On the phenomenological description of CP violation for K-mesons and its consequences

Autor(en): **Enz, C.P. / Lewis, R.R.**

Objekttyp: **Article**

Zeitschrift: **Helvetica Physica Acta**

Band (Jahr): **38 (1965)**

Heft VIII

PDF erstellt am: **13.09.2024**

Persistenter Link: https://doi.org/10.5169/seals-113624

On the Phenomenological Description of *CP* Violation for *K*-mesons and its Consequences

by **C. P. Enz** and **R. R. Lewis** *)**)

Institut de Physique Théorique, Université de Genève

(16. IX. 1965)

Résumé. Récemment une violation de CP a été mise en évidence par la découverte du mode de décomposition $\pi^+ + \pi^-$ du méson K à longue vie. Ce cas de violation de CP dans le système K^0, $\overline{K^0}$ est décrit phénoménologiquement par deux paramètres complexes, l'un mesurant la non-conservation de T, l'autre la non-conservation de CPT dans la matrice d'énergie. Le but de ce travail est de discuter en détail de ces paramètres et d'examiner les possibilités expérimentales de les déterminer. Des propositions d'expériences sont faites dans les domaines suivants: (i) interactions fortes (mesure de la phase de l'ocillation dans l'intensité), (ii) désintégrations (comparaison de l'oscillation dans le taux de désintégration pour un K^0 initial et un $\overline{K^0}$ initial), (iii) régénération cohérente dans la matière (mesure du taux de désintégration en fonction de la densité), iv) production de paires $K\,\overline{K}$ (détection de décompositions ou de réactions provenant de paires initiales $K\,\overline{K}$ à un moment cinétique pair, produites, par exemple, par des résonances).

Introduction

The recent discovery[1]) of the existence of a long-lived $\pi^+\pi^-$-decay mode with the mass and lifetime of the K_2^0, has seriously challenged the previous belief in exact CP-invariance. Since the original formulation of the theory of neutral kaons[2]) was based on this invariance, it has been necessary to reexamine the status of symmetry breaking interactions, especially for the K^0, $\overline{K^0}$ system[3]). Assuming that there are only two particles involved[4]), the phenomenological description of their behavior rests entirely on the energy matrix; the parameters necessary to specify this matrix constitute the observable properties of the system. It is the main purpose of this paper to present a careful discussion of these parameters, and to examine the experimental possibilities of determining them. Some of our proposals have been briefly presented elsewhere[5]).

If CP-invariance is assumed, there are only three observable quantities to determine: the widths and the mass difference of the two CP eigenstates, K_1^0 and K_2^0. If this symmetry is broken in such a way that TCP-invariance is preserved, as is the case in most field theoretic models[6]), then the energy matrix contains one additional complex parameter (r) which determines the admixture of K^0 and $\overline{K^0}$ in the 'mass eigenstates'. At this phenomenological level, it is also possible to describe a violation of TCP-invariance, whether by an intrinsic interaction or by a static external field[7]).

---

*) Permanent address: University of Michigan, Ann Arbor, Michigan, USA.

**) Supported in part by the Swiss National Fund.

Another independent complex parameter (ϱ) is thereby introduced. Determination of these parameters is therefore equivalent to establishing the validity of T, CP, and TCP for this system. This is not necessarily the same goal as determining the specific dynamical origin of the symmetry-breaking, since these parameters are defined solely by the total energy matrix, rather than by the decay amplitudes for any particular channel[8]). However, the determination of r and ϱ is an important step in discovering the origin of the CP-violation. One of the contributions of this work is a general discussion of the limitations imposed on the measurability of the energy matrix, and of the physical interpretation of the 'mass eigenstates'.

The main results presented are several proposals for experiments which could determine the parameters r and ϱ. We show that by using strong interactions to produce and detect kaons of definite strangeness (see *strong interactions of K-mesons*) one can measure the phase of the small oscillatory term, either in the total intensity (K^0 *and* $\overline{K}^0$) of an initially pure beam (K^0 *or* $\overline{K}^0$), or in the partial intensity (K^0 *or* $\overline{K}^0$) of an initially even mixture (K^0 *and* $\overline{K}^0$). The phase difference between the cases of TCP-conservation and of T-conservation is π, and the oscillations determine Re $(1 - r)$ in the first case and Im $(1 - \varrho)$ in the second. Similar expressions are derived for the decay rates for single channels, in particular for $2\,\pi$-decay (see *decays of K-mesons*). Here a comparison of the oscillations in the decay rates for initial K^0 and for initial $\overline{K}^0$ may also yield a test of TCP-versus T-conservation. In the case of TCP-conservation the oscillations have exactly opposite phase but slightly different amplitude, in the case of T-conservation they have exactly equal amplitude but a phase difference which is slightly different from π. In the case of coherent regeneration in matter (see *coherent effects in media*) we show that by measuring the decay rate for a particular channel as a function of density the phase of the decay amplitude may be determined. Finally we examine the possibility of obtaining information on r and ϱ from an investigation of $K\,\overline{K}$ states (see *kaon pair states*). We find that useful information with respect to TCP may be obtained from initial pair states of *even* angular momentum. One possibility here might be the detection of decay or reaction products of pairs from $K\,\overline{K}$ decays of resonances of even spin.

Form of the Energy Matrix

The propagation of a single free kaon can be described[2]) by the introduction of the two by two energy matrix of a meson at rest, Λ, with the equation of motion

$$i\,\frac{\partial}{\partial t}\,\psi(t) = \Lambda\,\psi(t)\,, \tag{1}$$

where t is the proper time*). In the absence of weak interactions, the conservation of strangeness and invariance under TCP (or CP or T) implies that the two particles

*) An alternative to this single particle formalism is the field theoretic approach, in which the K-meson propagator plays the central role rather than the energy matrix; see R. G. Sachs, Reference 15. We consider the use of single particle states more familiar and elementary. But we also recognize that some nettlesome questions are more readily understood using a formalism in which the kaons are only virtual. Example: How can a long-lived K_2^0 with well defined momentum, scatter from a crystal with zero momentum transfer and emerge as a K_1^0 with identical momentum but different mass? (coherent regeneration).

are stable and degenerate and that they have definite strangeness $S = \pm 1$. These states $| K^0 \rangle$ and $| \overline{K}^0 \rangle$ provide a standard representation for the subsequent calculations. The effect of the weak interactions, which may break each of these symmetries, is to destroy the degeneracy and the stability of these states. This can be described by the introduction of both a (hermitian) mass matrix $\boldsymbol{M}$ and a (hermitian) damping matrix $\boldsymbol{\Gamma}$, which are the hermitian and anti-hermitian parts of the energy matrix, $\boldsymbol{\Lambda} \equiv \boldsymbol{M} - i/2\, \boldsymbol{\Gamma}$. A suitable formalism for studying the form of $\boldsymbol{\Lambda}$ is provided by the non-relativistic perturbation theoretic result of WEISSKOPF-WIGNER[9]).

$$\Lambda_{S S'} = M\, \delta_{S S'} + \langle S \mid \mathcal{H}_W \mid S' \rangle + \sum_n \frac{\langle S \mid \mathcal{H}_W \mid n \rangle \langle n \mid \mathcal{H}_W \mid S' \rangle}{M - E_n + i\,\varepsilon} + \cdots$$

$$= \langle S \mid M + \mathcal{H}_W + \mathcal{H}_W \left[P \frac{1}{M - \mathcal{H}_S} - i\,\pi\,\delta(M - \mathcal{H}_S) \right] \mathcal{H}_W + \cdots \mid S' \rangle, \tag{2}$$

where $\mathcal{H}_W$ and $\mathcal{H}_S$ are the hamiltonians of the weak and strong interactions. The intermediate states $| n \rangle$ are understood to be eigenstates of the strong interactions, thus including the effect of final state interactions. If there is an external field present, it is to be included in $\mathcal{H}_W$.

It will be important for the subsequent discussion that the choice of the many arbitrary phase factors be made unambiguously (and intelligently!). We will choose the phases of the states so that the representation is *real*,

$$T \mid K^0 \rangle = \mid K^0 \rangle, \quad T \mid \overline{K}^0 \rangle = \mid \overline{K}^0 \rangle, \tag{3a}$$

which satisfies $T^2 = 1$. This does *not* fix the arbitrary phase in the definition of T for the kaon *field*. In addition we will define CP so that

$$CP \mid K^0 \rangle = \mid \overline{K}^0 \rangle, \quad CP \mid \overline{K}^0 \rangle = \mid K^0 \rangle, \tag{3b}$$

satisfying $(CP)^2 = 1$ and $[CP, T] = 0$. For the kaon *field* this commutativity restricts the phases of T and CP in such a way that their product is ± 1. If we represent $| K^0 \rangle$ by $\left| \begin{smallmatrix} 1 \\ 0 \end{smallmatrix} \right|$ and $| \overline{K}^0 \rangle$ by $\left| \begin{smallmatrix} 0 \\ 1 \end{smallmatrix} \right|$, then we must represent the strangeness operator by σ_3, CP by σ_1, and T by complex conjugation. In this standard representation, the form of $\boldsymbol{\Lambda}$ can easily be discovered once the symmetries of $\mathcal{H}_W$ and $\mathcal{H}_S$ are given. Of course, other choices of phases of the representation can be made[8]); but we must not lose sight of the fact that this implies other phases of the off-diagonal matrix elements of $\boldsymbol{\Lambda}$. The specific advantage of choosing a real representation is that an operator which commutes with T has *real* matrix elements, while an operator which anticommutes with T has *pure imaginary* matrix elements.

If CP is strictly conserved, then $\boldsymbol{\Lambda}$ must have the form

$$\boldsymbol{\Lambda}^{(0)} = \begin{vmatrix} \Lambda^{(0)}_{1,1} & \Lambda^{(0)}_{1,-1} \\ \Lambda^{(0)}_{1,-1} & \Lambda^{(0)}_{1,1} \end{vmatrix} \tag{4}$$

with four real parameters, which define the two complex eigenvalues

$$\lambda_1 = \Lambda^{(0)}_{1,1} + \Lambda^{(0)}_{1,-1} = m_1 - i\,\gamma_1/2\,, \quad \lambda_2 = \Lambda^{(0)}_{1,1} - \Lambda^{(0)}_{1,-1} = m_2 - i\,\gamma_2/2\,. \tag{5}$$

The corresponding CP eigenstates

$$| K_1^0 \rangle = (| K^0 \rangle + | \overline{K}^0 \rangle)/\sqrt{2}, \quad | K_2^0 \rangle = (|K^0 \rangle - | \overline{K}^0 \rangle)/\sqrt{2} \tag{6}$$

are independent of the energy matrix elements. Since $\boldsymbol{M}^{(0)}$ and $\boldsymbol{\Gamma}^{(0)}$ commute, the transformation of $\boldsymbol{\Lambda}^{(0)}$ to diagonal form can be made by a *unitary* matrix

$$\boldsymbol{U} = \frac{1}{\sqrt{2}} \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} . \tag{7}$$

The experimental determination of the parameters $m_1 - m_2, \gamma_1$, and γ_2 has required nearly a decade of effort; it is well established[10]) that the K_1^0 has a larger width ($\gamma_1 = 1{,}09 \pm 0{,}02 \cdot 10^{10}$ sec^{-1}) than the K_2^0 ($\gamma_2 = 1{,}78 \pm 0{,}21 \cdot 10^7$ sec^{-1}), and that the mass difference is comparable to γ_1. The precise value of $| m_1 - m_2 |$ is still subject to some discrepancy[11]) between different determinations, but appears to settling down to a value[12]) $| m_1 - m_2 | = (0{,}5 \pm 0{,}1) \cdot \gamma_1$; the sign is reported to be negative $(m_2 > m_1)$[13]). We can conclude, subject to small corrections due to symmetry-breaking contributions, that $\Gamma_{1,1}$, $\Gamma_{1,-1}$, and $-M_{1,-1}$ are all positive and of comparable magnitude.

Since CP-invariance implied symmetry of $\boldsymbol{\Lambda}$ both along and across the diagonal ($\boldsymbol{\Lambda}^{(0)} = \Lambda_{1,1}^{(0)} \cdot 1 + \Lambda_{1,-1}^{(0)} \cdot \sigma_1$), symmetry-breaking interactions will add terms either antisymmetric along the diagonal ($\sim \sigma_3$) or across the diagonal ($\sim \sigma_2$). The first of these violates TCP-invariance and preserves T-invariance; the second violates T and preserves TCP. In each case, two additional real parameters are added to $\boldsymbol{\Lambda}$. By straight-forward solution of the secular equations for the general case

$$\boldsymbol{\Lambda} = \begin{vmatrix} \Lambda_{1,1} & \Lambda_{1,-1} \\ \Lambda_{-1,1} & \Lambda_{-1,-1} \end{vmatrix} \tag{8}$$

we find for the eigenvalues and eigenvectors

$$\begin{aligned} \lambda_S &= \Lambda_{1,1} + r \varrho \Lambda_{1,-1} , \quad & | K_S \rangle &= (| K^0 \rangle + r \varrho | \overline{K}^0 \rangle)/\sqrt{2} \\ \lambda_L &= \Lambda_{1,1} - \frac{r}{\varrho} \Lambda_{1,-1} , \quad & | K_L \rangle &= (\varrho | K^0 \rangle - r | \overline{K}^0 \rangle)/\sqrt{2} , \end{aligned} \tag{9}$$

where we have defined

$$r = \sqrt{\Lambda_{-1,1}/\Lambda_{1,-1}}, \quad \varrho = \eta + \sqrt{1 + \eta^2}, \quad \eta = \frac{\Lambda_{-1,-1} - \Lambda_{1,1}}{2\sqrt{\Lambda_{1,-1} \Lambda_{-1,1}}} . \tag{10}$$

If we write these eigenvectors as

$$| K_S \rangle = \boldsymbol{V} | K^0 \rangle, \quad | K_L \rangle = \boldsymbol{V} | \overline{K}^0 \rangle \tag{11}$$

then

$$\boldsymbol{V}(r, \varrho) = \frac{1}{\sqrt{2}} \begin{vmatrix} 1 & \varrho \\ r\varrho & -r \end{vmatrix} \tag{12}$$

is *not* a unitary matrix. We note that neither $\boldsymbol{M}$ nor $\boldsymbol{\Gamma}$ are diagonalized by $\boldsymbol{V}$, but only the combination $\boldsymbol{\Lambda} = \boldsymbol{M} - i/2\, \boldsymbol{\Gamma}$. The complex parameter r measures the strength of the T-violating part of Λ, and ϱ measures the strength of the TCP-violating part. The square roots are defined so that r, ϱ are close to $+1$ for small CP-violations. It follows that the state $| K_S \rangle$ will have a smaller mass and larger width than $| K_L \rangle$; therefore for S and L one should read short- and longlived. We continue to use $| K_1^0 \rangle$ and $| K_2^0 \rangle$ to denote the CP-eigenstates.

It will be useful to have the inversion of the relations between λ_S, λ_L, r, ϱ and the energy matrix elements, at least for small CP-violations. If we write Λ as

$$\Lambda = \begin{vmatrix} \Lambda_{1,1}^{(0)} + \delta\Lambda_{1,1} & \Lambda_{1,-1}^{(0)} + \delta\Lambda_{1,-1} \\ \Lambda_{1,-1}^{(0)} - \delta\Lambda_{1,-1} & \Lambda_{1,1}^{(0)} - \delta\Lambda_{1,1} \end{vmatrix} \tag{13}$$

then to first order in $\delta\Lambda$ we obtain

$$\lambda_S \cong \Lambda_{1,1}^{(0)} + \Lambda_{1,-1}^{(0)} = \lambda_1 \,, \quad r \cong 1 - \delta\,\Lambda_{1,-1}/\Lambda_{1,-1}^{(0)}$$

$$\lambda_L \cong \Lambda_{1,1}^{(0)} - \Lambda_{1,-1}^{(0)} = \lambda_2 \,, \quad \varrho \cong 1 - \delta\,\Lambda_{1,1}/\Lambda_{1,-1}^{(0)} \,, \tag{14}$$

or inverting,

$$\delta\Lambda_{1,-1} \cong \frac{1}{2}\,(1-r)\,(\lambda_S - \lambda_L) \,, \quad \delta\Lambda_{1,1} \cong \frac{1}{2}\,(1-\varrho)\,(\lambda_S - \lambda_L) \,. \tag{15}$$

Notice that, since the degeneracy is already removed by the symmetry-preserving terms, there is no first order change in the eigenvalues due to the symmetry-breaking terms.

The energy matrix elements, and therefore λ_S, λ_L, ϱ, r, are not completely independent, but are restricted by the conditions that the eigenvalues of $\boldsymbol{M}$ and $\boldsymbol{\Gamma}$ be positive, as follows from Equation (2)*). This is known[14]) to place rather severe restrictions on the off-diagonal elements of $V^+ V$, and therefore on the strength of the symmetry-breaking terms. With large symmetry-breaking terms one cannot obtain the observed ratio of the two widths, $\gamma_1/\gamma_2 \cong 610$. The condition det $\boldsymbol{\Gamma} \geqslant 0$ leads to the inequality

$$\frac{|\,\varrho - \varrho^*\,|r|^2\,|^2}{(1 + |\varrho\,r|^2)\,(|\varrho|^2 + |r|^2)} \leqslant \frac{4\,\mathrm{Im}\,\lambda_S\,\mathrm{Im}\,\lambda_L}{|\lambda_S - \lambda_L^*|^2} \tag{16}$$

which provides a quantitative estimate of how large the deviation of r and ϱ from unity can become**). We find, for small $(1-r)$ and $(1-\varrho)$,

$$[\mathrm{Re}\,(1-r)]^2 + [\mathrm{Im}\,(1-\varrho)]^2 < 4\,\frac{\gamma_2}{\gamma_1} \cong 6{,}5 \cdot 10^{-3} \,. \tag{17}$$

Before proceeding to the discussion of experiments, we will consider some questions of physical interpretation and measurability. Although we could continue to work exclusively with the states K^0 and $\overline{K}^0$, and directly solve the original equation of motion for a given energy matrix, it is very useful instead to introduce the "mass

---

*) $\sum_{SS'} \Gamma_{SS'}\, x_S^*\, x_{S'} = 2\pi \cdot \langle \psi \,|\, \mathcal{H}_W\, \delta(M - \mathcal{H}_S)\, \mathcal{H}_W \,|\, \psi \rangle \geqslant 0$, where $|\psi\rangle \equiv \sum_S |\,S\rangle\, x_S$ and x_S is arbitrary.

**) There are other limits set by existing experiments, such as the fact that, to within the accuracy of the branching ratios, the fraction of kaons which decay after $\gamma_1 t \sim 5$ is $1/2$. This implies $(|\varrho|^4 + |\varrho|^2)/|1+\varrho^2|^2 \sim 1/2$ to within about 20%, setting limits on the real part of ϱ as well.

eigenstates' and 'mass eigenvalues'. But since the transformation to these states with $\boldsymbol{V}$ is *not* unitary, there are important differences in the physical interpretation of the eigensolution. For example, we note that the real and imaginary parts of the 'mass eigenvalues' are *not* the eigenvalues of $\boldsymbol{M}$ and $\boldsymbol{\Gamma}$. It is Re $(\lambda_S - \lambda_L)$ which is measured in the experiments designed to determine the 'mass difference'. The situation is not unique to K-meson physics, but presumably will occur whenever the mass and damping matrices of an unstable system fail to commute. For the neutral kaons, this pertains to the case in which CP-invariance is broken.

In dealing with the 'mass eigenstates' $| K_S \rangle$ and $| K_L \rangle$, it is useful to define some kind of adjoint states $\langle \tilde{K}_S |$ and $\langle \tilde{K}_L |$, and an inner product. If we use the ordinary hermitian adjoint, then it follows directly from the non-unitary character of $\boldsymbol{V}$ that the states are not orthogonal, $\langle K_S | K_L \rangle \neq 0$. This clearly makes it impossible to interpret $\langle K_S | \psi \rangle$ as the probability amplitude of obtaining the short-lived state in the state $| \psi \rangle$, since there would be a non-vanishing probability amplitude even for $| \psi \rangle = | K_L \rangle$. The customary completeness and orthonormality relations can be guaranteed by introducing[15]) instead the 'inverse states', defined by

$$\langle \tilde{K}_S | \equiv \langle K^0 | \boldsymbol{V}^{-1}, \quad \langle \tilde{K}_L | \equiv \langle \overline{K}^0 | \boldsymbol{V}^{-1}, \tag{18}$$

and given explicitly by

$$\langle \tilde{K}_S | = (\langle K^0 | r + \langle \overline{K}^0 | \varrho) \frac{\sqrt{2}}{r(1+\varrho^2)}, \quad \langle \tilde{K}_L | = (\langle K^0 | \varrho\, r - \langle \overline{K}^0 |) \frac{\sqrt{2}}{r(1+\varrho^2)} . \tag{19}$$

The major difference from the usual situation in quantum mechanics, is that the orthonormality conditions

$$\langle \tilde{K}_S | K_S \rangle = \langle \tilde{K}_L | K_L \rangle = 1, \quad \langle \tilde{K}_S | K_L \rangle = \langle \tilde{K}_L | K_S \rangle = 0 \tag{20}$$

leave undetermined not only the *phase* of the states, but also the *scale*. If we change basis by

$$| K_S \rangle \to c_S \, | K_S \rangle, \quad | K_L \rangle \to c_L \, | K_L \rangle \tag{21a}$$

then the 'inverse states' change according to

$$\langle \tilde{K}_S | \to \langle \tilde{K}_S | \, c_S^{-1}, \quad \langle \tilde{K}_L | \to \langle \tilde{K}_L | \, c_L^{-1} \tag{21b}$$

(not c^* but c^{-1}!), and the orthonormality is preserved for *any* value of c_S, c_L. As a result, a transition amplitude $\langle \tilde{K}_S | T | K_L \rangle$ is undefined to a complex factor (c_L/c_S), rather than simply to a phase factor; we can not consider $|\langle \tilde{K}_S | T | K_L \rangle|^2$ an observable[15]).

The resolution of these difficulties lies in the recognition that the 'mass eigenstates' have only an intermediate role, and cannot be prepared or observed (i.e., used as 'in' and 'out' states). If we insist that they appear only in the expansion of another initial state, thus occuring only in the combinations $| K_S \rangle \langle \tilde{K}_S |$ and $| K_L \rangle \langle \tilde{K}_L |$, then the normalization constants c_S and c_L do not appear and can be chosen arbitrarily. This criterion is implicit in the use of eigenstates of the K-meson propagator in the field theoretic treatment of SACHS.

To illustrate this point, we will give an example: the rate of $\pi^+ \pi^-$-decay of a long-lived kaon beam. For an initial K^0, making use of the identity

$$e^{-i\Lambda t} = | K_S \rangle e^{-i\lambda_S t} \langle \tilde{K}_S | + | K_L \rangle e^{-i\lambda_L t} \langle \tilde{K}_L | \tag{22}$$

and dropping the short-lived component, we have

$$R_{(K_L \to \pi^+ \pi^-)} = c \,|\langle 2\pi | \mathcal{H}_W | K_L \rangle e^{-i\lambda_L t} \langle \tilde{K}_L | K^0 \rangle|^2. \tag{23}$$

For an initial $\overline{K}^0$, the result is different

$$\overline{R}_{(K_L \to \pi^+ \pi^-)} = c \,|\langle 2\pi | \mathcal{H}_W | K_L \rangle e^{-i\lambda_L t} \langle \tilde{K}_L | \overline{K}^0 \rangle|^2. \tag{24}$$

We do not consider it possible to define the rate of decay $|\langle 2\pi | \mathcal{H}_W | K_L \rangle|^2$ unambiguously, irrespective of its initial state; the extra factors $\langle \tilde{K}_L | K^0 \rangle$ and $\langle \tilde{K}_L | \overline{K}^0 \rangle$ must be included to make the expression well-defined. In practice, these extra factors usually have a trivial effect for small CP violations, if we choose the normalization of the 'mass eigenstates' so that $| K_S \rangle$ and $| K_L \rangle$ differ from $| K_1^0 \rangle$ and $| K_2^0 \rangle$ only in first order of the symmetry-breaking terms. We have made this choice in Equation 9.

Next we consider the measurability of the complex parameters r and ϱ. In order to determine them, we generally must have another interaction of kaons with known matrix elements which can be used as a comparison. Consider first ϱ, which depends on the difference $\Lambda_{1,1} - \Lambda_{-1,-1}$. We expect that ϱ can be determined by utilizing another interaction which is different for K^0 and $\overline{K}^0$: the strong interactions involved in charge exchange scattering, hyperon production, etc. We shall see explicitly in the next section that such interactions permit the measurement of both the phase and magnitude of ϱ.

The measurement of r, which depends on the off-diagonal matrix elements $\Lambda_{1,-1} - \Lambda_{-1,1}$, is more subtle. Because of the conservation of strangeness, there is no *strong* interaction which couples K^0 to $\overline{K}^0$; this will be shown explicitly to imply that, using strong interactions alone, the magnitude of r can be measured, but not the phase. If weak interactions are used, a further difficulty intrudes: the phase of a weak interaction coupling K^0 to $\overline{K}^0$ is known only if T-invariance is valid. Therefore, if we have some a priori consideration which implies T-invariance of the decay amplitudes for a particular decay channel, then this channel can be used to measure the phase of r. Without such a priori considerations, which are usually associated with special models of the weak interaction, we cannot measure the phase of r.

Finally we comment that all this discussion concerns the problem of determining the energy matrix *in our standard representation*. A change of the relative phase of the K^0 and $\overline{K}^0$ states will change the phase of the off-diagonal elements of $\boldsymbol{\Lambda}$, thereby admixing the mass and damping matrices,

$$\Lambda_{1,-1} \to \Lambda_{1,-1}\, e^{i\phi} = \left(M_{1,-1} \cos\phi + \frac{1}{2}\,\Gamma_{1,-1} \sin\phi\right) + i\left(M_{1,-1} \sin\phi - \frac{1}{2}\,\Gamma_{1,-1} \cos\phi\right). \tag{25}$$

Thus, it is not possible to distinguish the effects of a symmetry-breaking term in $\boldsymbol{M}$ from one in $\boldsymbol{\Gamma}$, except with reference to a particular representation.

Strong Interactions of K-Mesons

The first test of any model of neutral kaons is the study of the subsequent strong interactions of a particle initially known to be either a K^0 or a $\overline{K^0}$ [16]); that is, reactions in which a kaon is both produced and later absorbed in strong interactions. There are clearly four relevant amplitudes, for the propagation of a particle of initial strangeness S_i having, after proper time t, a final strangeness S_f. We define

$$A(S_f, S_i) = \langle S_f \mid e^{-i\Lambda t} \mid S_i \rangle$$
$$= \langle S_f \mid K_S \rangle e^{-i\lambda_S t} \langle \tilde{K}_S \mid S_i \rangle + \langle S_f \mid K_L \rangle e^{-i\lambda_L t} \langle \tilde{K}_L \mid S_i \rangle . \tag{26}$$

By explicit calculation, we find

$$A(+1, +1) = \frac{1}{1+\varrho^2} (e^{-i\lambda_S t} + \varrho^2 e^{-i\lambda_L t})$$
$$A(+1, -1) = \frac{\varrho}{r(1+\varrho^2)} (e^{-i\lambda_S t} - e^{-i\lambda_L t})$$
$$A(-1, +1) = \frac{\varrho\, r}{1+\varrho^2} (e^{-i\lambda_S t} - e^{-i\lambda_L t})$$
$$A(-1, -1) = \frac{1}{1+\varrho^2} (\varrho^2 e^{-i\lambda_S t} + e^{-i\lambda_L t}) . \tag{27}$$

Several deductions can be drawn from these results. First we notice, as previously remarked, that r occurs only as a factor, and that therefore the phase of r cannot be determined in this way. Second, we notice that, in addition to the well-known oscillations in the intensity of the K^0 and $\overline{K^0}$, there are also small oscillations in the total number of mesons present (and therefore in the total rates of decay). For an initial K^0, we find

$$\mathcal{A}(t) = |A(+1, +1)|^2 + |A(-1, +1)|^2$$
$$= \frac{1}{|1+\varrho^2|^2} \Big\{ (1 + |\varrho\, r|^2)\, e^{-\gamma_S t} + [(\varrho^2 - |\varrho\, r|^2)\, e^{i\Delta m t}$$
$$+ \text{c. c.}]\, e^{-(\gamma_S + \gamma_L) t/2} + (|\varrho|^4 + |\varrho\, r|^2)\, e^{-\lambda_L t} \Big\}, \tag{28}$$

whereas for an initial $\overline{K^0}$, we find

$$\overline{\mathcal{A}}(t) = |A(+1, -1)|^2 + |A(-1, -1)|^2$$
$$= \frac{1}{|1+\varrho^2|^2} \left\{ \left(\left|\frac{\varrho}{r}\right|^2 + |\varrho|^4 \right) e^{-\gamma_S t} + \left[\left(\varrho^{*2} - \left|\frac{\varrho}{r}\right|^2 \right) e^{i\Delta m t} \right.\right.$$
$$\left.\left. + \text{c. c.} \right] e^{-(\gamma_S + \gamma_L) t/2} + \left(\left|\frac{\varrho}{r}\right|^2 + 1 \right) e^{-\gamma_L t} \right\}, \tag{29}$$

where $\Delta m = \operatorname{Re} (\lambda_S - \lambda_L)$, $\gamma_{S,L} = -2 \operatorname{Im} \lambda_{S,L}$. It has been noted earlier[14]) that the occurrence of oscillatory terms in the total intensity is a test of CP-violation. A further conclusion concerns the phase of these small oscillations, which, for either K^0 or $\overline{K^0}$, is

$$[\operatorname{Re} (1 - r) \cos (\Delta\, m\, t) + \operatorname{Im} (1 - \varrho) \sin (\Delta\, m\, t)]\, e^{-(\gamma_S + \gamma_L) t/2}$$

correct to first order in the symmetry-breaking terms. Thus TCP-conservation implies an oscillation $\sim \cos(\Delta m t)$ and T-conservation implies an oscillation $\sim \sin(\Delta m t)$. In view of the continuing breakdown of symmetry in the weak interactions, it seems very important to provide a test of TCP as well as CP and T; the above effect is one possible way. The amplitude is known from Equation 17 to be less than 8%, and is expected from the experiment of CHRISTENSON, CRONIN, FITCH, and TURLAY to be about a factor 10 smaller than this limit. To observe this effect by counting both the K^0 and the $\overline{K}^0$ intensity, one faces the experimental difficulties of obtaining adequate statistical significance as well as determining accurately the relative detection efficiency for K^0 and $\overline{K}^0$. As an alternative, one could attempt to measure the total rate of decay (the time derivative of this curve), by summing all the decay channels. The most sensitive region is then about $10\,\gamma_1^{-1}$, where the rates of decay of the two modes are comparable. In this case, the measurement of the absolute phase of the oscillatory term requires a precise knowledge of the mass difference.

It is important to realize that the same information can be obtained in another way, by observing the strong interactions of a K^0 (or a $\overline{K}^0$) in a beam which is known initially to have equal numbers of K^0 and $\overline{K}^0$ (incoherent mixture). Thus, for measurement of a K^0 after time t, we define

$$\begin{aligned}\mathcal{D}(t) &= |A(+1,+1)|^2 + |A(+1,-1)|^2 \\ &= \frac{1}{|1+\varrho^2|^2}\left\{\left(1+\left|\frac{\varrho}{r}\right|^2\right)e^{-\gamma_S t} + \left[\left(\varrho^2 - \left|\frac{\varrho}{r}\right|^2\right)e^{i\Delta m t} + \text{c.c.}\right]\right. \\ &\quad \left.\times\, e^{-(\gamma_S+\gamma_L)t/2} + \left(|\varrho|^4 + \left|\frac{\varrho}{r}\right|^2\right)e^{-\gamma_L t}\right\}, \end{aligned} \tag{30}$$

and for a $\overline{K}^0$,

$$\begin{aligned}\overline{\mathcal{D}}(t) &= |A(-1,+1)|^2 + |A(-1,-1)|^2 \\ &= \frac{1}{|1+\varrho^2|^2}\left\{(|\varrho r|^2 + |\varrho|^4)\,e^{-\gamma_S t} + [(\varrho^{*2} - |\varrho r|^2)\,e^{i\Delta m t}\right. \\ &\quad \left. +\,\text{c.c.}]\,e^{-(\gamma_S+\gamma_L)t/2} + (1+|\varrho r|^2)\,e^{-\gamma_L t}\right\} \end{aligned} \tag{31}$$

which have oscillatory terms

$$[\operatorname{Re}(1-r)\cos(\Delta m t) - \operatorname{Im}(1-\varrho)\sin(\Delta m t)]\,e^{-(\gamma_S+\gamma_L)t/2}\,.$$

In this case one need only determine that equal numbers of K^0 and $\overline{K}^0$ production events are counted in the sample, and the actual detection efficiency is not needed, except insofar as it depends on the momentum spread in the beam. It may be feasible to measure these curves, especially at higher energies where the increased laboratory lifetime enhances the relative probability of interaction versus decay.

The close similarity in form of $\mathcal{A}(t)$ and $\mathcal{D}(t)$ is an example of the close analogy to the theory of polarized spin 1/2 beams. The first expression, for $\mathcal{A}(t)$, would correspond to determining the asymmetry of an initially polarized beam. The second expression, for $\mathcal{D}(t)$, would correspond to determining the polarization of an initially unpolarized beam. The close relationship between these two is familiar in the theory of polarization.

We should make two further comments. Although we have stressed the measurement of the oscillatory terms, the hyperbolic terms also depend on the symmetry-breaking terms, containing Re $(1-r)$ and Re $(1-\varrho)$. These terms do not allow a test of T versus TCP however. Finally we repeat a comment of SACHS[15]), that there do exist oscillatory rates, such as $|A(+1,-1)|^2$ and $|A(-1,+1)|^2$, whose time dependence is independent of r and ϱ and which can therefore be used to determine the mass difference without corrections due to CP-violation.

Decays of K-Mesons

In the previous sections we have emphasized the measurement of the total energy matrix, whereas the measurement of K-decays normally gives information about the individual channels. To calculate the rate of decay into one particular state, it is necessary to introduce the decay amplitudes for K^0 and $\overline{K}^0$ into that state, and therefore the partial damping matrix. If we denote the amplitudes by

$$D_S^{(j)} = \langle j \mid \mathcal{H}_W \mid S \rangle ,$$

then the j^{th} partial damping matrix is defined by

$$\Gamma_{SS'}^{(j)} = D_S^{(j)*} D_{S'}^{(j)} , \tag{32}$$

where the appropriate kinematical factors have been absorbed into the definition of the amplitudes. The general form of the partial damping matrices is thus

$$\Gamma^{(j)} = \begin{vmatrix} \alpha_j & x_j + i\, y_j \\ x_j - i\, y_j & \beta_j \end{vmatrix} \tag{33}$$

with $\alpha\,\beta = x^2 + y^2$. If the j^{th} channel is CP-self-conjugate, as for the various $2\,\pi$ and $3\,\pi$ channels, then TCP-invariance implies $\alpha_j = \beta_j$. For any channel, if we continue to define the states as real, T-invariance implies $y = 0$*).

Using this general form and the definition (26) we can express the rate of decay of the beam into the j^{th} channel, as

$$R_j(t) \equiv |\langle j \mid \mathcal{H}_W\, e^{-i\Lambda t} \mid +1 \rangle|^2 = |\sum_S D_S^{(j)} A(S,+1)|^2$$

$$= \frac{1}{|1+\varrho^2|^2\,\alpha_j} \mid [\alpha_j + \varrho\, r(x_j + i\, y_j)]\, e^{-i\lambda_S t} + \varrho\,[\alpha_j\,\varrho - r(x_j + i\, y_j)]\, e^{-i\lambda_L t}] \mid^2 \tag{34}$$

for a single initial K^0 meson. If instead the initial state was a single $\overline{K}^0$, we obtain

$$\overline{R}_j(t) \equiv |\langle j \mid \mathcal{H}_W\, e^{-i\Lambda t} \mid -1 \rangle|^2 = |\sum_S D_S^{(j)} A(S,-1)|^2$$

$$= \frac{1}{|1+\varrho^2|^2\,\alpha_j\,|r|^2} \mid \varrho\,[\alpha_j + \varrho\, r\,(x_j + i\, y_j)]\, e^{-i\lambda_S t} - [\alpha_j\,\varrho - r\,(x_j + i\, y_j)]\, e^{-i\lambda_L t} \mid^2 . \tag{35}$$

*) In Reference 8, the relative phase of K^0 and $\overline{K}^0$ is chosen to make the dominant decay amplitude ($2\,\pi$, $I = 0$ channel) real, and so in that channel, $y = 0$ by convention. We prefer to use real states, for which the phase of a decay amplitude is determined by whether the interaction preserves T-invariance. The presence of final state interactions will give additional contributions to the phase of D_S.

For example, the rate of decay of the long-lived mode in the 2π-channel is, for a K^0,

$$R_{2\pi}(t) \xrightarrow[\gamma_S t \gg 1]{} \frac{|\varrho|^2 e^{-\gamma_L t}}{\alpha_{2\pi}|1+\varrho^2|^2} \, |\alpha_{2\pi}\varrho - r(x_{2\pi} + i\, y_{2\pi})|^2 \tag{36}$$

or, correct to first order in the symmetry-breaking terms, for either K^0 or $\overline{K}^0$ ($\alpha = x + \delta\alpha$),

$$R_{2\pi}(t) \cong x_{2\pi}\, e^{-\gamma_L t} \left| \frac{\delta\alpha_{2\pi} - i\, y_{2\pi}}{2\, x_{2\pi}} + \frac{\delta\Lambda_{1,-1} - \delta\Lambda_{1,1}}{\lambda_S - \lambda_L} \right|^2 . \tag{37}$$

This general result illustrates the fact that there are, at the phenomenological level, several independent origins possible for the CP violation.

The limitations on the measurability of the phase of r can be illustrated with the above formulas. If, in a particular model, there is a reason to assume CP-invariance for a given channel, then the phase of the oscillatory term in the decay rate in that channel depends on the phase of r. From a completely empirical view, making no assumption about the origin of the CP violation, we cannot determine that phase.

Finally, let us note that, if we compare the rates of decay of beams which are initially known to be either K^0 or $\overline{K}^0$, it is possible to obtain information about the total energy matrix without knowing the partial damping matrix. From Equations (34) and (35) we see that TCP-conservation implies that the oscillatory terms have exactly opposite *phase*, but amplitude ratio $|r|^{-4}$ while T-conservation implies that their *amplitudes* are the same whereas their phase difference is $-\varrho^*/\varrho$. On the other hand, the ratio of the rates for the long-lived component is

$$R_j(t)/\overline{R}_j(t) \xrightarrow[\gamma_S t \gg 1]{} |\varrho\, r|^2 \tag{38}$$

which can be used to determine Re $(2 - r - \varrho)$, but not to test T or TCP. Such experiments require the identification of the production vertex of the kaon. For the measurement of the oscillatory terms in the 2π-channel, the most sensitive region is around $\gamma_1 t \sim 12$, where the rates for the two modes are known to be comparable.

Coherent Effects in Media

The strong interactions to which we referred in the earlier section (i.e., charge exchange, hyperon production) are incoherent reactions, in the sense that an observable change of state of the medium results from the interaction. It was first pointed out by GOOD[17]) that it is also possible to experiment with those mesons which scatter in the medium without changing its state. These particles define a 'coherent wave' propagating through the medium according to the familiar laws of optics. Assuming only that these scattering processes conserve strangeness, we can write the form of the index of refraction for homogeneous matter as

$$n^2 = \mathbf{1} + 4\pi \frac{N}{k^2} \begin{vmatrix} f & 0 \\ 0 & \overline{f} \end{vmatrix} , \tag{39}$$

where N is the number density of scatterers and $f, \overline{f}$ are the forward elastic scattering amplitudes of the K^0 and the $\overline{K}^0$, respectively. The necessary modification of our mass matrix formulation is readily found from the dispersion law for kaons. If the kaon has velocity v, then relativistic covariance implies that the frequency and wave vector are given in terms of the mass by

$$\boldsymbol{\omega} = \gamma \boldsymbol{\Lambda}, \quad \boldsymbol{k} = \gamma v \boldsymbol{\Lambda}, \tag{40}$$

satisfying $\boldsymbol{\omega}^2 - \boldsymbol{k}^2 = \boldsymbol{\Lambda}^2$, with $\gamma = (1 - v^2)^{-1/2}$. The coherent scattering in matter alters the wave vector but not the frequency,

$$\boldsymbol{\omega}' = \boldsymbol{\omega}, \quad \boldsymbol{k}' = n\, \boldsymbol{k} \tag{41}$$

giving

$$\boldsymbol{\Lambda}'^2 = \boldsymbol{\omega}'^2 - \boldsymbol{k}'^2 = \boldsymbol{\Lambda}^2 - 4\pi N \begin{vmatrix} f & 0 \\ 0 & \bar{f} \end{vmatrix}. \tag{42}$$

Since $(n - 1)$ is very small, we retain only terms linear in the density. The final result is simply an additional contribution to the mass matrix

$$\boldsymbol{\Lambda}' = \boldsymbol{\Lambda} + \boldsymbol{\Lambda}_{SC} = \boldsymbol{\Lambda} - \frac{2\pi N}{M} \begin{vmatrix} f & 0 \\ 0 & \bar{f} \end{vmatrix}. \tag{43}$$

The wave equation corresponding to the dispersion law Equation (42) is the two-component Klein-Gordon equation, with mass $\boldsymbol{\Lambda}'$.

Since we have already discussed the eigenvalues and eigenvectors of the general energy matrix, the effect of adding $\boldsymbol{\Lambda}_{SC}$ to $\boldsymbol{\Lambda}$ is simply to make the parameters λ_S', λ_L', r', ϱ' depend on the density. To first order in the density, we find from our linearized connection formulas Equations (13), (14)

$$\lambda_S' = \lambda_S - \pi N (f + \bar{f})/M, \quad \lambda_L' = \lambda_L - \pi N (f + \bar{f})/M, \quad r' = r, \quad \varrho' = \varrho + 2\delta, \tag{44}$$

where $\delta = \pi N (f - \bar{f})/[M(\lambda_S - \lambda_L)]$. The primed quantities all refer to the beam inside matter and the unprimed quantities to the vacuum. The coherent scattering can be considered another source of CP-violation (since matter is not CP-invariant) which preserves T-invariance, and therefore appears to violate TCP-invariance. Since the degeneracy is already removed, the effect of the scattering is a small pertubation on the vacuum eigenstates, governed by the dimensionless parameter δ which is about 10^{-2} for condensed matter[18]). There is also a CP-conserving term $\pi N (f + \bar{f})/M$ in each of the eigenvalues, which decreases the lifetimes of both modes due to scattering out of the beam, but which does not alter the mass difference.

The theory of coherent regeneration of kaons in matter can be based on the calculation of the S-matrix elements according to

$$S_{fi} = \langle f \mid e^{-i(\boldsymbol{\Lambda} + \boldsymbol{\Lambda}_{SC})t} \mid i \rangle, \tag{45}$$

where t is the proper time elapsed in the matter. This result neglects the specularly reflected wave, whose amplitude is of first order in $(n - 1)$; it is a special case of the 'eikonal approximation' to the forward scattering amplitude of the medium. If we choose $\mid K_S \rangle$ and $\mid K_L \rangle$ as 'initial' states, $\langle \tilde{K}_S \mid$ and $\langle \tilde{K}_L \mid$ as 'final' states, introducing explicitly the matrix $\boldsymbol{V}(r, \varrho)$ which diagonalizes $\boldsymbol{\Lambda}$, we obtain

$$S_{fi} = \langle \tilde{K}_f \mid \boldsymbol{V} e^{-i\boldsymbol{\Lambda}'' t} \boldsymbol{V}^{-1} \mid K_i \rangle, \tag{46}$$

where

$$\Lambda'' = V^{-1}(\Lambda + \Lambda_{SC})\, V = \begin{vmatrix} \lambda_S & 0 \\ 0 & \lambda_L \end{vmatrix} - \frac{2\pi N}{M(1+\varrho^2)} \begin{vmatrix} f + \varrho^2 \bar{f}, & \varrho(f-\bar{f}) \\ \varrho(f-\bar{f}), & \varrho^2 f + \bar{f} \end{vmatrix} . \tag{47}$$

We remark that the symmetry of Λ'' is not due to any physical property, but is simply a consequence of our choice of normalization of $|K_S\rangle$ and $|K_L\rangle$.

The above result establishes the very weak dependence of regeneration phenomena on the violation of CP, first noted by GOOD[19]). In the above representation, Λ'' is independent of r, and only weakly dependent on ϱ, through terms of order $(1-\varrho)^2$ and $\delta\,(1-\varrho)$. Neglecting these terms, we can then calculate the regenerated amplitudes of $|K_S\rangle$ and $|K_L\rangle$ independently of the symmetry breaking terms. Of course, we must not lose sight of the fact that the 'mass eigenstates' themselves contain r and ϱ. Explicit evaluation of Equation (46) can easily be carried out and shown to lead to the formulas first derived by GOOD; we have no need for the explicit regeneration amplitudes here.

There are several significant observations to be drawn from these formulas:

1. The original motivation for this paper was to study the 'Stokes reversibility' of kaons in matter, as a possible test of T-invariance. This reversibility concerns the equality of transmission of particles travelling in opposite directions through an inhomogeneous medium. If we consider a double regenerator composed of two homogeneous pieces of different material, denoted by subscripts 1 and 2, then the transmission of K_L mesons from one side is

$$\langle \tilde{K}_L | S_1 S_2 | K_L \rangle = \langle \tilde{K}_L | S_1 | K_L \rangle \langle \tilde{K}_L | S_2 | K_L \rangle + \langle \tilde{K}_L | S_1 | K_S \rangle \langle \tilde{K}_S | S_2 | K_L \rangle \tag{48}$$

and the transmission from the other side is obtained by interchanging 1 and 2. However, since Equation (48) is explicitly invariant under the transformation (21) we can always choose Λ'' symmetric, as in Equation (47). It follows that the expression is symmetric in 1 and 2. Therefore, 'Stokes reversibility' is implied even in the absence of T-invariance; it is simply a consequence of the 'eikonal approximation'. This is probably well-known in optics, but was unknown to us! One can also show in a similar way that this property holds for K_S, K^0, and $\bar{K}^0$ mesons as well. There do not appear to be any interesting experiments of this type for testing T and TCP.

2. There is a very sensitive test of the violation of CP by measurement of the decay rates of the coherent beam in matter. Since the effect of both the weak interactions and the coherent scattering is to admix the states K^0 and $\bar{K}^0$, it is possible to compensate the one with the other, and therefore to determine the parameters of the decay amplitudes in terms of the known (or separately measurable) parameters in Λ_{SC}. Using Equations (37) and (44), the rate of 2π-decay for the long-lived mode in matter is

$$R'_{2\pi}(t) \simeq x_{2\pi}\, e^{-\gamma'_L t} \left| \delta + \left(\frac{\delta\alpha_{2\pi} - i\, y_{2\pi}}{2\, x_{2\pi}} + \frac{\delta \Lambda_{1,-1} - \delta \Lambda_{1,1}}{\lambda_S - \lambda_L} \right) \right|^2 . \tag{49}$$

By measuring this rate as a function of the density, one can determine the phase of the quantity

$$\frac{\delta\alpha_{2\pi} - i\, y_{2\pi}}{2\, x_{2\pi}} + \frac{\delta \Lambda_{1,-1} - \delta \Lambda_{1,1}}{\lambda_S - \lambda_L}$$

relative to the phase of δ. From the known rate of the long-lived mode in vacuum, we estimate that the critical density necessary for strong interference is about 1/10 of the normal density of condensed materials, which can be obtained by use of a stratified medium[5]) *). Such an experiment will give important evidence concerning the source of the CP-violating terms, although it cannot differentiate between a violation in the partial damping matrix for that channel ($\delta\alpha$, $y \neq 0$) and a violation in the total energy matrix ($\delta\Lambda_{1,1}$, $\delta\Lambda_{1,-1} \neq 0$).

Kaon Pair States

It is very natural to consider the self-conjugate system of a kaon pair ($K^0\,\overline{K}^0$) for a study of invariance under CP, T, and TCP. The wide variety of production mechanisms (such as meson resonance decay, antiproton annihilation, $\pi\,p$ and $p\,p$ reactions, etc.) and the many possibilities for observing decays and/or interactions of the kaons, make this subject very rich and varied. The subtle correlations in the two particle state due to the Einstein-Podolsky effect must also be correctly treated[20]).

We begin the discussion by noting that of the sixteen states of two kaons[21]), only the two zero strangeness states of neutral kaons, $K^0(k_1)\,\overline{K}^0(k_2)$ and $\overline{K}^0(k_1)\,K^0(k_2)$, need be considered since it is assumed that strangeness, charge, and the momenta are well specified in the production process.

The two particles are distinguished by their momenta. The states of interest are antisymmetric and symmetric combinations which we write in the center of mass system,

$$| \psi_{\mp} \rangle \equiv \frac{1}{\sqrt{2}} \, | \, K^0(k)\,\overline{K}^0(-k) \mp \overline{K}^0(k)\,K^0(-k) \rangle \,. \tag{50}$$

A further decomposition into partial waves could be made, but the only result of importance here is that, as a consequence of Bose statistics, the state $| \psi_- \rangle$ contains only odd L, and $| \psi_+ \rangle$ contains only even L. Only propagation in the vacuum is considered. The coherent effects of a scattering medium can be discussed as in the previous section, but it is complicated by the relative motion of the matter and the meson pair reference frames[22]).

The temporal evolution of the pair states will be assumed to be given by an energy matrix separable and symmetric in the two particles, $\mathcal{H}(1, 2) = \Lambda(1) + \Lambda(2)$. This neglects the weak final state interaction between the two particles, but includes the weak self-interaction. Since weak interactions result in a very short range force, this procedure is correct for pair states with an appreciable relative momentum. With this assumption, and with the definition (40) the time dependence of $| \psi_{\mp} \rangle$ is given by

$$| \psi_{\mp}(t) \rangle = e^{-(\omega(1)+\omega(2))t} \, | \psi_{\mp} \rangle \,, \tag{51}$$

where t is the time after production of the pair. With the help of Equations (22) and (19) we find for the time evolution of the one-kaon states

$$e^{-i\omega t} \, | \, K^0(k) \rangle = \frac{\sqrt{2}}{1+\varrho^2} \left(e^{-i\gamma\lambda_S t} \, | \, K_S(k) \rangle + \varrho \, e^{-i\gamma\lambda_L t} \, | \, K_L(k) \rangle \right)$$

$$e^{-i\omega t} \, | \, \overline{K}^0(k) \rangle = \frac{\sqrt{2}}{r(1+\varrho^2)} \left(\varrho \, e^{-i\gamma\lambda_S t} \, | \, K_S(k) \rangle - e^{-i\gamma\lambda_L t} \, | \, K_L(k) \rangle \right) . \tag{52}$$

*) A successful experiment of this type has recently been reported by V. L. Fitch, R. F. Roth, J. S. Russ, W. Vernon, see Reference 12.

This gives for Equation (51)

$$| \psi_-(t) \rangle = \frac{\sqrt{2}}{r(1+\varrho^2)} e^{-i\gamma(\lambda_S+\lambda_L)t} \mid K_L(k)\, K_S(-k) - K_S(k)\, K_L(-k) \rangle . \tag{53}$$

Note that in the center of mass system $\gamma = (1 - v^2)^{-1/2}$ is the same for both mesons. If we express this result in K_1, K_2 or in K^0, $\overline{K}^0$ then

$$\begin{aligned} | \psi_-(t) \rangle &= \frac{1}{\sqrt{2}} e^{-i\gamma(\lambda_S+\lambda_L)t} \mid K_2(k)\, K_1(-k) - K_1(k)\, K_2(-k) \rangle \\ &= e^{-i\gamma(\lambda_S+\lambda_L)t} \mid \psi_- \rangle \end{aligned} \tag{54}$$

which is *independent of* r, ϱ, and therefore identical with the results of a CP-invariant model. As a consequence we see that there cannot be simultaneous decay of both mesons into $\pi^+\pi^-$ from odd L.

The corresponding result for $| \psi_+ \rangle$ is more complicated; we find from (52)

$$\begin{aligned} | \psi_+(t) \rangle = \frac{\sqrt{2}}{r(1+\varrho^2)^2} \Big\{ & 2\,\varrho\big(e^{-i\gamma 2\lambda_S t} \mid K_S(k)\, K_S(-k) \rangle - e^{-i\gamma 2 \lambda_L t} \mid K_L(k)\, K_L(-k) \rangle\big) \\ & - (1-\varrho^2)\, e^{-i\gamma(\lambda_S+\lambda_L)t} \times \mid K_S(k)\, K_L(-k) + K_L(k)\, K_S(-k) \rangle \Big\} . \end{aligned} \tag{55}$$

Note that TCP conservation implies the absence of the last term in Equation (55). Written interms of K^0, $\overline{K}^0$ $| \psi_+(t) \rangle$ contains the states $| K^0(k)\, K^0(-k) \rangle$ and $| \overline{K}^0(k)\, \overline{K}^0(-k) \rangle$ with strangeness $\pm$ 2, in addition to $| \psi_+ \rangle$*). As is seen by applying Bose statistics these three states all contain only even L; they are the only pair states of neutral kaons with this property. The explicit expression is

$$\begin{aligned} | \psi_+(t) \rangle = \frac{e^{-i\gamma(\lambda_S \lambda + L)t}}{(1+\varrho^2)^2} \Big\{ & [(1-\varrho^2)^2 + 4\,\varrho^2 \cos\Delta\,\omega\, t] \cdot | \psi_+ \rangle + \sqrt{2}\, \frac{\varrho}{r}\, (1 - e^{i\Delta\omega t}) \\ & \times [(\varrho^2 + e^{-i\Delta\omega t}) \cdot | K^0(k)\, K^0(-k) \rangle + r^2(1 + \varrho^2 e^{-i\Delta\omega t}) \cdot | \overline{K}^0(k)\, \overline{K}^0(-k) \rangle] \Big\} , \end{aligned} \tag{56}$$

where $\Delta\omega \equiv \gamma\,(\lambda_S - \lambda_L)$.

Quantities of more direct physical significance are the decay rates of the states $| \psi_\mp \rangle$ whereby one kaon decays into channel j_1 at time t_1 and the other into j_2 at t_2,

$$R_\mp(j_1\, t_1 ;\, j_2\, t_2) \equiv |\langle j_1\, j_2 \mid \mathcal{H}_W(1) \cdot \mathcal{H}_W(2)\; e^{-i(\boldsymbol{\omega}(1)t_1 + \boldsymbol{\omega}(2)t_2)} \mid \psi_\mp \rangle|^2 . \tag{57}$$

With Equations (34), (35), (40), and (50) we find

$$R_-(j_1\, t_1 ;\, j_2\, t_2) = \frac{| e^{-i\gamma(\lambda_S t_1 + \lambda_L t_2)} |^2}{2\, | r |^2\, | 1+\varrho^2|^2\, \alpha_{j_1} \alpha_{j_2}} \mid a_{j_1}\, b_{j_2} - b_{j_1}\, a_{j_2}\, e^{i\Delta\omega(t_1 - t_2)} \mid^2 \tag{58}$$

---

*) The following elegant comment is due to Dr. P. K. KABIR: These results can be simply understood in terms of the analogy of kaons with spin 1/2 particles, $K^0 \sim$ spin up, $\overline{K}^0 \sim$ spin down. Then $| \psi_- \rangle$, $| \psi_+ \rangle$ are the singlet and the triplet $m = 0$ states, respectively. Since we have assumed separability, our energy matrix expresses the precession of each spin in a fictitious 'magnetic field'. But the singlet state is unaffected by a magnetic field, since the total spin is conserved, whereas the triplet state undergoes a precession, admixing the other two triplet states $K^0\, K^0$ and $\overline{K}^0\, \overline{K}^0$!

and

$$R_+(j_1\,t_1;\,j_2\,t_2) = \frac{|\,e^{-i\gamma(\lambda_S t_1 + \lambda_L t_2}\,|^2}{2\,|\,r\,|^2\,|\,1+\varrho^2\,|^4\,\alpha_{j_1}\,\alpha_{j_2}}\,|\,2\,\varrho\,(a_{j_1}\,a_{j_2}\,e^{-i\,\Delta\,\omega\,t_2} - b_{j_1}\,b_{j_2}\,e^{+i\,\Delta\,\omega\,t_1})$$

$$- (1-\varrho^2)\,(a_{j_1}\,b_{j_2} + b_{j_1}\,a_{j_2}\,e^{+i\,\Delta\,\omega(t_1-t_2)})\,|^2 \tag{59}$$

where $a_j \equiv \alpha_j + \varrho\,r\,(x_j + i\,y_j)$, $b_j \equiv \varrho\,\alpha_j - r\,(x_j + i\,y_j)$. The cross term $\sim (1-\varrho^2)$ (const. $+\,e^{i\,\Delta\,\omega(t_1-t_2)}$) in R_+ measures a possible violation of TCP. No such test can be made with R_-. For simultaneous decay of the two mesons into the same channel

$$R_-(j\,t;\,j\,t) = 0 \tag{58'}$$

and

$$R_+(j\,t;\,j\,t) = 2\,R_j(t)\cdot\overline{R}_j(t)\,. \tag{59'}$$

Equation (58′) generalizes an earlier observation: Symmetrical decay of a pair state with odd L is impossible, independent of CP violation. In particular the resonance decay $\phi \to K^0\,\overline{K}^0$ only yields asymmetrical events.

Instead of decays, strong interactions of the pair may be detected. The reaction rate for producing a K^0 at t_1 and an other K^0 at t_2 from $|\,\psi_+\,\rangle$, for example, is

$$|\,\langle +1, +1\,|\,e^{-i(\boldsymbol{\omega}(1)t_1+\boldsymbol{\omega}(2)t_2}\,|\,\psi_+\,\rangle|^2 = \frac{|\,e^{-i\gamma(\lambda_S t_1+\lambda_L t_2)}\,|^2}{2\,|\,r\,|^2\,|\,1+\varrho^2\,|^4}$$

$$\times\,|\,2\,(e^{-i\,\Delta\,\omega\,t_2} - \varrho^2\,e^{+i\,\Delta\,\omega\,t_1}) - (1-\varrho^2)\,(1 + e^{+i\,\Delta\,\omega(t_1-t_2)})\,|^2\,. \tag{60}$$

Similar expressions hold for production of $\overline{K}^0\,K^0$ or $\overline{K}^0\,\overline{K}^0$. Again they all contain a TCP-violating term $\sim (1-\varrho^2)$ (const. $+\,e^{i\,\Delta\,\omega(t_1-t_2)}$), whereas no such terms occur with $|\,\psi_-\,\rangle$.

As a final comment we assert that in spite of the correlations of K^0 and $\overline{K}^0$ in $|\,\psi_\mp\,\rangle$, we can use any incoherent mixture of these pair states as a source of mesons with equal mixture of K^0 and $\overline{K}^0$, as is required for measurement of Equations (30) or (31). It is only necessary to use *one* of the two particles produced in the pair, regardless of the state of the other particle. (However, it is vital to be certain that it is a $K^0\,\overline{K}^0$ production event!). To prove this, introduce the density matrix for such an initial state

$$\varrho = |\,\psi_-\,\rangle\,P_-\,\langle\,\psi_-\,| + |\,\psi_+\,\rangle\,P_+\,\langle\,\psi_+\,| = \frac{1}{4}\,(1 - \sigma(1)\cdot\sigma(2))\cdot P_-$$

$$+\,\frac{1}{4}\,(1 + \sigma(1)\cdot\sigma(2) - 2\,\sigma_z(1)\,\sigma_z(2))\cdot P_+ \tag{61}$$

in our standard representation. Here P_- and P_+ are the probabilities of producing $|\,\psi_-\,\rangle$ and $|\,\psi_+\,\rangle$. Now performing a trace over matrices (1), corresponding to summing over all decay products of meson (1), gives

$$\mathrm{tr}_1\,\varrho = \frac{1}{4}\,(P_- + P_+)\,. \tag{62}$$

which describes a mixture with equal weights for K^0 and $\overline{K}^0$, as desired. Summing instead over matrices (2) gives the same result, since ϱ is symmetric in (1) and (2).

Acknowledgments

The authors are pleased to acknowledge fruitful conversations on these subjects with their colleagues, especially with Dr. L. Horwitz and Prof. L. Wolfenstein. One of us (R.R.L.) is indepted to the Swiss National Fund for a grant supporting this work, and to Prof. J. M. Jauch for the hospitality of the Institut de Physique Théorique.

Bibliography

[1]) J. H. Christenson, J. W. Cronin, V. L. Fitch, R. Turlay, Phys. Rev. Letters *13*, 138 (1964); X. de Bouard et al., Phys. Letters *15*, 58 (1965); W. Galbraith et al., Phys. Rev. Letters *14*, 383 (1965).

[2]) M. Gell-Mann and A. Pais, Phys. Rev. *97*, 1387 (1955).

[3]) For some of the earliest discussions, see T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. *106*, 340 (1957); Steven Weinberg, Phys. Rev. *110*, 782 (1958); R. G. Sachs and S. B. Treiman, Phys. Rev. Letters *8*, 137 (1962); R. G. Sachs, Phys. Rev. *129*, 2280 (1963).

[4]) Several authors have suggested that there are more than two neutral kaons, J. L. Uretsky, Phys. Letters *14*, 154 (1965); K. Nishijima and H. Saffouri, Phys. Rev. Letters *14*, 205 (1965); for a discussion of the consequences, see also L. B. Okun and I. Ya. Pomeranchuk, Phys. Letters *16*, 338 (1965); P. K. Kabir and R. R. Lewis, Phys. Rev. Letters *15*, 306 (1965).

[5]) R. R. Lewis and C. P. Enz, Phys. Letters *16*, 73 (1965); R. R. Lewis, Phys. Rev. Letters *14*, 749 (1965).

[6]) R. G. Sachs, Phys. Rev. Letters *13*, 286 (1965); N. Cabibbo, Phys. Letters *12*, 137 (1964); T. N. Truong, Phys. Rev. Letters *13*, 358 (1964); L. Wolfenstein, Phys. Rev. Letters *13*, 562 (1964); and many others.

[7]) J. Bernstein, N. Cabibbo, and T. D. Lee, Phys. Letters *12*, 146 (1964); J. S. Bell and J. K. Perring, Phys. Rev. Letters *13*, 348 (1964); F. Guersey and A. Pais (unpublished).

[8]) T. T. Wu and C. N. Yang, Phys. Rev. Letters *13*, 380 (1964); T. D. Lee and L. Wolfenstein, Phys. Rev. *138*, B1490 (1965); L. Wolfenstein, CERN report TH.583 (1965).

[9]) V. F. Weisskopf and E. P. Wigner, Zs. für Physik *63*, 54 (1930); Ko Aizu, Nuovo Cimento *VI*, 1040 (1957).

[10]) A. H. Rosenfeld et al., Revs. Mod. Phys. *36*, 977 (1964).

[11]) T. Fujii, J. V. Jovanovitch, F. Turkot, G. T. Zorn, Phys. Rev. Letters *13*, 253 (1964).

[12]) V. L. Fitch, R. F. Roth, J. S. Russ, W. Vernon, Phys. Rev. Letters *15*, 73 (1965); J. H. Christenson, J. W. Cronin, V. L. Fitch, R. Turlay, Phys. Rev. (to be published).

[13]) F. S. Crawford, B. B. Crawford, R. L. Golden, G. W. Meisner, Bull. American Phys. Soc. *9*, 443 (1964).

[14]) T. D. Lee, R. Oehme, and C. N. Yang, see Reference 3.

[15]) R. G. Sachs, see Reference 3 and Annals of Phys. *22*, 239 (1964).

[16]) A. Pais and O. Piccioni, Phys. Rev. *100*, 1487 (1955).

[17]) M. L. Good, Phys. Rev. *106*, 591 (1957); *110*, 550 (1958).

[18]) R. H. Good and E. Pauli, Phys. Rev. Letters 8, 223 (1962).

[19]) See Reference 17 and also Malcolm C. Whatley, Phys. Rev. Letters *9*, 317 (1962).

[20]) T. B. Day, Phys. Rev. *121*, 1204 (1961).

[21]) M. Goldhaber, T. D. Lee, and C. N. Yang, Phys. Rev. *112*, 1796 (1958).

[22]) V. L. Lyuboshitz, E. O. Okonov, and M. I. Podgoretskii, Sov. Phys. JETP *20*, 1257 (1965). Several equations contain errors of sign.