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# The Extraordinary Hall Effect in Kondo Alloys

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*Abstract.* The extraordinary Hall effect is calculated in a dilute magnetic alloy, taking into account the logarithmic temperature dependence that occurs in Kondo systems. To do this, it was necessary to evaluate the vertex part $\Gamma$ in a magnetic field. This was achieved only by summing a simple ladder series of diagrams for $\Gamma$. The results are discussed in the low field and high temperature limits.

## I. Introduction

The Hall effect in dilute alloys was first discussed theoretically by Béal-Monod and Weiner [1] using the exchange interaction Hamiltonian $-J\mathbf{S}\cdot\boldsymbol{\sigma}$ with an isotropic band. The experimental results showed however that additional large effects were present [2]. These were calculated [3] by introducing an additional, skew, interaction which had been proposed earlier by Kondo [4] on quite general grounds.

The main result of these investigations was that the so-called *s–d* interaction has in reality a much more complicated form than the simple exchange term, a point emphasized also by Hirst [5] and, for a special case, by Coqblin and Schrieffer [6]. Quite recently an attempt has been made to explain the experimental Hall effect data through band structure effects, thus leaving the exchange Hamiltonian untouched [7].

We feel however, that, besides the Hall effect, for which a convincing amount of experimental data has been gathered by Fert and his group [8], recent measurements of the magneto-resistivity in *Au* RE alloys [9] and of the thermopower in $PrCu_5$ and $PrNi_2$ compounds [10] suggest strongly that one has indeed to generalize the simle $\mathbf{S}\cdot\boldsymbol{\sigma}$ interaction in order to understand the detailed behaviour of localized moments in metals.

Although most experimental results of the Hall effect in magnetic alloys have been obtained in systems whose Kondo temperature is vanishingly small, some of the earlier results [2], and those of Ref. [7], were obtained in so-called Kondo alloys.

The aim of this paper is to examine the extraordinary Hall effect taking into account the logarithmic temperature dependence which arises when one goes beyond lowest order in perturbation theory. It is well known that this is necessary to explain the low temperature behaviour of the longitudinal electrical resistivity.

Previous calculations of the EHE in these alloys [3] have only considered the lowest non-zero order in an expansion in powers of the *s–d* interaction and hence do not include any logarithmic terms.

We shall use the model introduced by Kondo [4] and which was used by us for our earlier calculations [3]. This model may be written (for a single impurity)

$$\mathscr{H}_{sd} = \mathscr{H}_{sd}^{(1)} + \mathscr{H}_{sd}^{(2)}$$

with

$$\mathscr{H}_{sd}^{(1)} = -J_{ex} \sum_{\substack{\mathbf{k}\,\mathbf{k}' \\ \alpha\,\alpha'}} \mathbf{S}\cdot\boldsymbol{\sigma}_{\alpha\alpha'} a_{\mathbf{k}\alpha} a_{\mathbf{k}'\alpha'} \tag{1}$$

is the usual form of the *s–d* interaction and

$$\mathscr{H}_{sd}^{(2)} = \lambda_1 \sum_{\substack{\mathbf{k}\,\mathbf{k}' \\ \alpha}} \frac{1}{k_f^2} i(\mathbf{k}\times\mathbf{k}')\cdot\mathbf{S} a_{\mathbf{k}\alpha}^{+} a_{\mathbf{k}'\alpha} \tag{2}$$

represents the interaction between the localized spin **S** and the $l = 1$ component of the electronic orbital angular momentum. $\alpha$ is the magnetic quantum number of the conduction electron spin and the $\boldsymbol{\sigma}$ are the Pauli spin matrices. We shall assume that the coupling constant $\lambda_1$ is much less than $J_{ex}$, so that we only need calculate to first order in $\lambda_1$. The spin-independent part of the interaction has been omitted from the calculations for the sake of simplicity.

Using the above interaction, we shall evaluate the Hall resistivity $\rho_H$ at zero frequency by a method that is essentially that introduced by Götze and Wölfle [11] and used by Giovannini [3] for this problem. This method relates the longitudinal and transverse resistivities, $\rho$ and $\rho_H$, to the longitudinal and transverse conductivities $\sigma_{xx}^{(1)}$ and $\sigma_{xy}^{(1)}$ where the superscript (1) implies that the conductivities are calculated formally to first order in the concentration. Then

$$\left.\begin{aligned} \rho &= \left(\frac{m}{ne^2}\right)^2 z^2 \sigma_{xx}^{(1)}(z) \\ \rho_H &= \left(\frac{m}{ne^2}\right)^2 z^2 \sigma_{xy}^{(1)}(z). \end{aligned}\right\} \tag{3}$$

$\sigma^{(1)}$ behaves as $1/z^2$ as $z \to 0$ so that $\rho$ and $\rho_H$ tend to finite values. It should be emphasized that equation (3) does not represent an expansion in the interaction strength (indeed, we shall sum a certain class of diagram for $\sigma_{xy}^{(1)}$ to all orders) but rather an expansion in the concentration of impurities. We admit that it is doubtful that equation (3) is generally valid, but it forms a plausible and very convenient basis for calculations. In particular, the problem is thereby reduced to a single impurity problem.

As we shall see, the calculation of $\rho_H$ reduces to the evaluation of the renormalized vertex function of the interaction $\mathscr{H}_{sd}^{(1)}$ in a magnetic field. This task is quite intractable (even without a magnetic field) and so we restrict ourselves to summing that part of the series for the vertex which can be summed analytically; this corresponds to the simple ladder approximation that has been discussed by Mattuck [12]. Although this certainly omits terms (parquet graphs') of the same order as those we include, it might be hoped, as Mattuck suggested is the case for the resistivity, that we obtain roughly the right sort of temperature dependence. We can also compare $\rho$ and $\rho_H$ calculated in the same approximation.

The calculation is performed diagrammatically using the Abrikosov [13] pseudofermion technique to describe the localized spins. The restriction to lowest order in the

concentration permits us to consider a single impurity, which in turn implies that just one continuous impurity line be present in a diagram. Diagrams with more than one such line vanish due to additional factors $e^{-\beta\lambda}$ which arise, where $\lambda$ is the energy assigned to a spin to ensure that the occupancy of a spin site is unity.

The calculation in this paper includes the Kondo effect only to the extent that it directly affects the transport properties. We have excluded the modifications that arise from the renormalization of the spins S, that is to say we have not taken account of the modification of the susceptibility due to the Kondo effect. Since the Hall effect depends explicitly on the effects of a magnetic field, this might be serious. Nevertheless, as the influence of the Kondo state upon the transport properties (as manifested by the electrical resistivity) and upon the magnetic properties (as manifested by the susceptibility) appear to be fairly distinct problems we think it reasonable at the beginning to attack them separately. In this paper, therefore, we only consider the former problem.

We now turn our attention to the calculation of the vertex function in a magnetic field, and in Section III we shall use this result to evaluate the Hall resistivity.

## II. The Vertex Function

The approximation for the vertex function $\Gamma$ that we shall use includes only those diagrams in which the conduction electron Greens functions and the localized spin Greens functions are either always parallel or always antiparallel. We omit all diagrams which mix the two. $\Gamma$ therefore splits naturally into two components, which we shall call $\Gamma^p$ ($p$ for particle) and $\Gamma^h$ ($h$ for hole). Note that $\Gamma$ is a matrix in the two spin spaces of the conduction electrons and of the localized spins, as well as depending upon frequency.

The diagrams for $\Gamma^p$ and $\Gamma^h$ are shown in Figure 1, where the external lines are included for clarity. The external frequencies are labelled in each case in such a way that $\Gamma$ depends only upon the single frequency $\omega_m$. In the next section we shall see that we need to calculate $\Gamma^p$ with the frequency continued to the real axis such that

$$i\omega_m \to y + \lambda - i\delta \tag{4}$$

and to calculate $\Gamma^h$ for

$$i\omega_m \to y + \lambda + i\delta. \tag{5}$$

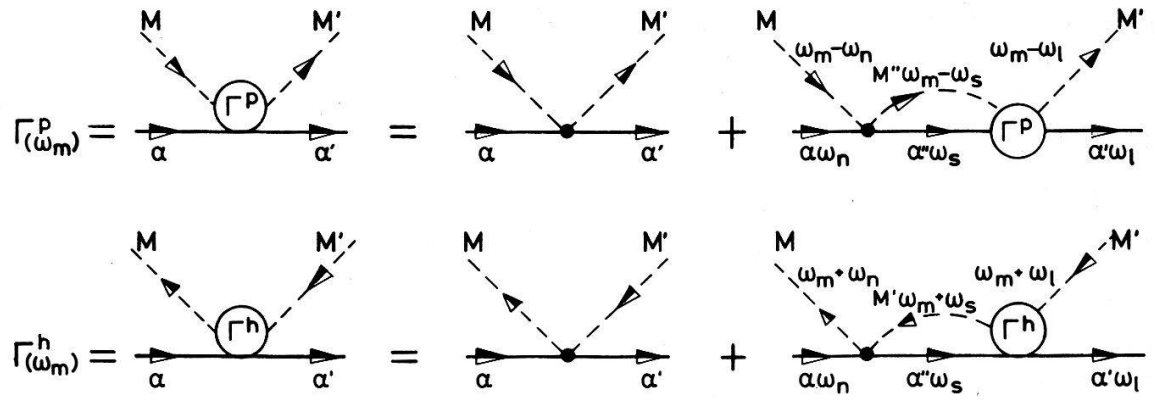


Figure 1

The magnetic field $H$ is included in the Greens function which now depends upon the magnetic quantum numbers $\alpha$ (for the conduction electrons) and $M$ (for the localized spins):

$$G_\alpha(\mathbf{k}, \omega_n) = \frac{1}{i\omega_n - \varepsilon_\kappa - \alpha v_e} \tag{6}$$

for the conduction electrons.

$$\mathcal{G}_M(\omega_n) = \frac{1}{i\omega_n - \lambda - M v_i}$$

for the localized spins.

$$v_e = g_e \mu_B H$$
$$v_i = g_i \mu_B H.$$

At this point we recall that, following Abrikosov [13], at the end of the entire calculation we must multiply the result by

$$e^{\beta\lambda} \frac{\sinh \frac{1}{2}\beta v_i}{\sinh(S + \frac{1}{2})\beta v_i}$$

and take the limit $\lambda \to \infty$.

The integral equation for $\Gamma^p$ is

$$\Gamma^p_{\substack{\alpha\alpha' \\ MM'}}(\omega_m) = -J_{ex}\boldsymbol{\sigma}_{\alpha\alpha'}\mathbf{S}_{MM'} + \sum_{\alpha''M''} \boldsymbol{\sigma}_{\alpha\alpha''}\cdot\mathbf{S}_{MM''}F_{M''}(\omega_m)\Gamma^p_{\substack{\alpha''\alpha' \\ M''M'}}(\omega_m) \tag{7}$$

where $F_M(\omega_m)$ is the kernel

$$\begin{aligned} &-J_{ex}T\sum_{\omega_s}\sum_{\mathbf{k}} G_\alpha(\mathbf{k}, \omega_s)\mathcal{G}_M(\omega_m - \omega_s) \\ &= -N(o)J_{ex}\int d\varepsilon\, T\sum_{\omega_s} \frac{1}{i\omega_s - \varepsilon - \alpha v_e}\,\frac{1}{i(\omega_m - \omega_s) - \lambda - M v_i} \\ &= -\tfrac{1}{2}N(o)J_{ex}\int d\varepsilon\, \frac{\tanh\beta(\varepsilon + \alpha v_e)/2 + \tanh\beta(M v_i + \lambda)/2}{\varepsilon + \alpha v_e + M v_i - i\omega_m + \lambda} \end{aligned} \tag{8}$$

where the frequency sum has been carried out in the usual way. If it is now assumed that the bandwidth $D$ is much greater than $v_e$, we can shift the energy integral by $\alpha v_e$ and neglect the change in the integral's limits. $F$ does not then depend upon $\alpha$, as our notation has anticipated. We now perform the continuation according to equation (4) and obtain

$$\begin{aligned} F_M(y + \lambda - i\delta) = -\tfrac{1}{2}N(o)J_{ex}\Bigg\{ &\oint d\varepsilon\, \frac{\tanh\frac{\varepsilon}{2} + \tanh\beta(\lambda + M v_i)/2}{\varepsilon + M v_i - y} \\ &- i\pi(\tanh\beta(y - M v_i)/2 + \tanh\beta(\lambda + M v_i)/2)\Bigg\} \equiv F(y - M v_i), \end{aligned} \tag{9}$$

where the function $F(x)$ is defined as

$$F(x) = -\tfrac{1}{2} J_{ex} N(o) \left\{ \ln \frac{D^2}{x^2 + 4T^2} - i\pi \left( 1 + \tanh \beta \frac{x}{2} \right) \right\}. \tag{10}$$

The real part of the integral has been approximated for $D \gg x, T$ and in the imaginary part we have anticipated the limit $\lambda \to \infty$. In the following we shall write

$$F(y - Mv_i) \equiv F_M.$$

Because of the magnetic field the usual ansatz, $\Gamma = \Gamma_0 + \Gamma_1 \vec{\sigma} \cdot \vec{S}$ will not work. However, the fact that the kernel $F$ does not depend upon electron spin suggests the following general form (dropping the frequency variable)

$$\Gamma^p_{\substack{\alpha\alpha' \\ MM'}} = \Gamma^{p0}_{MM'} \delta_{\alpha\alpha'} + \sigma^i_{\alpha\alpha'} \Gamma^{pi}_{MM'}, \tag{11}$$

where the $\sigma^i$ are the Pauli matrices and the repeated index $i$ implies summation. The four matrices $\Gamma^0$, $\Gamma^i$ remain to be determined.

Using the following relations

$$\sigma^i \sigma^j = \delta_{ij} + i\varepsilon_{ijk} \sigma^k$$

$$\varepsilon_{ijk} A^j B^k = (\mathbf{A} \times \mathbf{B})^i$$

and writing $\Gamma^i$ as a vector in real space, equations (11) and (7) yield (dropping for the moment the superscript $p$)

$$\Gamma^0_{MM'} = \sum_{M''} \mathbf{S}_{MM''} \Gamma_{M''M'} F_{M''} \tag{12}$$

$$\Gamma_{MM'} = -J_{ex} \mathbf{S}_{MM'} + \sum_{M''} \{ i(\mathbf{S}_{MM''} \times \Gamma_{M''M'}) + \mathbf{S}_{MM''} \Gamma^0_{M''M'} \} F_{M''}. \tag{13}$$

These equations are straightforwardly but laboriously solved in terms of the matrices $\Gamma^0$, $\Gamma^z$ and $\Gamma^{\pm} = 1/\sqrt{2}\,(\Gamma^x \pm i\Gamma^y)$.

Substituting for the matrix elements of $\mathbf{S}$,

$$S^+_{MM'} = \delta_{M',M-1} A_{M-1}$$

$$S^-_{MM'} = \delta_{M',M+1} A_M$$

$$S^z_{MM'} = M \delta_{MM'},$$

where $A_M = \sqrt{\tfrac{1}{2}(S - M)(S + M + 1)}$ one finds

$$\Gamma^{p\pm}_{MM'} = \Gamma^{p\pm}_M \delta_{M',M\mp1}$$

$$\Gamma^{p0}_{MM'} = \Gamma^{p0}_M \delta_{MM'}$$

$$\Gamma^{pz}_{MM'} = \Gamma^{pz}_M \delta_{MM'}$$

with

$$\left.\begin{aligned}
\Gamma_M^{p+} &= \frac{-J_{ex}A_{M-1}}{1+MF_M-(M-1)F_{M-1}-S(S+1)F_MF_{M-1}} = \Gamma_{M-1}^{p-} \\
\Gamma_M^{p-} &= \frac{-J_{ex}A_M}{1-MF_M+(M+1)F_{M+1}-S(S+1)F_MF_{M+1}} = \Gamma_{M+1}^{p+} \\
\Gamma_M^{p0} \pm \Gamma_M^{pz} &= \frac{-J_{ex}[S(S+1)F_{M\pm1} \pm M]}{1 \mp MF_M \pm (M \pm 1)F_{M\pm1} - S(S+1)F_MF_{M\pm1}}.
\end{aligned}\right\} \tag{14}$$

The calculation for $\Gamma^h$ is similar with the kernel replaced by

$$F'_M(\omega_M) = -J_{ex}\sum_{\omega_s}\sum_{k} G_\alpha(\mathbf{k},\omega_s)\,G_M(\omega_m+\omega_s)$$

followed by the continuation equation (5), leading to

$$F'_M(y+\lambda+i\delta) = -(F(y-Mv_l))^*.$$

Furthermore the equation for $\Gamma^h$ corresponding to equation (7) has the matrices $S$ transposed. The result is that

$$\left.\begin{aligned}
\Gamma_M^{h\pm}(y+\lambda+i\delta) &= (\Gamma_M^{p\pm}(y+\lambda-i\delta))^* \\
\Gamma_M^{hz}(y+\lambda+i\delta) &= \Gamma_M^{pz}(y+\lambda-i\delta))^* \\
\Gamma_M^{h0}(y+\lambda+i\delta) &= -(\Gamma_M^{p0}(y+\lambda-i\delta))^*.
\end{aligned}\right\} \tag{15}$$

Since the bare interaction is included in both $\Gamma^p$ and $\Gamma^h$ we have that

$$\Gamma = \Gamma^p + \Gamma^h + J_{ex}\boldsymbol{\sigma}\cdot\mathbf{S}. \tag{16}$$

These results will be used in the next section to calculate $\rho_H$.

## III. The Hall Resistivity

As discussed in the Introduction, $\rho_H$ will be calculated by way of the transverse conductivity $\sigma_{xy}^{(1)}$ evaluated formally for a single impurity. $\sigma_{xy}^{(1)}$ is related to the retarded $x$-current–$y$-current correlation function by

$$\sigma_{xy}(z) = \frac{1}{z}X_{xy}(z) \tag{17}$$

where

$$X_{xy}(z) = \int_0^\infty dt\, e^{izt}\langle[j_x(t),j_y(o)]\rangle. \tag{18}$$

Two factors greatly reduce the number of diagrams contributing to $X_{xy}(z)$. Firstly the fact that we consider a single impurity requires that there be only one impurity

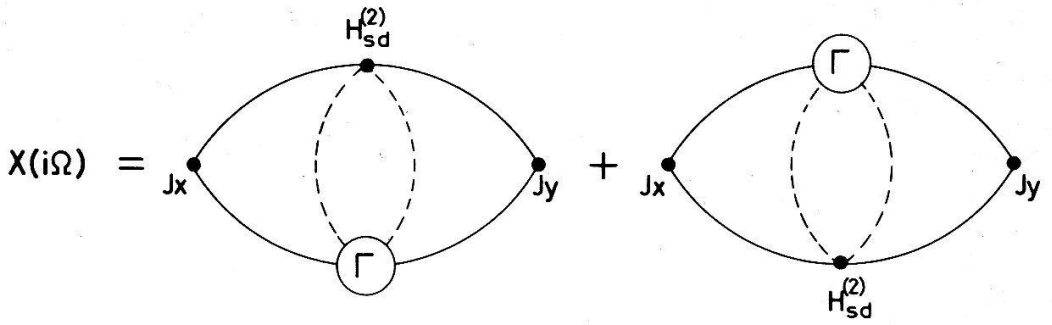


Figure 2

line in each diagram. Secondly, the restriction to first order in $\lambda$, together with the form of the interaction $\mathscr{H}_{sd}^{(2)}$ requires that the electronic Greens function on one side of the correlation function bubble contain just one vertex $\mathscr{H}_{sd}^{(2)}$ and no vertices $\mathscr{H}_{sd}^{(1)}$. Otherwise the bubble vanishes due to the angular integration of the momenta. This also implies that the only component of $\mathscr{H}_{sd}^{(2)}$ that contributes is proportional to $k_x$, $k_y'$, $S_z$, which is conveniently diagonal in $M$. The general form of the diagrams contributing to $X_{xy}(i\Omega)$ are shown in Figure 2. These two diagrams yield equal contributions to $\rho_H$.

Using equation (16) for $\Gamma$, the fully labelled diagrams for $X$ are shown in Figure 3.

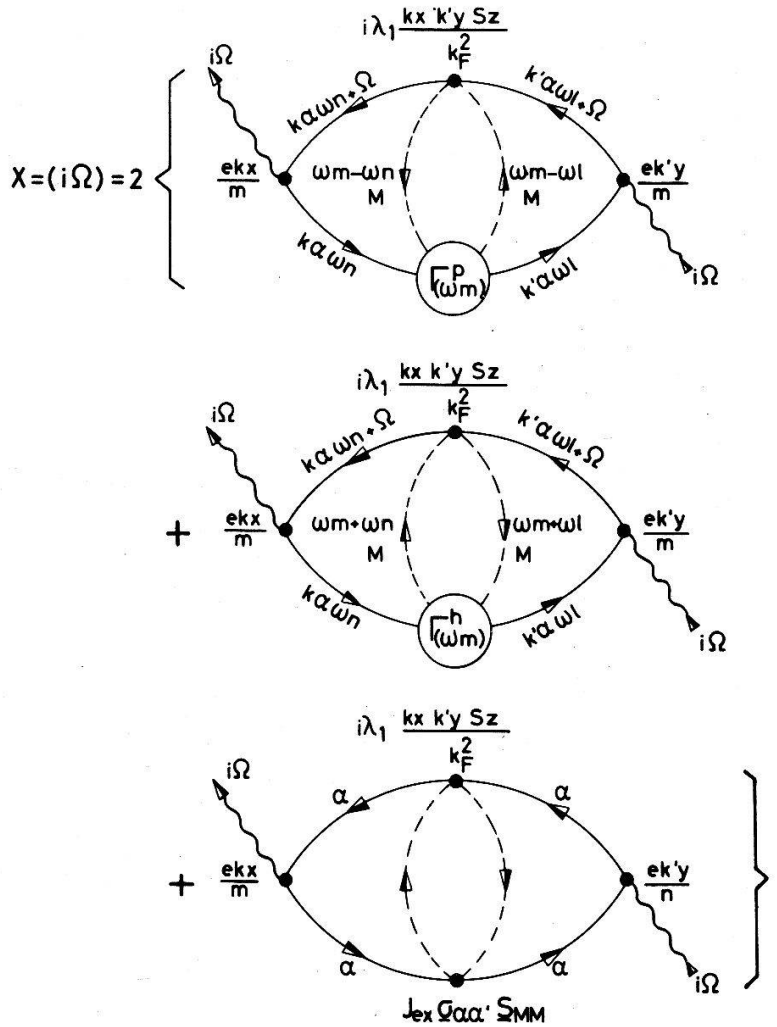


Figure 3

Let us consider the first diagram of Figure 3, which we shall call $X^p$. We have

$$X^p = \frac{e^2}{m^2} \sum_{\alpha M} \sum_{\mathbf{k}\mathbf{k}'} T^3 \sum_{\omega_l \omega_m \omega_n} \frac{i\lambda_1 k_x^2 k_y'^2}{k_f^2} \cdot M\Gamma^p_{\substack{\alpha\alpha \\ MM}}(\omega_m) \times G_\alpha(\mathbf{k}, \omega_n) G_\alpha(\mathbf{k}, \omega_n + \Omega)$$

$$\times G_\alpha(\mathbf{k}', \omega_l)\, G_\alpha(\mathbf{k}', \omega_l + \Omega)\, \mathcal{G}_M(\omega_m - \omega_1)\, \mathcal{G}_M(\omega_m - \omega_e). \tag{19}$$

The sum over $\mathbf{k}$, for example, gives

$$\sum_k \frac{k_x^2}{m} G_\alpha(\mathbf{k}, \omega_n)\, G_\alpha(\mathbf{k}, \omega_n + \Omega) = \frac{n}{2} \int d\varepsilon \, \frac{1}{(i\omega_n - \varepsilon - \alpha v_l)(i(\omega_n + \Omega) - \varepsilon - \alpha v_l)}$$

$$= ni\pi(i\Omega)^{-1} \qquad \text{if } -i\Omega < i\omega_n < 0$$

$$= 0 \qquad \text{otherwise.} \tag{20}$$

The resulting sum over $\omega_n$ we write as

$$T \sum_{\omega_n = -i\Omega}^{0} \frac{1}{i(\omega_m - \omega_n) - Mv_i - \lambda}$$

$$= -i\Omega T \sum_{\omega_n = 0}^{\infty} \frac{1}{\{i(\omega_m - \omega_n + \Omega) - Mv_i - \lambda\}\{i(\omega_m - \omega_n) - Mv_i - \lambda\}}$$

$$\equiv -i\Omega \mathcal{L}_M(i\omega_m). \tag{21}$$

Then we have

$$X^p(i\Omega) = -i\pi^2 \lambda_1 \left(\frac{nl}{k_f}\right)^2 \sum_{\alpha M} T \sum_{\omega_m} [\mathcal{L}_M(i\omega_m)]^2 M\Gamma^P_{\substack{\alpha \\ M}}{}^{\alpha}_{M}(\omega_m). \tag{22}$$

The only dependence on electron spin occurs in $\Gamma$, and we see from equation (11) that only $\Gamma^0$ contributes to $X$ since $\operatorname{Tr}\sigma^i = 0$. For the same reason the third diagram of Figure 3 vanishes.

Let us consider $L_M(z)$ as defined in equation (21) as a function of the complex variable $z$. Performing the frequency sum in the standard manner, we have

$$L_M(z) = \frac{1}{4\pi i} \int_{-\infty}^{+\infty} dx \tanh\frac{\beta x}{2} \, \frac{1}{(x - z - i\Omega + Mv_i + \lambda)} \, \frac{1}{(-z + x + Mv_i + \lambda)}$$

$$+ \frac{\beta}{2} \Delta(z) \frac{1}{i\Omega} \tanh \beta(Mv_i + \lambda)/2 \tag{23}$$

where

$$\Delta(z) = 1 \quad \text{if} \quad -i\Omega < \operatorname{Jm} z < 0$$

$$= 0 \qquad \text{otherwise.}$$

$L_M(z)$ thus has a cut along the real axis and the line $\mathrm{Im}\, z = -i\Omega$. Furthermore $\Gamma(z)$ has a cut along the real axis.

The frequency sum in equation (22) is now transformed into an integral, remembering that $\omega_m$ is an even frequency:

$$T \sum_{\omega_m} \{L_M(i\omega_m)\}^2 \Gamma(\omega_m) = \frac{1}{4\pi i} \int_C dz \coth \frac{\beta z}{2} \{L_M(z)\}^2 \Gamma(z) \tag{24}$$

where the contour $C$ is indicated in Figure 4. We now consider the function $L_M(z)$ along each of the integration paths of $C$ bearing in mind that according to equations (3) and (17) we seek terms that behave as $1/\omega$, where $\omega$ is the real external frequency.

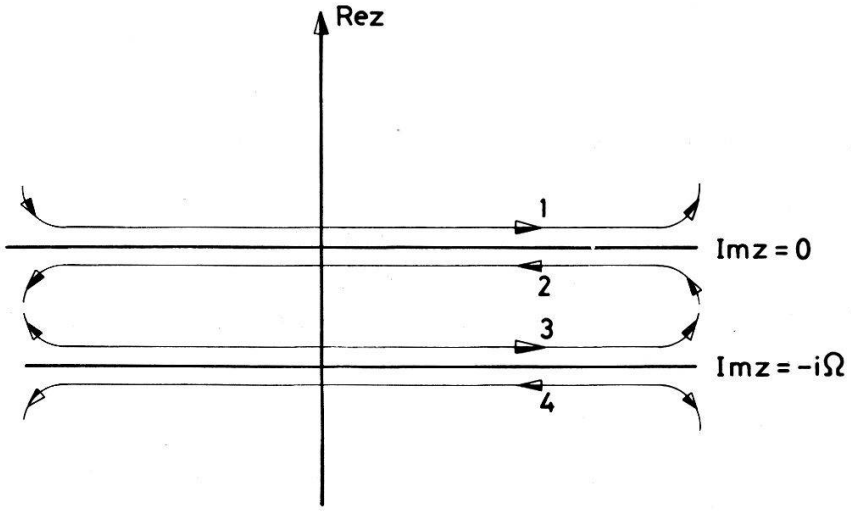


Figure 4

*Along 1*

The following continuations are performed:

$$z = y + i\delta, \quad z + i\Omega = y + \omega + i\delta.$$

Then $\Delta(z) = 0$ and, from equation (23),

$$L_M(y) = \frac{1}{4\pi i} \int_{-\infty}^{+\infty} dx \tanh \frac{\beta x}{2} \frac{1}{(x - y - \omega - i\delta + Mv_i + \lambda)} \frac{1}{(x - y - i\delta + Mv_i + \lambda)}$$

$$= \frac{1}{4\pi i} \frac{1}{\omega} \left\{ ln \left\{ \frac{(y + \omega - Mv_i - \lambda)^2 + \Gamma^2}{(y - Mv_i - \lambda)^2 + \Gamma^2} \right\} \right.$$

$$\left. + i\pi \{\tanh \beta(y + \omega - Mv_i - \lambda)/2 - \tanh \beta(y - Mv_i - \lambda)/2\} \right.$$

The lowest order term is independent of $\omega$, so we neglect this contribution.

*Along 4*

As above, there is no contribution.

*Along 2*

Here, $z = y - i\delta$, $z + i\Omega = y + \omega + i\delta$

$\Delta(z) = 1$

and

$$L_M(y) = \frac{1}{4\omega}\{\tanh\beta(y + \omega - Mv_i - \lambda)/2 + \tanh\beta(y - Mv_i - \lambda)/2 + 2\tanh\beta(Mv_i + \lambda)/2\}. \tag{25}$$

*Along 3*

$z = y - \omega - i\delta z + i\Omega = y + i\delta.$

$\Delta(z) = 1$

$$L_M(y) = \frac{1}{4\omega}\{\tanh\beta(y - \omega - Mv_i - \lambda)/2 + \tanh\beta(y - Mv_i - \lambda)/2 + 2\tanh\beta(Mv_i + \lambda)/2\}. \tag{26}$$

If equations (26) and (25) are substituted into equations (24) and (22), and the result is expanded to lowest order in $\omega$, we find

$$X^p(\omega) = -i\pi^2\left(\frac{nl}{k_f}\right)^2 \lambda_1 \sum_M M \frac{\beta}{16\pi i}\frac{1}{\omega}\int_{-\infty}^{+\infty} dy\{\tanh\beta(y - Mv_i - \lambda)/2 + \tanh\beta(Mv_i + \lambda)/2\}^2 \frac{1}{\sinh^2\frac{\beta y}{2}}\Gamma_M^{p^o}(y - i\delta)$$

$$= \frac{1}{\omega}\pi\beta\left(\frac{nl}{4k_f}\right)^2 \lambda_1 \sum_{\alpha M} M \int_{-\infty}^{+\infty} dy \operatorname{sech}^2\beta(Mv_i + \lambda)/2 \operatorname{sech}^2\beta \times (y - Mv_i - \lambda)/2\,\Gamma_M^{p^o}(y - i\delta) \tag{27}$$

where in the last line we have used the relation

$$\tanh A + \tanh B = \frac{\sinh(A + B)}{\cosh A \cosh B}.$$

On shifting the frequency $y$ by an amount $\lambda$ it is seen that $\Gamma^p_{\alpha M}$ is to be evaluated at a frequency $y + \lambda - i\delta$, as we did in Section II. An entirely similar calculation for the second diagram of Figure 3, $X^h$, yields equation (27) with $\Gamma^p$ replaced by $\Gamma_M^{h^o}(y + i\delta)$, which again we anticipated in Section II.

From equation (15) we see that

$$\Gamma^{p^o}(y - i\delta) + \Gamma^{h^o}(y + i\delta) = 2\mathrm{Jm}\,\Gamma^{p^o}(y - i\delta).$$

After multiplying by the factor $e^{\beta\lambda}[\sinh^{\beta v_i/2}]/[\sinh(S+\frac{1}{2})\,\beta v_i]$, taking the limit $\lambda \to \infty$ and using equation (3) we find

$$\rho_H = \frac{\sinh \beta v_i/2}{\sinh(S+\frac{1}{2})\,\beta v_i}\,\frac{4\pi\lambda_1}{(ev_f)^2}\sum_M M e^{-\beta M v_i}\int_{-\infty}^{+\infty} dy f'(y)\,\mathrm{lm}\,\Gamma_M^{p^o}(y+Mv-i\delta) \tag{28}$$

where $f'(y)$ is the derivative of the Fermi function, and we have dropped $\lambda$ in the argument of $\Gamma$ since, as seen in the previous section $\Gamma(y+\lambda-i\delta)$ does not depend upon $\lambda$.

We may now write

$$\rho_H = \frac{\sinh \beta v/2}{\sinh(S+\frac{1}{2})\,\beta v}\,\frac{2\pi\lambda_1 J_{ex}^2 c}{(ev_f)^2}\sum_M M\{e^{-\beta M v}A(v) - e^{\beta M v}A(-v)\} \tag{29}$$

$$A(v) = \int_{-\infty}^{+\infty} dy f'(y)\,\mathrm{lm}\,\frac{S(S+1)F(y-v)+M}{1-MF(y)+(M+1)F(y-v)-(S(S+1)F(y)F(y-v)}.$$

Since the calculation up till now has been for a single impurity, we have multiplied equation (29) by the concentration $c$ (the volume has been taken equal to one). This is our general result for $\rho_H$, in terms of a rather unmanageable integral.

At high temperatures, $T \gg T_k$, we can expand $A$ in powers of $F$, i.e. a perturbation expansion is valid. We find to lowest order

$$\rho_H = \frac{\pi^2 c\lambda_1 N(o) J_{ex}^2}{(ev_f)^2}\Bigg\{S(S+1)\tfrac{1}{2}\left[(2S+1)\coth(2S+1)\frac{\beta v}{2} - \coth\frac{\beta v}{2}\right]$$
$$-\left(\coth\frac{\beta v}{2} - \frac{\frac{1}{2}\beta v}{Sh^2\frac{\beta v}{2}}\right)$$
$$\times\left[S(S+1) - \tfrac{1}{2}\left((2S+1)\coth(2S+1)\frac{\beta v}{2}\coth\frac{\beta v}{2} - \coth^2\frac{\beta v}{2}\right)\right]\Bigg\}. \tag{30}$$

The first and second square brackets are the average values of $M$ and $M^2$ respectively. The function in curly brackets has a saturation value of $S^3$, and an initial slope (as a function of $\beta v/2$) $\frac{2}{3}S^2(S+1)^2 - \frac{2}{9}S(S+1)$.

At more general temperatures ($T > T_k$) we obtain an expression for the Hall coefficient $R_H$ (defined as $d\rho_H/dH|_{H=0}$) by expanding equation (29) in powers of $v$. This still leaves the problem of doing the integral for $A(v)$, but we circumvent this difficulty by treating the derivative of the Fermi function as a $\delta$-function and take the zero frequency value of the integrand. Since both the functions $F(y)$ and $f'(y)$ vary on a scale of $T$ this is certainly not very accurate, but it is reasonable to expect that this frequently though often surreptitiously used approximation will give the correct general behaviour.

We shall also assume that the imaginary part of the expression $1 + F(o) - S(S+1)$-$F(o)^2$ is small compared with the real part. This requires that $T$ is not too close $T_k$ (defined as the temperature at which the real part vanishes). We remark that according to Mattuck [9] $\mathrm{lm}\,F(o) = 0$, but this is incorrect according to equation (10). This implies that there is actually no divergence in physical quantities at any temperature. The result

of the expansion gives

$$R_H = g\mu_B \frac{1}{3T} S(S+1)\, c \frac{\pi^2 \lambda_1 N(o) J_{ex}^2}{(ev_f)^2} \times \left\{ \frac{S(S+1)(1+S(S+1)L_0^2) - 1}{(1+L_0 - S(S+1)L_0^2)^2 + \frac{\pi^2 J_{ex}^2 N(o)^2}{4}(1-2S(S+1)L_0)^2} \right\} \tag{31}$$

where

$$L_0 = -J_{ex} N(o) \ln \frac{D}{2T}.$$

We have retained that component of the denominator which comes from the small imaginary part of $1 + F(o) - S(S+1)F_0^2$ in order to show explicitly that the result does not diverge. Such terms have been neglected in the numerator. Mattuck and Cheung's result [9] for the zero field resistivity $\rho$ corresponds to the above approximation for $\Gamma$, but because the imaginary part of the kernel was not correctly evaluated they do not have the term in the denominator that prevents the divergence. If we are not too close to $T_k$ this is of no importance. They found (generalizing to spin $S$)

$$\rho = \frac{3\pi J_{ex}^2 c}{8(ev_f)^2} \left\{ \frac{2(1+S(S+1)L_0^2)}{(1+L_0 - S(S+1)L_0^2)^2} - 1 \right\} S(S+1).$$

We see that $\rho$ and $\rho_H$ 'diverge' in the same way, and therefore that the low field Hall angle $\theta_H$ should have no really anomalous behaviour. In fact

$$\theta_H = \frac{HR_H}{\rho} \simeq \frac{g\mu_B H}{3T} \pi\lambda_1 N(o) \frac{8}{3} \times \left\{ \frac{S(S+1)(1+S(S+1)L_0^2) - 1}{2(1+S(S+1)L_0^2) - (1+L_0 - S(S+1)L_0^2)^2} \right\} \tag{32}$$

which for $L_0 \ll 1$ becomes

$$\theta_H = \frac{g\mu_B H}{3T} \frac{8\pi\lambda_1}{3} N(o)\{S(S+1) - 1\}. \tag{33}$$

## IV. Discussion

Although our quantitative results should not, perhaps, be taken too literally due to the nature of the approximations used, we can nevertheless draw the following general conclusions.

Firstly, the temperature dependence of $\rho_H$ is essentially that of $\rho$ multiplied by the susceptibility (which in our calculation is just $1/T$). This means that the Hall angle is just proportional to the susceptibility, the bracketed function in equation (32) being very slowly varying.

Secondly, according to equations (32) and (33) there is a possibility of $\rho_H$ having different signs depending only upon the spin. For example, equation (33) predicts that the hypothetical case $S = \frac{1}{2}$ has opposite sign to $S > \frac{1}{2}$. This occurs because of the competing effects of the magnetic field's action upon the level population on the one hand, and its action upon the scattering on the other.

We remark that equation (33) relates the unknown parameter $\lambda_1$ very simply to the Hall angle.

The available experimental data on the Hall effect [2, 7] in Kondo alloys does not permit a detailed comparison with our theory. Such a comparison would require accurate measurements of that part of the zero field Hall coefficient due solely to skew scattering as a function of temperature. The existing results indicate in general an increase in $R_H$ as $T_k$ is approached from above, but the proportion of this increase due to the susceptibility (proportional to $1/T$ in our approximation) and that due to logarithmic terms is uncertain.

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