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# Influence of the Chemical Environment on $\beta$ -Decays

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(26. XII. 70)

*Abstract.* The influence of a change of the electron screening on  $\beta^-$ -decay is discussed. The results are used to calculate the change of the half-life of various  $\beta$ -decays in different chemical environments.

## I. Introduction

It has been claimed that in various measurements an influence of the chemical environment on the half-life of  $\beta^-$ -decays could be proved. Quite strong effects, in some cases even changes up to 4%, were reported [1, 2]. In this paper we want to show by an approximate calculation that such a change of the half-life can only be of the order of magnitude of  $10^{-4}$  to  $10^{-5}$ . A possible exception is the very low energy transition  ${}^3\text{H} \rightarrow {}^3\text{He}$  where the effect may be of the order of  $10^{-3}$ .

We assume that the decay rate changes due to the change of the Fermi function  $F(Z, W)$  caused by different values of the screening potential in different chemical environments. The Fermi function is computed by a modified WKB method and the results are compared with a non-relativistic calculation using a Hulthen potential.

## II. Beta Decay Theory

### (a) Relativistic Approximation

The  $ft$ -value of a beta transition is defined as the product of the statistical function  $f(Z, W_0)$  and the half-life  $t_H$

$$f \cdot t_H = \frac{f \cdot \ln 2}{\Lambda} = \text{const.} \quad (1)$$

It depends only on nuclear matrix elements and on the  $\beta$ -decay constants and is therefore not changed by the chemical environment. The transition probability is denoted by  $\Lambda$ , while the statistical function  $f(Z, W_0)$  is defined through the endpoint energy  $W_0$  of the beta particle by

$$f(Z, W_0) = \int_1^{W_0} p W (W_0 - W)^2 F^\pm(Z, W) dW. \quad (2)$$

The momentum and energy of the emitted electron are  $p$  and  $W$ , while  $F^\pm(Z, W)$  is the Fermi function in the screened potential. This function  $F_c^\pm(Z, W)$  for a pure Coulomb field is given by

$$F_c^\pm(Z, W) = 2(1 + \gamma) [\Gamma(2\gamma + 1)]^{-2} (2pR)^{2(\gamma-1)} e^{\mp \xi \frac{W}{p}} \left| \Gamma\left(\gamma + i\alpha Z \frac{W}{p}\right) \right|^2, \quad (3)$$

where  $\gamma$  and  $\xi$  are connected with the fine structure constant by  $\gamma = (1 - \alpha^2 Z^2)^{1/2}$  and  $\xi = \pi\alpha Z$ . The charge number of the nucleus is  $Z$ , while  $R$  is the nuclear radius for which we assume  $R = 1.2 \cdot A^{1/3}$  fm. We denote the difference between the screened electrostatic potential and the pure Coulomb potential at the nucleus by  $V^s$  and call it screening potential. The quantity  $W^s$  is defined by  $W^s = W - V^s$ . The modified WKB approximation<sup>1)</sup>, according to Rose [3] and Good, Jr. [4], replaces the pure Coulomb Fermi function  $F_c^\pm(Z, W)$  by

$$F^\pm(Z, W) \simeq \frac{p^s W^s}{p W} F_c^\pm(Z, W^s). \quad (4)$$

The screened statistical function  $f^{scr}(Z, W_0)$  is then obtained by numerical integration.

For the change of the decay rate we then obtain

$$\frac{\Delta A}{A} = \frac{f_I - f_{II}}{f_I}, \quad (5)$$

where the subscripts I and II denote the different chemical environments which are manifested in a change of the screening potential  $V^s$ .

#### (b) Non-relativistic Approximation

A Hulthen potential  $V_H(r)$  with variable screening parameter  $\lambda$  is assumed to act on the electron

$$V_H(r) = \frac{Z\alpha\lambda e^{-\lambda r}}{1 - e^{-\lambda r}} \xrightarrow{r \rightarrow 0} \frac{Z\alpha}{r} - \frac{Z\alpha\lambda}{2} + \dots \quad (6)$$

For small  $r$  the potential takes Coulombian form with an energy shift of  $1/2 Z\alpha\lambda = V^s$ .

The Schrödinger equation can be solved analytically for  $l = 0$  which leads to the explicit Fermi function  $F_s(Z, W)$  [5]

$$F_s(Z, W) = \left| \frac{\Gamma(1 + i\nu) \Gamma(1 - i\xi)}{\Gamma(1 - 2i\kappa)} \right|^2 \quad (7)$$

with

$$\nu = \frac{(p^2 + a^2)^{1/2}}{\lambda} - \frac{p}{\lambda} \quad \xi = \frac{(p^2 + a^2)^{1/2}}{\lambda} + \frac{p}{\lambda}$$

and

$$\kappa = \frac{p}{\lambda} \quad a = 2Z\alpha\lambda \quad p = (W^2 - 1)^{1/2}.$$

<sup>1)</sup> The WKB results were compared to a solution of the Dirac equation in a self-consistent field by Matese and Johnson [6]. The agreement of these detailed calculations with the simple WKB approximation is very good. It is well worth noticing that the WKB method depends only on the well determined constant  $V^s$ .

### III. Results and Discussion

For various atoms and various states of ionicity<sup>2)</sup> the screening potential  $V^s$  can be computed by means of the Hartree-Fock method yielding the approximate result  $V^s \simeq 1.45 Z^{4/3} \alpha^2$ . Such calculations show also that the change of  $V^s$  is always smaller than  $\alpha^2 = 27.21$  eV if an electron is added to the atom. We denote the difference of the screening potential  $V^s$  in two chemical environments by  $\Delta I \alpha^2$ . Thus we get for the relative change of the transition probability

$$\frac{\Delta A}{A} = \frac{\Delta I}{1.45 Z^{4/3}} \frac{f^{scr} - f^{unscr}}{f^{unscr}}. \quad (8)$$

In the non-relativistic approximation the screening potential  $V^s$  is connected with the parameter  $\lambda$  of the Hulthen potential by  $V^s = Z \alpha \lambda/2$ .

In the following we try to estimate the characteristic parameter  $\Delta I$  for the two different chemical states, i.e. for solid NaCl and for an aqueous solution of NaCl. The solid state forms a ionic crystal with  $\text{Na}^+$  and  $\text{Cl}^-$  at the lattice points. The total screening potential is the sum of the potential  $V^s(\text{Na}^+)$  from the  $\text{Na}^+$  ion and  $V^s(\text{latt})$  from the surrounding ions. The value  $V^s(\text{Na}^+)$  can be computed by a Hartree-Fock program while  $V^s(\text{latt})$  according to [8] is given by

$$V^s(\text{latt}) = \alpha_M \frac{e^2}{a} \quad (9)$$

where  $\alpha_M = 1.747$  is the Madelung constant and  $a = 2.814 \text{ \AA}$  is the lattice constant. The contribution  $V^s(\text{latt})$  is of the order of 8.9 eV. In the aqueous solution, the  $\text{Na}^+$  ion is surrounded by six water molecules which form a dipole layer around the  $\text{Na}^+$  ion.

The centers of the positive and negative charge of the water molecule are  $r_+$  and  $r_-$  respectively. It is convenient to write for the water dipole moment  $\delta (r_+ - r_-)$ , where the charge  $\delta$  is of the order of the electron charge. The contribution of the six water dipoles to the screening potential is thus given by

$$V^s(\text{dip}) = 6 e \delta \left( \frac{1}{r_-} - \frac{1}{r_+} \right). \quad (10)$$

From the experimental value of the dipole moment of  $1.86 \times 10^{-18}$  esu and from  $r_- = 1.05 \text{ \AA}$  and  $r_+ = 1.60 \text{ \AA}$  we obtain for the dipole screening potential a value of 20 eV. In the difference of the screening potentials between liquid and solid states, the explicit value of  $V^s(\text{Na}^+)$  cancels. We get

$$V^s(L) - V^s(S) = 11 \text{ eV} = 0.4 \alpha^2 \quad (11)$$

which leads finally to the relative change in the transition probability

$$\frac{\Delta_L - \Delta_S}{\Delta_L} = -0.030/_{00}. \quad (12)$$

We also have investigated the dependence of the relative change  $\Delta A/A$  as a function of  $Z$  and the energy  $W_0$ , both for electron and positron emitters, which is shown in Figures 1 and 2. The parameter  $\Delta I$  is assumed to be unity. It is noted that the effects

<sup>2)</sup> For example for  $\text{Na}^+$  and  $\text{Na}^0$  we found  $\Delta I = 0.4 \alpha^2$ .

become largest for low  $Z$  and low  $W_0$  in the case of electron emission, and for high  $Z$  and low  $W_0$  for positron emission. In the latter case, the electron capture might be dominant and must also be considered.

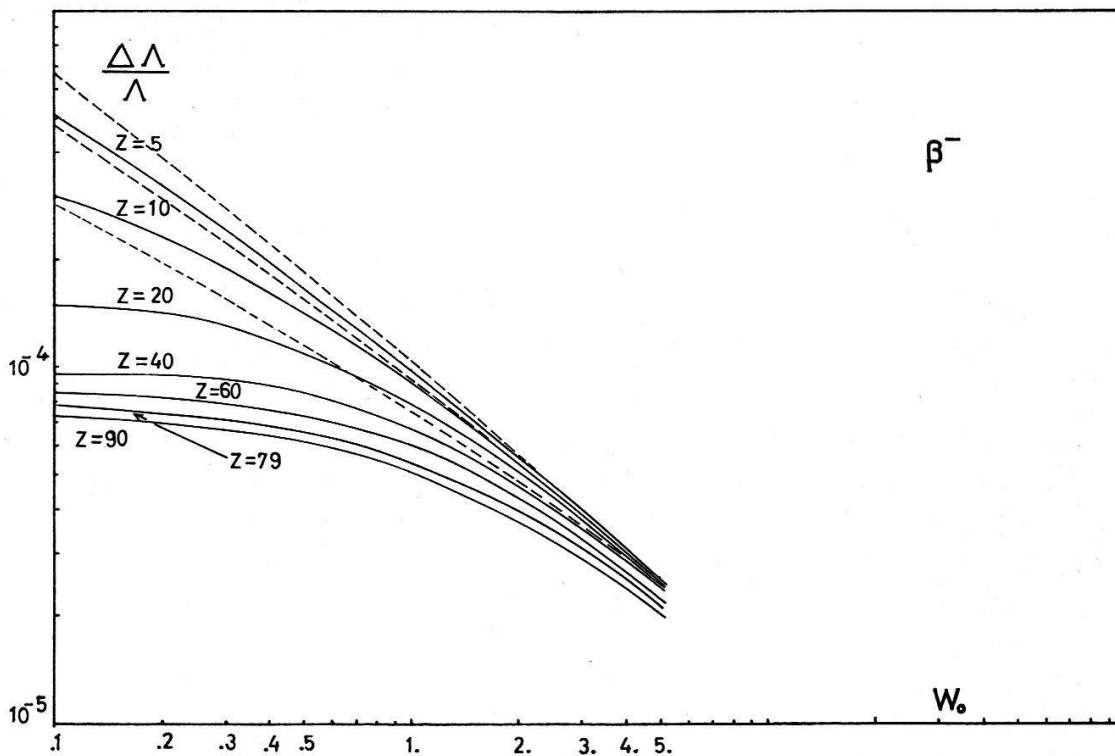


Figure 1

The change  $\Delta\Lambda/\Lambda$  is shown as a function of  $Z$  and  $W_0$  for  $\beta^-$ -decays for  $\Delta I = 1$ . The dashed lines are calculated with the approximation for low  $Z$  given by D. H. Wilkinson [8]. These lines correspond to  $Z = 5, 10$  and  $20$ .

In Table 1 we collected the results of some  $\beta^-$ -decays. Especially for low  $Z$  and low  $W_0$ , the simple non-relativistic calculation shows a remarkable agreement with the relativistic one. From the above considerations it seems to us that the reported changes of the half-life [1, 2] in the  $\beta^-$ -decay of  $^{24}\text{Na}$  and  $^{131}\text{I}$  are not correct. The conditions seem to be more favourable for the low energy transition  $^{35}\text{S} \rightarrow ^{35}\text{Cl}$ , especially since  $^{35}\text{S}$  can be produced in different valence states (+6 to -2). Also the very low energy (18 keV) transition  $^3\text{H} \rightarrow ^3\text{He}$  should give rather large effects in different chemical surroundings.

Table 1

The change  $\Delta\Lambda/\Lambda$  for some selected  $\beta^-$ -decays for  $\Delta I = 1$ . The entries show the transition, the charge number of the daughter nucleus, the transition energy, the relativistic and unrelativistic statistical factor. The last few entries indicated the change of the transition probability according to the relativistic and the unrelativistic approximation

Transition	$Z$	$W$ ( $m_e c^2$ )	$f$ (rel.)	$f$ (n. rel.)	$\Delta\Lambda/\Lambda$ ‰ rel.	$\Delta\Lambda/\Lambda$ ‰ n. rel.
$^3\text{H} \rightarrow ^3\text{He}$	2	1.036	2.603 (-6)	2.85 (-6)	2.75	3.0
$^{24}\text{Na} \rightarrow ^{24}\text{Mg}$	12	3.724	2.453 (+1)	2.02 (+1)	0.07	0.03
$^{35}\text{S} \rightarrow ^{35}\text{Cl}$	17	1.326	1.190 (-2)	1.15 (-2)	0.11	0.1
$^{131}\text{I} \rightarrow ^{131}\text{Xe}$	54	2.186	5.906 (0)	1.91 (0)	0.07	0.008

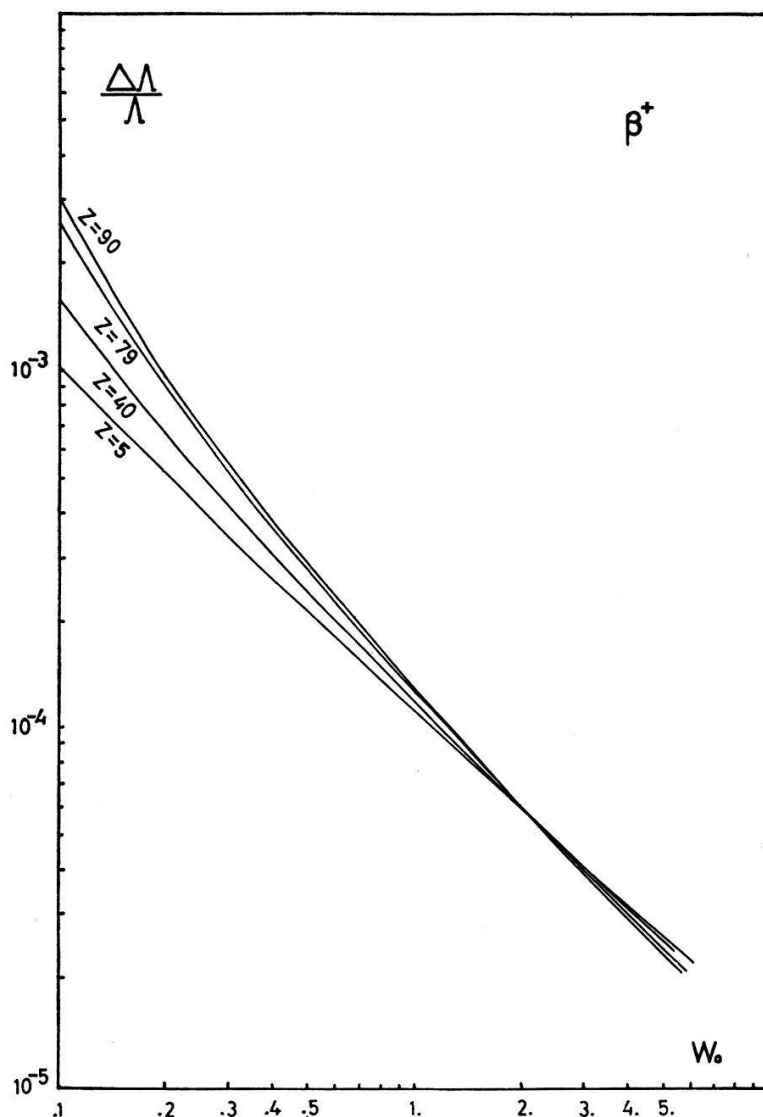


Figure 2  
The change in half-life  $\Delta\lambda/\lambda$  is shown as a function of  $Z$  and  $W_0$  for  $\beta^+$ -decays for  $\Delta I = 1$ .

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