Acceleration of charged particles in the electromagnetic field of pulsars

Autor(en): **Fischer, W. / Straumann, N.**

Objekttyp: **Article**

Zeitschrift: **Helvetica Physica Acta**

Band (Jahr): **45 (1972)**

Heft 7

PDF erstellt am: **13.09.2024**

Persistenter Link: https://doi.org/10.5169/seals-114429

Helvetica Physica Acta
Vol. 45, 1972.

Birkhäuser Verlag Basel

Acceleration of Charged Particles in the Electromagnetic Field of Pulsars

by **W. Fischer**

SIN Zürich[1])

and **N. Straumann**

SIN and University of Zürich

(31. VII. 72)

Abstract. The acceleration of charged particles in the wave zone of the electromagnetic field of pulsars is studied numerically and compared with previous approximate analytic solutions.

1. Introduction

It has been suggested by Gunn and Ostriker [1] that pulsars are probably sources for cosmic rays. The copious x- and gamma-ray emission of the Crab nebula could then be attributed to synchrotron radiation of electrons which are continuously injected from the central pulsar into the nebula with energies up to $\simeq 10^{13}$ eV.

Because of a possible extended corotating magnetosphere [2, 3] it is not clear whether the charged particles are strongly accelerated already in the near field zone or only further out in the wave zone. The fields in the near and intermediate zones will be strongly affected by a corotating magnetospheric plasma and the motion of a charged particle from the surface of the star into the wave zone will be extremely complicated. In [1] Gunn and Ostriker have studied the motion of a test particle in the wave zone of the low frequency dipole field of a pulsar which arises if the magnetic axis is oblique to the axis of rotation. Any interaction of the plasma and the radiation field in the wave zone was thereby neglected. They solved the strongly nonlinear equations of motion in a simple approximation which could, however, lead *a priori* to a large over-estimate of the resulting energies of the accelerated particles. (The approximation of Gunn and Ostriker will be repeated further below.) For this reason, we found it worthwhile to solve the equations of motion numerically.

In this paper we present the solution of the numerical integration and compare it with the approximation obtained by Gunn and Ostriker. A similar study of the acceleration of charged particles in the near field zone of a rotating magnetic (vacuum) dipole field is in progress.

---

[1]) Now: SIN Schweizerisches Institut für Nuklearforschung CH 5234 Villigen.

2. Equations of Motion

Consider an electromagnetic field travelling in the z-direction with the following non-vanishing components (the z-direction corresponds to a radial direction from the pulsar)

$$E_x = B_y = \frac{z_0}{z} E(t - z). \tag{1}$$

If we absorb the factor e/m of a test particle in this field in the function $E(t - z)$, then the equations of motion are ($c = 1$)

$$\ddot{t} = \frac{z_0}{z} E(t - z)\, \dot{x}, \quad \ddot{x} = \frac{z_0}{z} E(t - z)(\dot{t} - \dot{z})$$

$$\ddot{y} = 0, \qquad \ddot{z} = \frac{z_0}{z} E(t - z)\, \dot{x}, \tag{2}$$

where the dot indicates differentiation with respect to the eigentime s. We immediately conclude that $\ddot{t} - \ddot{z} = 0$. If we choose as initial conditions $\dot{x} = \dot{y} = \dot{z} = 0$ for $s = 0$, then

$$\dot{t} = 1 + \dot{z} \tag{3}$$

and

$$\dot{y} = 0.$$

From $\dot{t}^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2 = 1$ we obtain then

$$\dot{x}^2 = 2\dot{z} \tag{4}$$

and hence the following equation for $z(s)$ results

$$\ddot{z} = \frac{z_0}{z} E(s) \sqrt{2\dot{z}}. \tag{5}$$

Beside $\dot{z} = 0$ for $s = 0$ we choose as a further initial condition $z = z_0$.

We put

$$E(s) = \omega_0 \sin(\omega s)$$

and

$$z_0 = 1/\omega \quad \text{(radius of the velocity-of-light cylinder)},$$

where ω is the frequency of the rotating pulsar and ω_0 the formal gyrofrequency corresponding to the magnetic field at the distance z_0.

Equation (5) then reads

$$\ddot{z} = \frac{\omega_0}{\omega} \sin(\omega s) \frac{\sqrt{2\dot{z}}}{z}. \tag{6}$$

The 'nonlinearity parameter' ω_0/ω is extremely large; if the surface field of a pulsar is $\simeq 10^{12}$ Gauss, then $\omega_0/\omega \sim 4 \times 10^{11}$ for electrons, if $\omega \simeq 1.9 \times 10^2$ sec^{-1}. This leads to a tremendous acceleration in a very short eigentime s. This can already be seen by considering a plane wave, i.e. by neglecting the damping $1/z$ in (1). In that case, the equation corresponding to (6) can immediately be solved, giving

$$\dot{z}(s) = \frac{1}{2}\left(\frac{\omega_0}{\omega}\right)^2 [1 - \cos(\omega s)]^2. \tag{7}$$

For $0 < s < \pi/\omega$ the derivative $\dot{z}(s)$ which is, according to (3), equal to the relativistic γ-factor minus one, increases from zero to a maximum $2(\omega_0/\omega)^2$—a truly tremendous number. Such high energies are, however, not reached if the amplitude of the wave decreases like $1/z$. Within the eigentime $s = \pi/\omega$ the particle already moves very far away and soon feels only a weakened field. For the same reason, the particle is only modestly decelerated during $\pi/\omega < s < 2\pi/\omega$, whereas for a strict plane wave the particle is again brought to rest, of course. Similarly, during later periods the energy will no more be changed much.

Gunn and Ostriker have integrated equation (6) approximately by replacing $\sin(\omega s)$ by $\sin\phi_0$, ϕ_0 = constant. The solution of this simplified equation is immediately found to be

$$\dot{z} = \left[\frac{3}{\sqrt{2}}\frac{\omega_0}{\omega}\sin\phi_0 \ln\frac{z}{z_0}\right]^{2/3}. \tag{8}$$

Gunn and Ostriker have taken $\sin\phi_0 \simeq 1$. This choice is, however, not obvious to us. The effective phase ϕ_0 could, *a priori*, be very small (consider as an example again a plane wave). Furthermore, the monotonic increase in (8) with z is unrealistic.

3. Numerical Solution of the Equations of Motion and Results

For the numerical solution of (6), we had to go through 10^5–10^6 integration steps. Thanks to its stability, we have used the integration method of Hamming [4]. This method is not self-starting. The starting solution has been constructed with an iterative procedure, taking as input the solution (7) for a plane wave. This then leads to three precise starting values needed for the Hamming integration.

The results of the numerical integration are shown in Figure 1 and Figure 2 for two different fields, corresponding to a magnetic field strength of 10^{12} Gauss and 1.5×10^9 Gauss, respectively, at the surface of the pulsar. The energy obviously reaches a maximum for $s = \pi/\omega$. After a few periods, the energy reaches practically an asymptotic value. On the other hand, the approximate solution (8) steadily increases, although only logarithmically, and overshoots the correct value by an order of magnitude.

4. Radiation Damping

One might ask whether the radiation damping changes the acceleration noticeably. It is, however, easy to see that this is not the case. From the well-known formula

$$\frac{dE_{\mathrm{rad}}}{ds} = \frac{2e^2}{3}(\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2 - \ddot{t}^2)\,\dot{t}$$

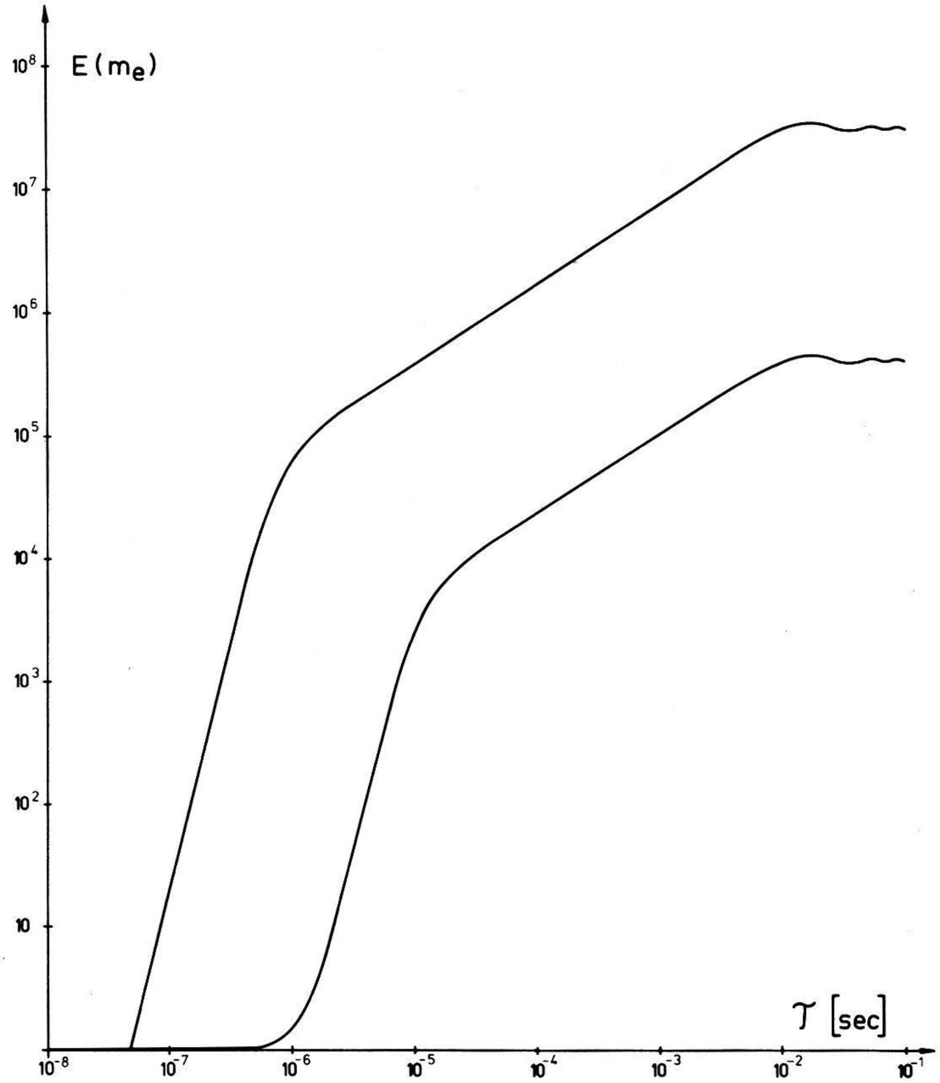

Figure 1
Energy $E(m_e)$ of the accelerated electrons in units of the electron mass as a function of eigentime τ. The upper (lower) curve corresponds to a surface field of 10^{12} (1.5×10^9) Gauss and a rotation frequency $\omega \simeq 1.9 \times 10^2\ \text{sec}^{-1}$.

one finds, using (3) and (4), that

$$\frac{dE_{\text{rad}}}{ds} = \frac{2e^2}{3} \ddot{z}^2 \frac{1+\dot{z}}{2\dot{z}} \cdot$$

Hence

$$\frac{dE_{\text{rad}}(s)/ds}{dE(s)/ds} = \frac{e^2}{3m} \ddot{z} \left(1 + \frac{1}{\dot{z}}\right) \sim 10^{-26} \ddot{z} \lll 1,$$

since $\ddot{z}$ never becomes larger than $\sim 10^{11}$ ($c = 1$). The radiation loss is small because the acceleration of the test particle is mainly longitudinal (see equation (4)).

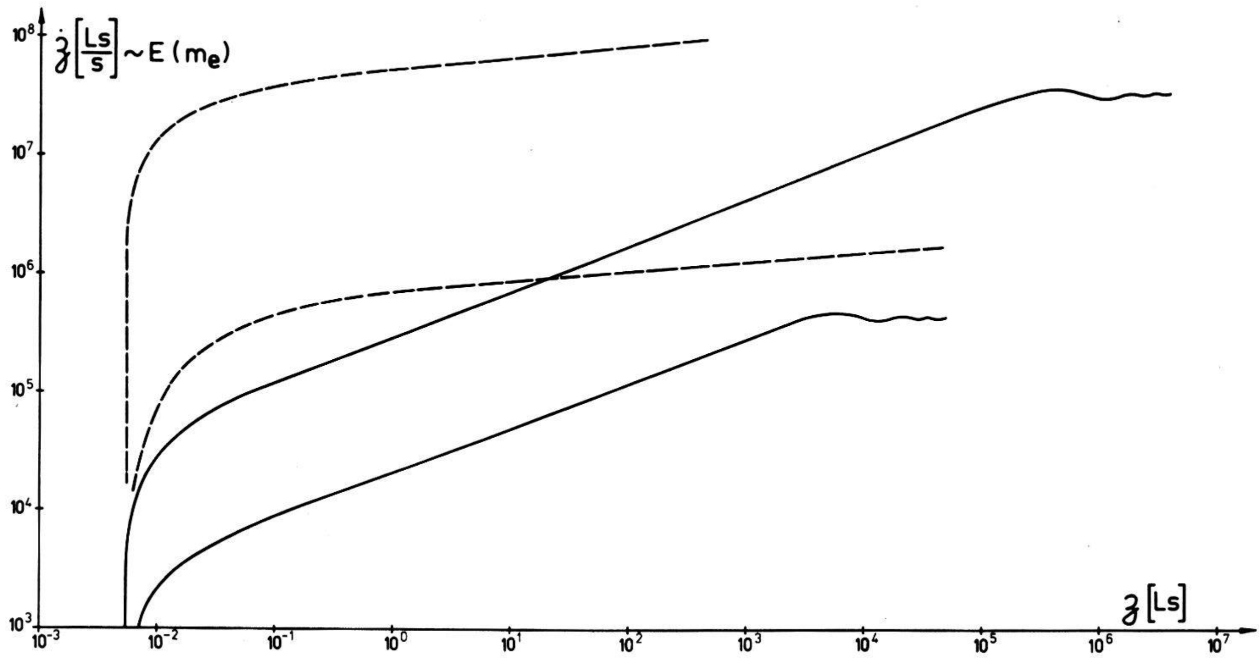

Figure 2
$dz/d\tau = E(m_e) - 1$ as a function of the distance z in units of light seconds. The dotted curves represent the approximate solution of Gunn and Ostriker [1].

Acknowledgements

We thank Professor M. Fierz for numerous interesting discussions and contributions related to this work. We also thank the members of the SIN theory group for their interest and encouragement.

REFERENCES

[1] J. E. GUNN and J. P. OSTRIKER, Phys. Rev. Letters *22*, 728 (1969).
[2] P. GOLDREICH and W. H. JULIAN, Ap. J. *157*, 869 (1969).
[3] J. M. COHEN and E. T. TONTON, Ap. Letters *7*, 213 (1971).
[4] R. W. HAMMING, J. Assoc. Comp. Mach. *6*, 37 (1959).