**Zeitschrift:** Helvetica Physica Acta
**Band:** 37 (1964)
**Heft:** VII-VIII

**Artikel:** Operational meaning of higher-order optical coherence

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**DOI:** https://doi.org/10.5169/seals-113513

# Operational Meaning of Higher-Order Optical Coherence

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(26. IX. 1964)

*Abstract.* The definition of higher-order optical coherence given by Glauber is interpreted in terms of the moments of the joint probability distribution of the numbers of photons to be counted in different modes. This makes it clear why normal-ordered operators need to be used for the former while not for the latter.

We consider coherence of light between modes of the radiation field whose harmonic oscillator annihilation and creation operators are $a_i$ and $a_i^+$. When expressed in terms of modes rather than space-time points, the definition given by GLAUBER[1]) is that light is coherent to the $n^{th}$ order if for each mode $i$ there is a complex number $z_i$ such that

$$\langle a_{i_1}^+ \dots a_{i_m}^+ a_{i_{m+1}} \dots a_{i_{2m}} \rangle = z_{i_1}^* \dots z_{i_m}^* z_{i_{m+1}} \dots z_{i_{2m}} \tag{1}$$

for every $m \leqslant n$. We may be interested in coherence between modes characteristic of the light source or of devices such as slits which single out various parts of the beam. Coherence to the $n^{th}$ order over a complete set of modes is nevertheless independent of the choice of modes.

We consider photons counted in modes whose annihilation and creation operators are $b_r$ and $b_r^+$. These modes are characteristic of the detectors. Their annihilation operators are complex linear combinations

$$b_r = \sum_i u_{ri}\, a_i \tag{2}$$

of the annihilation operators for the source or slit modes. The moments of the joint probability distribution of the numbers $n_r$ of photons to be counted in the detection modes $r$ are expectation values of products of the operators $b_r^+ b_r$[2]). For example

$$\overline{n_r n_s} = \langle b_r^+ b_r b_s^+ b_s \rangle = \delta_{rs} \langle b_r^+ b_r \rangle + \langle b_r^+ b_s^+ b_r b_s \rangle$$
$$= \delta_{rs} \sum_{ij} u_{ri}^* u_{rj} \langle a_i^+ a_j \rangle + \sum_{ijkm} u_{ri}^* u_{sj}^* u_{rk} u_{sm} \langle a_i^+ a_j^+ a_k a_m \rangle .$$

We go to normal-ordered operators, expand in the operators $a_i$ and $a_i^+$, and include only $a_i$ and $a_i^+$ for excited modes.

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When light is coherent to the $n^{th}$ order, the moments of order $\leqslant n$ of the joint probability distribution of the numbers $n_r$ of photons to be counted in detection modes $r$ are the same as for a classical field[2]) with amplitude coefficients $z_i$ in the modes $i$. This property is sufficient to establish $n^{th}$ order coherence if the expectation values (1) can be determined by measuring the joint moments of the $n_r$ for different choices of detection modes $r$ and for different combinations of the modes $i$ excited.

By considering photons counted in modes $r$ that are different from the modes $i$ between which we consider coherence, and by using the expansion (2), we get the joint moments of the $n_r$ to involve all the expectation values (1), not only those having indices $i_1 \ldots i_m$ paired with $i_{m+1} \ldots i_{2m}$ which are joint moments of the numbers of photons to be counted in the modes $i$. Using normal-ordered operators allows us to include just the $a_i$ and $a_i^+$ for excited modes. For the example of light coming through a system of slits, we include only $a_i$ and $a_i^+$ for modes corresponding to the open slits and omit the $a_i$ and $a_i^+$ for modes that are blocked off. Commutator terms from normal-ordering in the $n^{th}$ order joint moments of the $n_r$ bring in expectation values (1) for $m < n$.

This is analogous to the definition of first order coherence in the classical theory[3]) where the mutual coherence function for two spacetime points is related by the linear propagation of the field to the intensities at different points.

I am grateful to F. Ghielmetti and E. C. G. Sudarshan for helpful discussions and to Professor Mercier and the staff of the Institut für theoretische Physik, Universität Bern for their hospitality.

## References

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