An expression for relative strength of chiral and SU(3) breaking interactions

Autor(en): **Acharya, R.**

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An Expression for Relative Strength of Chiral and SU(3) Breaking Interactions[1])

by **R. Acharya**

Institut für theoretische Physik der Universität Bern, Bern (Switzerland)

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Abstract. An explicit expression is derived as a function of π, K, $\varkappa$, K^* and K_A masses for the parameter $\varepsilon_8/\varepsilon_0$ which measures the strength of SU(3) breaking interaction relative to chiral symmetry breaking. The departure of the numerical value of this parameter from its SU(2) ⊗ SU(2) symmetric value is determined and the result is compared with the estimate given by GELL-MANN, OAKES and RENNER.

In a recent paper, GELL-MANN, OAKES and RENNER [1] have investigated the behaviour under SU(3) ⊗ SU(3) of the hadron energy density and the related question of the transformation properties of the axial vector and the strangeness changing vector currents under the same group. This investigation is similar inspirit to but somewhat different in its approach from the earlier work of GLASHOW and WEINBERG [2] on broken chiral symmetry. The various commutators of charges and current divergences are derived in terms of a single constant $\varepsilon_8/\varepsilon_0$ which describes the strength of the SU(3) breaking interaction relative to the chiral symmetry braking interaction. The authors of Ref. [1] find that the constant is *not* small as was originally suggested by GELL-MANN [3] but instead close to the value $(-\sqrt{2})$ corresponding to an exact SU(2) ⊗ SU(2) symmetry and massless pions.

In the present note, we shall work within the same framework of SU(3) ⊗ SU(3) to derive an explicit expression ior $\varepsilon_8/\varepsilon_0$ in terms of the π, K, $\varkappa$, K^* and K_A masses. During the course of this deiivation, we shall utilize a modified form of the second WEINBERG sum rule for strangeness changing currents [4]. This sum rule has proved to be quite effective in yielding a fairly trustworthy prediction of the mass of the kappa meson [5].

We begin by defining

$$H^{00}(x) = \bar{H}^{00}(x) + \varepsilon_0\, S_0(x) + \varepsilon_8\, S_8(x) \tag{1}$$

where $\bar{H}^{00}(x)$ is that part of the hadron energy density which is invariant under SU(3) ⊗ SU(3) and the second and the third terms in equation (1) break this symmetry; under SU(3), the two terms transform as a singlet and an octet respectively.

The non-vanishing vacuum expectation values

$$\lambda_0 = \langle 0 |\, S_0(0)\, | 0 \rangle \tag{2}$$

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and

$$\lambda_8 = \langle 0 | \, S_8(0) \, | 0 \rangle \tag{3}$$

introduce symmetry breaking into the picture,

It is then possible to derive the following three equations which relate the symmetry breaking parameters ε_0, ε_8, λ_0 and λ_8 [6]:

$$\varepsilon_0 \, \lambda_0 = \frac{1}{2} \int_0^\infty d\mu^2 \left\{ \varrho^\pi(\mu^2) + 2 \, \varrho^K(\mu^2) - \frac{2}{3} \, \varrho^\varkappa(\mu^2) \right\} \mu^2 \tag{4}$$

$$\varepsilon_0 \, \lambda_8 + \varepsilon_8 \, \lambda_0 = \sqrt{2} \int_0^\infty d\mu^2 \left\{ \varrho^\pi(\mu^2) - \varrho^K(\mu^2) - \frac{1}{3} \, \varrho^\varkappa(\mu^2) \right\} \mu^2 \tag{5}$$

$$\varepsilon_8 \, \lambda_8 = \frac{4}{3} \int_0^\infty d\mu^2 \, \varrho^\varkappa(\mu^2) \, \mu^2 \tag{6}$$

where $\varrho^\pi(\mu^2)$ etc. stand for usual spin zero spectral functions.

In what follows, we shall assume the PCAC (PCVC) assumption in its strong form:

$$\varrho^{\pi, K, (\varkappa)}(\mu^2) = F^2_{\pi, K, \varkappa} \, \delta \, (\mu^2 - m^2_{\pi, K, \varkappa}) \tag{7}$$

where F_π etc. are defined by the one-particle matrix element in the standard fashion.

Equations (4), (5), (6) and (7) give

$$a \, \frac{\varepsilon_0}{\varepsilon_8} + b \, \frac{\varepsilon_8}{\varepsilon_0} = c \tag{8}$$

with

$$a = \frac{4}{3} \, m^2_\varkappa \, F^2_\varkappa \tag{9}$$

$$b = \frac{1}{2} \left\{ m^2_\pi \, F^2_\pi + 2 \, m^2_K \, F^2_K - \frac{2}{3} \, m^2_\varkappa \, F^2_\varkappa \right\} \tag{10}$$

$$c = \sqrt{2} \left\{ m^2_\pi \, F^2_\pi - m^2_K \, F^2_K - \frac{1}{3} \, m^2_\varkappa \, F^2_\varkappa \right\}. \tag{11}$$

Solving equation (8) for $\varepsilon_8/\varepsilon_0$, we obtain

$$\frac{\varepsilon_8}{\varepsilon_0} = \frac{c \pm \sqrt{c^2 - 4\,a\,b}}{2\,b}. \tag{12}$$

The relation

$$F^2_K = F^2_\pi \tag{13}$$

which, incidentally, holds to all orders in SU(3) breaking interaction under the pole dominance hypothesis of spin one and spin zero spectral functions [7], enables us to rewrite equation (12) as follows:

$$\frac{\varepsilon_8}{\varepsilon_0} = -\sqrt{2} \left\{ \frac{(m^2_K - m^2_\pi) \, (F^2_\pi/F^2_\varkappa) + (1/3) \, m^2_\varkappa \mp \sqrt{m^4_\varkappa - 2 \, (m^2_K + m^2_\pi) \, m^2_\varkappa \, (F^2_\pi/F^2_\varkappa) + (m^2_K - m^2_\pi)^2 \, (F^4_\pi/F^4_\varkappa)}}{(m^2_\pi + 2 \, m^2_K) \, (F^2_\pi/F^2_\varkappa) - (2/3) \, m^2_\varkappa} \right\}. \tag{14}$$

We observe that in the limit $m_\pi \to 0$, equation (14) reduces to the following expression:

$$\frac{\varepsilon_8}{\varepsilon_0} \xrightarrow[m_\pi \to 0]{} -\sqrt{2} \left\{ \frac{(m_K^2 (F_\pi^2/F_\varkappa^2) + (1/3)\, m_\varkappa^2) \mp (m_\varkappa^2 - m_K^2 (F_\pi^2/F_\varkappa^2))}{2\, m_K^2 (F_\pi^2/F_\varkappa^2) - (2/3)\, m_\varkappa^2} \right\}. \tag{15}$$

If we choose the upper (minus) sign in equation (15), then $\varepsilon_8/\varepsilon_0$ tends smoothly to its SU(2) ⊗ SU(2) value of $(-\sqrt{2})$ as m_π goes to zero. If we, instead, choose the lower (plus) sign, then

$$\frac{\varepsilon_8}{\varepsilon_0} \xrightarrow[m_\pi \to 0]{} -\sqrt{2} \left[\frac{(4/3)\, m_\varkappa^2}{2\, m_K^2 (F_\pi^2/F_\varkappa^2) - (2/3)\, m_\varkappa^2} \right]. \tag{16}$$

The solution displayed in equation (16) will tend to the SU(2) ⊗ SU(2) limit provided

$$m_\varkappa^2 = m_K^2 \frac{F_\pi^2}{F_\varkappa^2}. \tag{17}$$

Equation (17) is precisely the relation that one obtains from considerations based on the 'super' validity of asymptotic symmetry[2]). However, as has been pointed out in [4], the assumption of the super validity of asymptotic SU(2) ⊗ SU(2) symmetry for strangeness changing currents is *not* compatible with experiment. In view o this situation, we shall reject equation (16) and hence the plus sign in equation (14),

The ratio $F_\pi^2/F_\varkappa^2$ may be determined as a function of the masses by appealing to the modified form of the second WEINBERG sum rule introduced in [4]:

$$\frac{F_\pi^2}{F_\varkappa^2} = \frac{m_\varkappa^2 - m_{K^*}^2}{m_{K_A}^2 - 2\, m_{K^*}^2 + m_K^2}. \tag{18}$$

We, now, eliminate $F_\pi^2/F_\varkappa^2$ from equations (14) and (18) to obtain the desired final result. *We observe that the parameter $\varepsilon_8/\varepsilon_0$ is expressed solely in terms of the masses.*

We take the values $m_\pi = 140$, $m_K = 490$, $m_{K^*} = 890$, $m_{K_A} = 1250$ and $m_\varkappa = 1130$ (all masses in Mev) into consideration to get

$$\varepsilon_8/\varepsilon_0 \approx -1.56\,. \tag{19}$$

This estimate is somewhat higher (about 25%) than the one given in [1][3]). However, it is appropriate to remark here that equation (14) implies quite generally that $-\varepsilon_8/\varepsilon_0 > \sqrt{2}$ provided the kappa meson mass satisfies the following inequality:

$$m_\varkappa^2 < \frac{3}{2}\,(m_\pi^2 + 2\, m_K^2)\, \frac{F_\pi^2}{F_\varkappa^2}. \tag{20}$$

This inequality is, indeed, satisfied with the above-mentioned choice of the masses. *An accurate experimental determination of the kappa and K_A masses is called for before one can pin down the ratio $\varepsilon_8/\varepsilon_0$ more precisely.*

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[2]) The 'super' validity of asymptotic symmetry asserts that $q^2\, F(q^2)$ and $q^4\, G(q^2)$ *both* vanish as $q^2 \to \infty$ where $\Delta_{K^*}^{\mu\nu}(q) - \Delta_{K_A}^{\mu\nu}(q) = F(q^2)\, \delta_{\mu\nu} + G(q^2)\, q_\mu\, q_\nu$; for details see Reference [4].

[3]) The authors of Reference [1] find that $\varepsilon_8/\varepsilon_0 \approx -1.25$.

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