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On the Ratio of Wave Function Renormalization Constants of π and K Mesons

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Abstract. The validity of Khuri's conjecture on the ratio of the wave function renormalization constants of π and K mesons is investigated within the framework of asymptotic SU(3) symmetry. It is shown that the result holds only if certain strong asymptotic conditions are satisfied. These asymptotic requirements also lead to the degeneracy of A_1 and K_A masses under the pole dominance hypothesis for spin one spectral functions.

A few years ago, KHURI [1] applied the method of Fubini and Furlan to the equal-time canonical commutations of the renormalized π and K fields and arrived at the result that the ratio of the wave function renormalization constants of the kaon and the pion is approximately equal to the fourth power of the ratio of the respective physical masses: $Z_K/Z_\pi \sim m_K^4/m_\pi^4$. Later on, KHURI pointed out that although the derivation of this result was not quite rigorous, the final answer might be true quite generally.

In this note, we investigate the validity of Khuri's relation within the framework of asymptotic symmetry considerations of OKUBO et al. [2]. We show that Khuri's result follows if asymptotic SU(3) symmetry is 'super-valid' in a precise sense, to wit:

$$\lim_{q\to\infty} \{\Delta^{\pi}_{A_{\mu\nu}}(q) - \Delta^{K}_{A_{\mu\nu}}(q)\} \sim q^{-4-\epsilon}.$$

This very condition, however, also leads to several other sum rules for spin one (and spin zero) spectral functions. One obtains, in this manner, a 'positive moment' sum rule, in addition to the first and second Weinberg sum rules. The assumption of saturation of this sum rule by spin one mesons, leads to the degeneracy of A_1 and K_A mesons.

We proceed to derive Khuri's result from asymptotic symmetry considerations. Before we do this, it is perhaps appropriate to recall that the conventional statement of asymptotic symmetry [2] merely leads to the first Weinberg sum rule:

$$\int_0^\infty dm^2 \left[\frac{\varrho_\pi^{(1)}(m^2) - \varrho_K^{(1)}(m^2)}{m^2}\right] = \int_0^\infty dm^2\, [\varrho_K^{(0)}(m^2) - \varrho_\pi^{(0)}(m^2)] \qquad (1)$$

where $\varrho^{(1)}_{\pi,K}(m^2)$ and $\varrho^{(0)}_{\pi,K}(m^2)$ respectively denote the spin one and spin zero spectral functions.

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Strictly speaking, the second Weinberg sum rule

$$\int_0^\infty dm^2\,[\varrho_\pi^{(1)}(m^2) - \varrho_K^{(1)}(m^2)] = 0 \tag{2}$$

is *not* a consequence of the usual form of asymptotic symmetry.

Let us now impose the stronger asymptotic requirement

$$\lim_{q\to\infty} \{\Delta^\pi_{A_{\mu\nu}}(q) - \Delta^K_{A_{\mu\nu}}(q)\} \sim \frac{1}{q^{4+\epsilon}} \tag{3}$$

where $\Delta^{\pi,K}_{A\mu\nu}(q)$ stands for the propagators of π and K type axial vector currents.

Following beaten tracks, we define[2])

$$\Delta^\pi_{A_{\mu\nu}}(q) - \Delta^K_{A_{\mu\nu}}(q) \equiv \delta_{\mu\nu}\,F(q^2) + q_\mu\,q_\nu\,G(q^2)\,. \tag{4}$$

The invariant functions $F(q^2)$ and $G(q^2)$ possess Källén-Lehmann spectral representations:

$$F(q^2) = \int_0^\infty dm^2\,\frac{\varrho_\pi^{(1)}(m^2) - \varrho_K^{(1)}(m^2)}{q^2+m^2} \tag{5}$$

$$G(q^2) = \int_0^\infty dm^2\,\frac{\varrho_\pi^{(1)}(m^2) - \varrho_K^{(1)}(m^2)}{m^2\,(q^2+m^2)} + \int_0^\infty dm^2\,\frac{\varrho_\pi^{(0)}(m^2) - \varrho_K^{(0)}(m^2)}{q^2+m^2}\,. \tag{6}$$

Let us consider the large q^2 limit of $F(q^2)$ and $G(q^2)$:

$$F(q^2) \underset{q^2\to\infty}{\to} \frac{1}{q^2}\int_0^\infty dm^2\,[\varrho_\pi^{(1)}(m^2) - \varrho_K^{(1)}(m^2)] - \frac{1}{q^4}\int_0^\infty dm^2\,[\varrho_\pi^{(1)}(m^2) - \varrho_K^{(1)}(m^2)]\,m^2 + 0\left(\frac{1}{q^6}\right) \tag{7}$$

$$\begin{aligned} G(q^2) \underset{q^2\to\infty}{\to} &\frac{1}{q^2}\left\{\int_0^\infty dm^2\,\frac{\varrho_\pi^{(1)}(m^2) - \varrho_K^{(1)}(m^2)}{m^2} + \int_0^\infty dm^2\,[\varrho_\pi^{(0)}(m^2) - \varrho_K^{(0)}(m^2)]\right\} \\ &- \frac{1}{q^4}\left\{\int_0^\infty dm^2\,[\varrho_\pi^{(1)}(m^2) - \varrho_K^{(1)}(m^2)] + \int_0^\infty dm^2\,[\varrho_\pi^{(0)}(m^2) - \varrho_K^{(0)}(m^2)]\,m^2\right\} \\ &+ \frac{1}{q^6}\left\{\int_0^\infty dm^2\,[\varrho_\pi^{(1)}(m^2) - \varrho_K^{(1)}(m^2)]\,m^2 \right. \\ &\left. + \int_0^\infty dm^2\,[\varrho_\pi^{(0)}(m^2) - \varrho_\pi^{(0)}(m^2)]\,m^4\right\} + 0\left(\frac{1}{q^8}\right). \end{aligned} \tag{8}$$

Equation (3) implies $F(q^2) \sim q^{-4-\epsilon}$ and $G(q^2) \sim q^{-6-\epsilon}$. As a consequence, one arrives at the following sum rules besides equation (1):

[2]) We are ignoring the Schwinger term in equation (4), in view of equation (1).

spin one:

$$\int_0^\infty dm^2\, [\varrho_\pi^{(1)}(m^2) - \varrho_K^{(1)}(m^2)] = 0 \tag{9}$$

$$\int_0^\infty dm^2\, [\varrho_\pi^{(1)}(m^2) - \varrho_K^{(1)}(m^2)]\, m^2 = 0 \tag{10}$$

spin zero:

$$\int_0^\infty dm^2\, [\varrho_\pi^{(0)}(m^2) - \varrho_K^{(0)}(m^2)]\, m^2 = 0 \tag{11}$$

$$\int_0^\infty dm^2\, [\varrho_\pi^{(0)}(m^2) - \varrho_K^{(0)}(m^2)]\, m^4 = 0\,. \tag{12}$$

We recognize equation (9) as the second WEINBERG sum rule [3]; equation (10) is the FRISHMAN sum rule [4]. It is easy to see that equation (11) expresses the equality of the vacuum expectation values of π and K type σ-terms [5]. We are concerned here with equation (12): this is precisely Khuri's relation since PCAC (for both π and K mesons) implies the following equality[3]):

$$m^4\, \varrho_{\pi,\,k}^{(0)}(m^2) = F_{\pi,k}^2\, m_{\pi,k}^4\, \sigma_{\pi,k}(m^2) \tag{13}$$

where $\sigma_{\pi,K}(m^2)$ are the Källén-Lehmann spectral functions for pion and kaon renormalized field propagators:

$$W_{\pi,k}(q^2) = \int_0^\infty dm^2\, \frac{\sigma_{\pi,k}(m^2)}{q^2 + m^2} \tag{14}$$

$$Z_{\pi,k}^{-1} = \int_0^\infty dm^2\, \sigma_{\pi,k}(m^2)\,. \tag{15}$$

Equations (12), (13) and (15) imply Khuri's result:

$$\frac{Z_k}{Z_\pi} = \frac{F_k^2}{F_\pi^2}\, \frac{m_k^4}{m_\pi^4}\,. \tag{16}$$

In view of the near equality of pion and kaon decay constants, equation (16) reduces to KHURI's original conjecture. Unfortunately, however, equations (9) and (10) imply $m_{K_A} = m_{A_1}$ unless one is prepared to give up pole dominance of spin one spectral functions. Of course, one could argue at this point that pole dominance is a bad approximation in the case of the 'positive' moment sume rule such as equation (10). For instance, pole dominance of *spin zero* spectral functions in equation (12), would lead to the catastrophic result ($Z_\pi = Z_K = 1$)

$$\frac{F_\pi^2}{F_k^2} \approx \frac{m_k^4}{m_\pi^4}\,. \tag{17}$$

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[3]) Equation (13) follows from PCAC and the existence of spectral forms for $\Delta_{\mu\nu}^A(q)$ and $W(q^2)$. I am indebted to Professor N. FUCHS for a communication.

But, we hasten to point out that the validity of equation (17) is not necessarily restricted to pole dominance. To see this, let us consider the renormalized field propagators, equation (14) and demand the super validity of asymptotic SU(3) in this case:

$$\lim_{q^2 \to \infty} q^2 \{W_\pi(q^2) - W_k(q^2)\} = 0 . \tag{18}$$

Equation (18) has the immediate consequence

$$Z_\pi = Z_k . \tag{19}$$

Equations (16) and (19) again lead to equation (17)! In other words, the *simultaneous* application of the hypotheses of super validity of asymptotic SU(3) symmetry to current *and* field propagators leads to equation (17) which is in violent disagreement with experiment. The discrepancy can be avoided only if pion and kaon masses are degenerate in which case one is forced to the conclusion that super validity of asymptotic SU(3) symmetry of both field and current propagators implies exact symmetry[4]). *Thus, within the framework of asymptotic symmetry, one must reject equation* (18) *in favour of equation* (3) *in order to maintain Khuri's result, thereby introducing a special preference to current propagators.*

Finnaly, we wish to point out that although Khuri's relation may not be *generally* valid, it is nonetheless possible, in principle, to construct a *specific* model with 'built-in' Khuri relation: a model in which $\partial_\mu A_\mu^{\pi,k} = \mu_0^2 \phi_0^{\pi,k}$ where μ_0 is the common bare mass of π and K and $\phi_0^{\pi,k}$ the corresponding unrenormalized field operator, will automatically satisfy Khuri's relation provided one postulates the validity of the PCAC hypothesis

$$\partial_\mu A_\mu^{\pi,k} = F_{\pi,k} m_{\pi,k}^2 \phi^{\pi,k}; \qquad \phi^{\pi,k} = \frac{1}{\sqrt{Z_{\pi,k}}} \phi_0^{\pi,k} . \tag{20}$$

The gradient-coupling model is one such example [7]. In this model, both equations (11) and (12) lead to Khuri's result, in view of the existence of the following sum rule [8]:

$$\frac{Z_{\pi,k}^{-1}}{\mu_0^2} = \int_0^\infty dm^2 \frac{\sigma_{\pi,k}(m^2)}{m^2} . \tag{21}$$

The gradient-coupling model, however, possesses pathological features and has been shown to be internally inconsistent [9]. In view of this, it is not particularly a good example. The only other example that immediately comes to mind is the σ-model which is known to be renormalizable [10]. The internal consistency of this model also has been challenged [11]. We conclude by remarking that in the absence of a specific model which is internally consistent and which satisfies Khuri's relation, the 'super' validity of asymptotic SU(3) symmetry which leads to Khuri's result, remains untested and an open question[5]).

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[4]) Asymptotic symmetry implies exact symmetry under certain conditions, see [6]; equation (3) of this reference should read $\lim_{q \to \infty} \{S'_{F_n}(q) - S'_{F_p}(q)\} \sim q^{-2-\epsilon}$.

[5]) For the view that SU(2) × SU(2) symmetry is probably better satisfied at infinite energy than SU(3) symmetry, see, S. Okubo, International Theoretical Conference on Particles and Fields, University of Rochester, August 1967.

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[11] See reference [9].