = Q - i\,\alpha$. We shall consider cases where the attenuation constant is small compared to the wave number Q. Thus $|\,\alpha\,| \ll Q$, and $Q \simeq \omega/v$, where v is the phase velocity of the composite wave. In this approximation

$$\lambda_1\,\lambda_2 = \frac{\varrho_0\,q\,k^2}{m\,\varepsilon\,v_0\,u_t}\left(\frac{\gamma_0}{\gamma}\right)^4\left(1 - \frac{\gamma}{\beta_e}\right). \tag{31}$$

As a first approximation let $\gamma = \omega/v$ on the right hand side of Equation (33) and solve the resulting quadratic expression for γ. The two roots γ_1 and γ_2 are found to be

$$\gamma_1 = \frac{\beta_t + \beta_e \pm \beta_c}{2} + i\,\sqrt{\lambda_1\,\lambda_2 - \left[\frac{\beta_t - (\beta_e \pm \beta_c)}{2}\right]^2}\,, \tag{32}$$

$$\gamma_2 = \frac{\beta_t + \beta_e \pm \beta_c}{2} - i\,\sqrt{\lambda_1\,\lambda_2 - \left[\frac{\beta_t - (\beta_e \pm \beta_c)}{2}\right]^2}\,, \tag{33}$$

when the square root in these expressions is real γ_1 corresponds to an exponentially growing wave and γ_2 to an exponentially damped wave. If $\lambda_1 \lambda_2$ is positive the square root will always be real when $\beta_t = \beta_e \pm \beta_c$. This condition may be written

$$u_t = \frac{v_0}{1 \pm \frac{\omega_c}{\omega}}. \tag{34}$$

This is what one would expect intuitively, since $v_0/1 \pm (\omega_c/\omega)$ is just the phase velocity of the fast (+ sign) or slow (– sign) cyclotron wave. When this phase velocity is equal to the elastic wave velocity conditions are favorable for a strong interaction.

Consider the case when Equation (34) is satisfied. From Equation (31) it is seen that $\lambda_1 \lambda_2 > 0$ when $\gamma/\beta_e < 1$, or $v_0 < v = u_t$. This condition is satisfied by the fast cyclotron wave, where $v_p = v_0/1 + (\omega_c/\omega)$ for negative carriers from Equation (5). In this case the gain, or negative attenuation constant, is

$$-\alpha = \omega_p \left(\frac{u_t}{c'}\right)^2 \sqrt{\frac{k^2}{v_0 u_t}\left(1 - \frac{v_0}{u_t}\right)}, \tag{35}$$

where $\omega_p^2 = \varrho_0 q/m \varepsilon$ and c' is the velocity of light in the medium.

III. Discussion

The negative attenuation constant calculated in Equation (38) is seen to involve a factor $(u_t/c')^2$ which reduces its magnitude appreciably below the values calculated for interactions involving longitudinal effects. This reduction is characteristic of transverse waves. For example, for values of the constants in Equation (38) of

$$u_t = 2 \times 10^3 \text{ m/sec}, c' = 10^8 \text{ m/sec}, k^2 = 0.01, v_0 = 2 \times 10^2 \text{ m/sec}, \omega_p = 10^{14}/\text{sec}$$

one finds $-\alpha \simeq 6$/meter. The frequency enters only by virtue of the fact that Equation (37) must be satisfied. Hence for a given drift velocity higher frequencies require higher cyclotron frequencies, and hence larger magnetic fields. The variation of $-\alpha$ as a function of drift velocity at a fixed frequency is indicated in Figure 1. If the frequency is varied and v_0 is continuously adjusted to satisfy (37) the behavior shown in Figure 2 is obtained.

The carrier-lattice interaction can be included in the analysis by means of a collision time approximation. The effect of this is to replace ω by $(\omega + i/\tau)$ in the equations of motion. In the region of long scattering times this will not qualitatively alter the resultant behavior.

In order to take account of the thermal motion it is necessary to average Equation (35) over the appropriate velocity distribution. At low carrier concentrations and not too low temperatures the distribution may be taken as a Maxwellian, displaced corresponding to the drift velocity in the z-direction. The thermal velocity in the direction of the magnetic field will have the effect of limiting the magnitude of the interaction at low drift velocities or as thereby removing the asymptotic divergencies indicated in Figures 1 and 2.

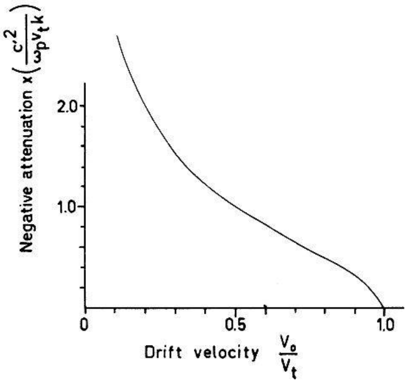

Figure 1

Negative attenuation, $-\alpha$, as a function of drift velocity, v_0, for fixed frequency (v_t is the sound velocity for transverse waves).

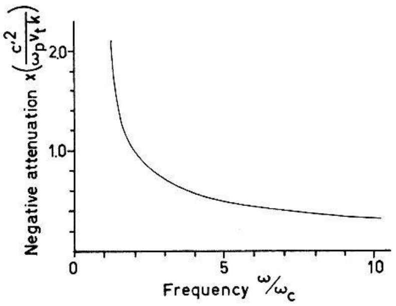

Figure 2

Negative attenuation, $-\alpha$, as a function of frequency with optimum drift velocity (ω_v is the cyclotron frequency).

The external electric field which gives rise to the carrier drift velocity can have a direct effect on the helicon dispersion relations. However, this effect will not be large provided the fractional energy gained by the particles from the external field in one wavelength is not large.

In the linearized theory used here the transverse fields do not, in the first approximation, alter the parallel drift velocity, and hence spatial bunching does not occur. Thus the effects of carrier diffusion, which are so important in the case of the longitudinal space charge waves, do not enter in the same sense here. Thermal motion is, of course, still important, but the interaction is dependent primarily on a bunching of carriers in phase, rather than in space.

Because of the reduction of the piezoelectric effects by a factor of $(u/c')^2$ with respect to the longitudinal effects, the helicon-phonon may not prove promising as a practical way of achieving phonon amplification. One of the major problems in work with microwave ultrasonics has been the very high transduction losses encountered with conventional piezoelectric or magnetostrictive techniques at high frequencies. It is possible that the phenomena discussed here may be useful as a means of phonon generation at ultramicrowave frequencies. As is the case for waves interacting with the longitudinal space-charge waves, the gain and loss here for forward and backward traveling waves is not reciprocal, and oscillations can build up when the background losses are not too large.

The present discussion has been limited to an interaction between the carriers and the lattice via the piezoelectric effect. However, in non-piezoelectric materials, such as the semi-metals, similar interactions with cyclotron modes may occur via the deformation potential. Dumke and Haering[10]) have considered the case of crossed E and H fields in semi-metals and find that significant amplification should be obtainable by phonon interaction with the space-charge waves via the deformation potential.

Similarly, in the case of optical phonons in a compound semiconductor a strong interaction with the cyclotron waves may exist.

As a means of studying basic crystal properties the process described may make it possible to observe phenomena such as acoustic cyclotron resonance under circumstances which would not be favorable for conventional techniques. Experimentally it may be much easier to detect the increase in ampltitude of a low level elastic wave which occurs when the cyclotron resonance conditions are satisfied and the carriers give up energy to the lattice, rather than look for the small fractional increase in attenuation at cyclotron resonance that occurs in the conventional arrangement with no *DC* electric field applied.

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