The reconstruction of quantum fields from Euclidean Green's functions at arbitrary temperatures

Autor(en): **Fröhlich, Jürg**

Objekttyp: **Article**

Zeitschrift: **Helvetica Physica Acta**

Band (Jahr): **48 (1975)**

Heft 3

PDF erstellt am: **10.08.2024**

Persistenter Link: https://doi.org/10.5169/seals-114679

Helvetica Physica Acta
Vol. 48, 1975.

Birkhäuser Verlag Basel

The Reconstruction of Quantum Fields from Euclidean Green's Functions at Arbitrary Temperatures

by **Jürg Fröhlich**

Department of Mathematics, Princeton University, Princeton, N.J. 08540, USA

(10. III. 75)

Abstract. This note announces and describes results on the reconstruction of equilibrium states on the Borchers field algebra of quantum fields at arbitrary temperature from Euclidean Green's functions. These results are applied to a large class of models of a self-interacting, local, neutral, scalar fields in two space-time dimensions for which the Euclidean Green's functions in the infinite volume limit can be constructed at arbitrary temperatures. They can be (and have been) applied to non-relativistic field theories at positive temperature.

1. Introduction

In quantum theory a system of particles or fields (in thermal equilibrium) at some arbitrary $T = \beta^{-1} \geqslant 0$ is usually described in terms of some $*$ algebra of operators $\mathfrak{A}$ containing the observables of the system and a state – i.e. a normalized, positive, linear expectation functional on $\mathfrak{A}$ – ω^β *invariant* under time- (and space-) translations. Let $\{A(t) : t \in \mathbb{R}\} \subset \mathfrak{A}$ represent the time evolution of an operator $A \in \mathfrak{A}$ in the Heisenberg picture. All interesting information about such a system can in principle be calculated from the expectation values

$$\left\{\omega^\beta\left(\prod_{j=1}^{n} A_j(t_j)\right) : A_j \in \mathfrak{A},\quad j = 1, \ldots, n \in \mathbb{N}\right\}. \tag{1}$$

Under the usual regularity assumptions on the expectation values (1), one can reconstruct [1] from these expectation values a Hilbert space $\mathscr{H}^\beta$, a unitary time-translation group $\{T_t^\beta : t \in \mathbb{R}\}$ on $\mathscr{H}^\beta$, a cyclic, time-translation invariant vector Ω^β and a representation π^β of $\mathfrak{A}$ such that

$$\omega^\beta\left(\prod_{j=1}^{n} A_j(t_j)\right) = \left(\Omega^\beta, \prod_{j=1}^{n} T_{t_j}^\beta \pi^\beta(A_j) T_{t_j}^{\beta *} \Omega^\beta\right)_{\mathscr{H}^\beta} \tag{2}$$

The following further assumptions on $(\omega^\beta, \mathfrak{A}, T_t^\beta)$ are standard:

A1) For temperature $\beta^{-1} = 0$ the usual spectrum condition tells us that $T_t^\infty = e^{iHt}$, where H is the Hamiltonian of the system and is a positive, selfadjoint operator on $\mathscr{H}^\infty$.

A2) For temperature $\beta^{-1} > 0$ the usual condition characterizing an equilibrium state ω^β is the famous Kubo–Martin–Schwinger boundary condition [2, 3]: For arbitrary

A and B in $\mathfrak{A}$ there exists a function $F^{\beta}_{AB}(\zeta)$ holomorphic on $\{z|0 < \operatorname{Im} z < \beta\}$ such that

$$F^{\beta}_{AB}(t) = \omega^{\beta}(AB(t)) \quad \text{and} \quad F^{\beta}_{AB}(t + i\beta) = \omega^{\beta}(B(t)A) \tag{3}$$

in the distributional sense. Condition (3) implies the time-translation invariance of ω^{β}; see Ref. [3]. A state ω^{β} in the sense of definition (1) on some $*$ algebra $\mathfrak{A}$ satisfying the KMS condition (3) is called an *equilibrium state on* $\mathfrak{A}$.

The assumptions A1) and A2) have in common that they imply that the distributions

$$F^{\beta}(t_1, \ldots, t_n) \equiv \omega^{\beta}\left(\prod_{j=1}^{n} A_j(t_j)\right), \quad \beta^{-1} \geqslant 0,$$

are the boundary values of functions $F^{\beta}(\zeta_1, \cdots, \zeta_n)$ holomorphic on the tubular domain [3, 4]

$$\mathfrak{T}^{\beta}_n = \{(\zeta_1, \ldots, \zeta_n) : 0 < \operatorname{Im} \zeta_1 < \cdots < \operatorname{Im} \zeta_n < \beta\}. \tag{4}$$

The points $\mathscr{E}^{\beta}_n \equiv \{(it_1, \ldots, it_n) : 0 < t_1 < \cdots < t_n < \beta\} \subset \mathfrak{T}^{\beta}_n$ are called *Euclidean points.*

We set $\mathfrak{S}^{\beta}_n(t_1, \ldots, t_n) \equiv F^{\beta}(it_1, \ldots, it_n)$ on $\mathscr{E}^{\beta}_n$ and call it the n-point *Euclidean Green's – or Schwinger function.* In the case of systems of interacting Bosons or Bose fields it has turned out to be very convenient to try to construct *first* the *Schwinger functions* $\{\mathfrak{S}^{\beta}_n(t_1, \ldots, t_n)\}_{n=0}^{\infty}$ by means of *path space methods* [5, 6] and then try to reconstruct the distributions $F^{\beta}(t_1, \ldots, t_n)$ from these Schwinger functions by analytic continuation in the time variables and passing to the boundary. For an account of the success of this approach, see Refs. [5, 6, 7]. For the case of quantum fields various conditions on the Schwinger functions $\{\mathfrak{S}^{\beta}_n\}_{n=0}^{\infty}$ which guarantee that these can be analytically continued back to the real times have been isolated by Nelson [8], by Osterwalder and Schrader [9] and in Ref. [10] for the case where $\beta^{-1} = 0$ and in a forthcoming article (Ref. [11]) for the case where $\beta^{-1} > 0$. In this note we sketch the conditions found in Ref. [11] and apply the results of Refs. [8–11] to Bose quantum field models in a two-dimensional space-time [6, 12]. For $\beta^{-1} > 0$ our techniques and results extend to the case of charged Bose quantum fields with non-vanishing charge density and to the Yukawa model in two space-time dimensions provided the coupling constant is sufficiently small. (These latter models might describe condensation or phase transition phenomena.) This note describes results detailed in Refs. [10, 11, 14]. The methods and results described below and in Ref. [11], in particular Theorems 2 and 3, can also be applied to non-relativistic, quantum mechanical many body systems at positive temperature [17, 11]. One of the reasons why we discuss two-dimensional, relativistic quantum field models in this paper is that the construction of the Euclidean Green's functions at positive temperatures is particularly easy for these models [7]. Apart from that our motivation for the (probably controversial) study of quantum fields at positive temperatures is that it provides some insight into interesting problems of truly physical nature:

i) If the fundamental theory of nature has the form of a quantum field theory the concept of a field theoretic description of systems at positive temperatures – like solids or stars – has to be proven consistent and successful.
ii) Broken symmetries of quantum field models at $\beta^{-1} = 0$, like gauge symmetries, can become unbroken as the temperature is increased, i.e. phase transitions can occur [13]. The study of quantum fields at positive temperatures might therefore

provide a new insight into the nature of dynamical symmetry breaking and the Higgs mechanism [13].

The Poincaré symmetry of relativistic systems at $\beta^{-1}=0$ is always broken for $\beta^{-1}>0$. (The rest frame of the heat bath is distinguished.) It should, however, still be reflected in certain special properties of equilibrium states satisfying condition (3). Such properties can be searched for and eventually be found by studying simple quantum field models like the ones investigated in this note. (One reflection of Lorentz invariance is the existence of a transfer matrix in space direction.)

Acknowledgments

I thank R. Høegh-Krohn for a preprint of Ref. [7] and correspondence which greatly stimulated this analysis.

2. The Models

The models we study describe a neutral, self-interacting, local, relativistic Bose quantum field φ in a two-dimensional space-time. They can be formally characterized by the following formal, non-linear field equations

$$(\Box + m^2)\,\varphi(t,x) = -\!:V'(\varphi(t,x)):,$$

where V is either a polynomial of even degree with positive leading coefficient or a convex superposition of exponentials and : : denotes normal ordering.

For weak coupling and $\beta^{-1}>0$ our results can be extended to charged Bose fields in states of non-vanishing charge density (chemical potential) or the Yukawa model in two dimensions. The models of main interest in this note can be defined rigorously by a Euclidean, scalar action perturbing the Schwinger functions of the free field (rather than by the above field equations). The free field is completely determined, at arbitrary temperature β^{-1}, by its Schwinger functions. In the case of a scalar Bose field φ these Schwinger functions are the moments of a Gaussian measure $d\nu_0^\beta$ on a distribution space $\mathscr{S}_\beta'$ with mean 0 and covariance $\mathfrak{S}_2^\beta$ which is the free two-point Schwinger function at temperature β^{-1}. For convenience we transform the 'time'-variables: $\tau = t - \beta/2$. The Euclidean points in terms of the τ variables are

$$\mathscr{E}_n^\beta = \left\{(\tau_1,\ldots,\tau_n): -\frac{\beta}{2} < \tau_1 < \cdots < \tau_n < \frac{\beta}{2}\right\}.$$

The two-point function $\mathfrak{S}_2^\beta(\xi,\eta)$, $\xi = \langle \tau, x\rangle$, $\eta = \langle \sigma, y\rangle$, is the kernel of the operator $(-\Delta_\beta + m^2)^{-1}$ on the Hilbert space $L^2(S_\beta \times \mathbb{R})$, where $S_\beta = \{\tau: -\beta/2 \leqslant \tau \leqslant \beta/2\}$ and Δ_β is the Laplacian with *periodic* boundary conditions (b.c.) at $\tau = \pm\beta/2$. We let $\mathscr{S}_\beta$ be the space of real C^∞ test functions on $S_\beta \times \mathbb{R}$ with $f(x,-\beta/2) = f(x,\beta/2)$ which together with all their derivatives decrease rapidly as $x \to \pm\infty$; $\mathscr{S}_\beta'$ denotes its dual. The measure $d\nu_0^\beta$ can be defined on the σ-algebra generated by the Borel cylinder sets of $\mathscr{S}_\beta'$ [11, 14].

The Schwinger functions of a free, scalar, neutral field at temperature β^{-1} in a box with rigid walls at $x = \pm l/2$ are the moments of the Gaussian measure $d\nu_0^{\beta,l}$ with mean 0 and covariance $(-\Delta_{\beta,l} + m^2)^{-1}$, where $\Delta_{\beta,l}$ is the Laplacian on $L^2(S_\beta \times [-l/2, l/2])$ with *Dirichlet b.c.* at $x + \pm l/2$ and periodic b.c. at $\tau = \pm\beta/2$. Wick normal ordering with respect to $d\nu_0^\infty$ [15] is denoted by : :.

Let $V(x)$ be any of the following functions

a1) $V(x) = \lambda\left(\sum_{m=1}^{2n-1} a_m x^m + x^{2n}\right), \quad (\lambda > 0);$

a2) $V(x) = \sum_{m=1}^{n} a_m x^{2m} + \mu x, \quad a_n > 0, n \in \mathbb{N};$

b1) $V(x) = \int d\mu(\alpha) \cos h(\alpha x)$ [16],

b2) $V(x) = \int d\mu(\alpha)\, e^{\pm\alpha x}$,

b3) $V(x) = \int d\mu(\alpha) \cos(\alpha x)$[1], where μ is a finite, positive measure with support in $[0, -4/\sqrt{\pi})$.

The function V such as defined in b2) and b3) is studied in this note for the first time.

For $\tau \leqslant \beta$ we set

$$U_{l,\tau}(V) = \int_{-l/2}^{l/2} dx \int_{-\tau/2}^{\tau/2} d\tau' : V(\Phi(\tau', x)):.$$

It is known [6, 7, 11, 12, 15, 16] that the following are well-defined measures on $\mathscr{S}'_\beta$:

$$\alpha) \qquad d\nu^\beta_{V,l,\tau}(\Phi) \equiv \frac{e^{-U_{l,\tau}(V)}\, d\nu_{\beta,l}{}^0(\Phi)}{\int_{\mathscr{S}'_\beta} e^{-U_{l,\tau}(V)}\, d\nu_{\beta,l}{}^0(\Phi)}.$$

For $\tau = \beta$ this measure is the path space expression corresponding to the Gibbs state

$$\operatorname{Tr}(e^{-\beta H_l}\cdot)/\operatorname{Tr} e^{-\beta H_l} \quad \text{(see Refs. [7, 11])} \tag{5}$$

where H_l is the quantum field Hamiltonian in a box of length l.

$$\gamma) \qquad d\nu^\beta_{V,l,\tau}(\Phi) \equiv \frac{e^{-U_{l,\tau}(V)}\, d\nu^\beta_0(\Phi)}{\int_{\mathscr{S}'_\beta} e^{-U_{l,\tau}(V)}\, d\nu^\beta_0(\Phi)}.$$

Theorem 1 [11, 14]. (Existence of the thermodynamic limit.)
For all $\beta^{-1} \geqslant 0$ and all f in $\mathscr{S}_\beta$

$$Z^\beta_V(f) = \lim_{l\to\infty} \lim_{\tau\to\beta} \int_{\mathscr{S}'_\beta} d\nu^\beta_{V,l,\tau}(\Phi)\, e^{i\Phi(f)}$$

exists for

$d\nu^\beta_{V,l,\tau}$ as in α) and V as in a_2), b_1);
$d\nu^\beta_{V,l,\tau}$ as in γ) and V as in b_1), b_2), b_3), and for V as in a_1) with λ arbitrary if $\beta^{-1} > 0$ and λ such that the Glimm–Jaffe–Spencer cluster expansion [6] converges if $\beta^{-1} = 0$.

---

[1]) This model is essentially equivalent to a *neutral* system of infinitely many *classical* point particles of charge ± 1 interacting through a Yukawa potential in two space diemensions; see Ref. [14].

Under these conditions there exists a measure $d\nu_V^\beta$ on $\mathscr{S}'_\beta$ such that

$$Z_V^\beta(\zeta f) = \int d\nu_V^\beta(\Phi) e^{i\zeta\Phi(f)}$$

is entire analytic in ζ for arbitrary, fixed f in $\mathscr{S}_\beta$. The funcational $Z_V^\beta(f)$ is the characteristic functional of the measure $d\nu_V^\beta$ and determines $d\nu_V^\beta$ completely [11].

Remarks: Our proof of this theorem [11, 14] is based on the cluster expansion [6] (and references given there), the GKS and FKG correlation inequalities of Ref. [15], results of Refs. [7, 16], the existence of a *transfer matrix in space-direction* for the case where $d\nu_{V,l,\tau}^\beta$ is given by γ) [7] and $\beta^{-1} > 0$ (a fact which reflects the *Lorentz symmetry*) and uniform bounds on $\int_{\mathscr{S}'_\beta} d\nu_{V,l,\tau}^\beta(\Phi) e^{i\zeta\Phi(f)}$ proven in Ref. [14]. From Theorem 1 it obviously follows that the moments of the measures $d\nu_V^\beta$ exist and are tempered distributions. The nth moment is denoted by $\mathfrak{S}_V^{\beta,n}(\xi_1,\ldots,\xi_n)$. It is the n-point Schwinger function of the field theory with V as an interaction.

Theorem 2 [10, 11]. (Reconstruction of the quantum theory from Schwinger functions.)

1) For $\beta^{-1} = 0$ the moments $\{\mathfrak{S}_V^{\infty,n}\}_{n=0}^\infty$ are the Euclidean Green's functions of a uniquely determined state ω_V^∞ on the polynomial $*$ algebra generated by a *tempered*, scalar quantum field $\varphi(t,x)$. (This algebra can be defined abstractly as the *tensor algebra* over $\mathscr{S}(\mathbb{R}^2)$ in the sense of Borchers [1]; see also Ref. [4].) The field φ is *local*. The distributions

$$\mathscr{W}_V^{\infty,n}(t_1, x_1,\ldots, t_n, x_n) = \omega_V^\infty\left(\prod_{i=1}^n \varphi(t_j, x_j)\right)$$

satisfy all Wightman axioms [4] (except possibly the cluster properties in the case where V is as in a_2), b_2), b_3)).

2) For $\beta^{-1} > 0$ the moments $\{\mathfrak{S}_V^{\beta,n}\}_{n=0}^\infty$ are the Euclidean Green's functions of a uniquely determined equilibrium state ω_V^β on the polynomial $*$ algebra [1] generated by a *tempered, local*, scalar quantum field $\varphi(t,x)$ at temperature β^{-1}. The distributions

$$\mathscr{W}_V^{\beta,n}(t_1, x_1, \ldots, t_n, x_n) = \omega_V^\beta\left(\prod_{i=1}^n \varphi(t_j, x_j)\right)$$

satisfy (1) (temperedness), (3) (the KMS boundary condition), (4) (analyticity in the time variables), locality [1, 4, 11] and *exponential cluster properties* (i.e. *no* long-range correlations) in space directions. The limit state ω_V^β is independent of whether we choose the measures defined in α) or in γ); i.e. it *does not* depend on boundary conditions!

For all $\beta \leqslant \infty$ the states ω_V^β are *space-translation-invariant*.

Proof: The first part is proven in Ref. [10]; the second part follows easily from Theorem 1 and the results of Refs. [11, 17]. The proof is sketched below:

The second part of this theorem is based on techniques and results of Ref. [7] and a reconstruction theorem proven in Ref. [11]. Theorem 2,2) is merely a rigorous version of a conjecture in Ref. [7]. Apart from technical conditions the following properties of Z_V^β and $\{\mathfrak{S}_V^{\beta,n}\}_{n=0}^\infty$ are the main ingredients for verifying the hypotheses of the reconstruction theorems of Refs. [10] and [11]:

(i) Osterwalder–Schrader positivity [9]

Let n be an arbitrary positive integer, $f_1, \ldots, f_n$ arbitrary test functions in $\mathscr{S}_\beta$ with $\operatorname{supp} f_j \subset [0, \beta/2]$, $j = 1, \ldots, n$, and $c_1, \ldots, c_n$ arbitrary complex numbers. Then

$$\sum_{i,j=1}^{n} \bar{c}_i c_j Z_V^\beta (f_j - f_{i,\vartheta}) \geqslant 0, \tag{6}$$

where $f_\vartheta(\tau, x) = f(-\tau, x)$, $f \in \mathscr{S}_\beta$. (This form of O.S. positivity is used in Refs. [10] and [11].)

(ii) Exponential Z^β-bound [10, 14]

Let χ_λ be the characteristic function of the interval $[-\lambda, \lambda]$. Let f be an arbitrary, real, integrable function such that $\operatorname{supp} f \subseteq [-\beta/2, \beta/2]$ and $\|f\|_1 \leqslant \mathscr{K}(V)$, for some $\mathscr{K}(V) < \infty$. Then

$$Z_V^\beta \;\; (\text{i}f \otimes \chi_\lambda) \leqslant e^\lambda, \quad \text{for all real } \lambda. \tag{7}$$

Inequality (7) can be derived from the Glimm–Jaffe ϕ-bounds; see Ref. [14].

(iii) Bounds on moments of $d\nu_V^\beta$ [11, 14] *and Hölder property* [17, 11]

Let f be in $\mathscr{S}_{\text{real}}(\mathbb{R}^1)$. Then the reconstruction theorem of Ref. [11] requires that

$$(|||f|||_{\beta,V,n})^n \equiv \int d\nu_V^\beta(\Phi) \prod_{j=1}^{n} \Phi(\delta_{-(\beta/2)+[(j\beta)/n]} \otimes f) \tag{8}$$

must be bounded above by $|f|_{\mathscr{S}}^n(n!)^L$, for some Schwarz space norm $|\cdot|_{\mathscr{S}}$ and some finite, positive integer L. The significance of this bound is explained in Ref. [11].

Remark: For the reconstruction theorem of Ref. [11] to be valid any finite L is good enough. It follows however from (7) that for the models considered in this note we can set $L = 1$. This is a simple consequence of the Cauchy integral formula applied to $Z_V^\beta(\zeta(f \otimes \chi_\lambda))$.

Let $(\zeta_1, \ldots, \zeta_n)$ be complex numbers in $\mathfrak{T}_n^\beta$ and let m_j be the smallest, even, positive integer such that

$$\frac{\beta}{m_j} < \tfrac{1}{2} \min(\operatorname{Im}(\zeta_{j+1} - \zeta_j), \operatorname{Im}(\zeta_j - \zeta_{j-1})),$$

where $\operatorname{Im} \zeta_0 = \beta - \operatorname{Im} \zeta_n$, $\operatorname{Im} \zeta_{n+1} = \beta - \operatorname{Im} \zeta_1$. Then

$$|\mathscr{W}_V^{\beta,n}(\zeta_1, f_1, \ldots, \zeta_n, f_n)| \leqslant \prod_{j=1}^{n} |||f_j|||_{\beta,V,m_j}. \tag{9}$$

This inequality is called the Hölder property, since in the case where one considers a system in a finite box of length l it follows from the Hölder inequalities applied to the trace (5) [17]. Inequalities (8) and (9) (or (7) and (9)) imply temperedness of the boundary value of $\mathscr{W}_V^{\beta,n}$, for all n. Inequality (9) is very basic for equilibrium states [11, 17]; we conjecture that it follows from the KMS condition (3). The reader familiar with Ref. [17] will have no difficulty in understanding (9) and hence Theorem 2,2). Q.E.D.

Definition: Let ω^β be a space-translation-invariant state on the polynomial * algebra $\mathfrak{A}$ generated by a tempered quantum field $\varphi(t, x)$ (Borchers, Refs. 1, 4). For

B in $\mathfrak{A}$ let $B(x)$ denote the space translate of B by the vector x. We say that the state ω^β is clustering if

$$\omega^\beta(AB(x)) \to \omega^\beta(A)\,\omega^\beta(B), \quad \text{as } |x| \to \infty,$$

for all A and B in $\mathfrak{A}$.

Theorem 3 [10, 11]. (Non-existence of the interaction picture.)

1) For $V \neq 0$ as in $a_{1/2}$) or $b_{1/2/3}$) and all $\beta^{-1} \geqslant 0$ the field theory obtained from the state ω_V^β constructed in Theorem 2 is always 'physically different (or distinguishable)' in a mathematical sense made precise in Refs. [10] and [11] from the free field theory obtained from $\omega_{V=0}^{\beta'}$, all $(\beta')^{-1} \geqslant 0$. In particular, the representation of the (time 0–) canonical commutation relations in Weyl's form determined by the state ω_V^β is inequivalent and disjoint from the one determined by $\omega_0^{\beta'}$, all $\beta^{-1} \geqslant 0$, $(\beta')^{-1} \geqslant 0$.

2) The field theories obtained from two states $\omega_{V_1}^{\beta_1}$ and $\omega_{V_2}^{\beta_2}$ such as constructed in Theorem 2 are 'physically different' in a mathematical sense made precise in Refs. [10] and [11] *unless* $\beta_1 = \beta_2$ *and* $V_1(x) = V_2(\pm x + \text{const.}) + \text{const.}$!; all positive $\beta_1 \leqslant \infty$, $\beta_2 \leqslant \infty$.

3) For V as in a_1) and b_1) ω_V^β is *clustering* for all $\beta \leqslant \infty$. For $\beta < \infty$ ω_V^β is clustering for all V. The precise formulation and the proof of Theorem 3 are based on the following properties of the measures $d\nu_V^\beta$:

i) Let B be some compact rectangle contained in $[-\beta/2, \beta/2] \times \mathbb{R}$ and let $\mathfrak{M}(B)$ be the class of functions on $\mathscr{S}'_\beta$ obtained by weakly closing the functions $\{e^{i\phi(f)} | f \in \mathscr{S}_\beta, \operatorname{supp} f \subset B\}$ on $L^2(\mathscr{S}'_\beta, d\nu_0^\beta)$. Let $\sum_B$ be the minimal $\mathfrak{S}$-algebra on $\mathscr{S}'_\beta$ such that all functions in $\mathfrak{M}(B)$ are measurable with respect to $\sum_B$. Let $d\nu|_{\Sigma_B}$ denote the restriction of a measure $d\nu$ on $\mathscr{S}'_\beta$ to $\sum_B$. We set

$$F_{V,l,\tau,B}^\beta(\phi) = \frac{d\nu_{V,l,\tau}(\phi)|\sum_B}{d\nu_0^\beta(\phi)|\sum_B}.$$

Then the family of functions on $(\mathscr{S}'_\beta, \sum_B)$ defined by

$$\{F_{V,l,\tau,B}^\beta\} \quad 0 \leqslant \tau \leqslant \beta,\ 0 \leqslant l < \infty$$

is essentially compact in $L^1(\mathscr{S}'_\beta, d\nu_0^\beta|\sum_B)$ for V as in $a_{1/2}$) or $b_{1/2/3}$) and B an arbitrary, compact rectangle. Thus $d\nu_V^\beta|\sum_B$ exists. It is *equivalent* to $d\nu_0^\beta|\sum_B$ [11, 14].

ii) The limit measures $d\nu_V^\beta$ are $\mathscr{S}_\beta$*-quasi-invariant*, i.e. the Radon–Nikodym derivatives

$$\frac{d\nu_V^\beta(\phi + g)}{d\nu_V^\beta(\phi)}$$

exist for all g in $\mathscr{S}_\beta$ [14]. Using i) and ii) we can prove:

iii) The measures $d\nu_{V_1}^\beta$ and $d\nu_{V_2}^\beta$ are *mutually singular* unless $V_1 = V_2$ (non-commutative generalizations of this statement are used in the statement and proof of Theorem 3). Theorem 3 and iii) prove the *non-existence of the interaction picture*.

Concluding remarks: As already mentioned, the techniques and results reported here and in Ref. [11] can be applied to a wider class of models than the ones studied here. There are reasons to believe that in *two space-time dimensions* ther exist relativistic Bose quantum field models showing phase transitions at $\beta^{-1} = 0$, as the coupling constant is varied[2]), models of relativistic, charged Bose quantum fields with Bose–Einstein condensation at $\beta^{-1} > 0$ as the chemical potential of the conserved charge is varied and, may be, models of relativistic Bose and spin-$\frac{1}{2}$ Fermi quantum fields (like Yukawa$_2$) showing a phase transition, as β is varied. (Theorem 2,2) has been established for the Yukawa model in two dimensions for small coupling constants by the present author [11].)

A general discussion of phase transitions and their connection with the breakdown of ergodicity of the measures $d\nu_V^\beta$ under different groups for $P(\phi)_2$ is contained in Ref [14].

Theorem 4: Let V be as in $a_{1/2}$) or $b_{1/2/3}$), $\beta^{-1} = 0$, and let $d\nu_V^\infty$ be an Osterwalder–Schrader positive [9, 10, 11], Euclidean invariant infinite volume interacting measure [14]. Then *all* the components of $d\nu_V^\infty$ *ergodic* under 'time' translations $\{T_t | t \in \mathbb{R}\}$ are:

Osterwalder–Schrader positive and 'time'-reflection invariant,
Euclidean invariant,
$\mathscr{S}$-quasi-invariant,
locally equivalent to the free measure $d\nu_0^\infty$ in the sense of Remark (i) following Theorem 3.
Two measures $d\nu_{V_1}^\infty$, $d\nu_{V_2}^\infty$ are mutually singular (disjoint) unless $V_1 = V_2$ and determine 'physically different' [10, 14] quantum field theories (in the sense that the representations of the canonical commutation relations are disjoint, or in the sense of Borchers classes; see Ref. [4]) *unless* $V_1(x) = V_2(\pm + a) + b$ for a and b some constants.

Corollary: *All* components of $d\nu_V^\infty$ *ergodic* under $\{T_t | t \in \mathbb{R}\}$ determine different Poincaré-invariant, *pure* physical phases.

The proof of Theorem 4 is contained in the second reference listed under Ref. [14]. Results similar to, but slightly weaker than Theorem 4 (of course with the exception of Euclidean invariance) have been obtained for the case where $\beta^{-1} > 0$ in Ref. [11].

For other properties of pure phases and phase transitions at $\beta^{-1} = 0$ (and arguments for the existence of more than one pure phase in ϕ_2^4) see also Refs. [18] and [19].

We would like to remind the reader of the fact that reconstruction theorems such as Theorem 2.2 can be derived from the general results of Refs. [9, 10, 11] for very general field theories, e.g. models of nuclear matter or solids in three space-dimensions, etc. (See Ref. [17]). Apart from our applications to models in two space-time dimensions the main result of this paper is the observation that combining Osterwalder–Schrader positivity (6), the KMS condition (3) restricted to the Euclidean points, inequalities on the Schwinger functions described under (8) and the Hölder property (9) suffices to reconstruct an equilibrium state ω^β from the Schwinger functions.

References

[1] H.-J. BORCHERS, Nuovo Cimento *24*, 214 (1962).
[2] A. A. ABRIKOSOV, L. P. GOR'KOV and I. Y. DZYALOSHINSKII, *Quantum Field Theoretical Methods in Statistical Physics* (Pergamon Press, London–New York–Paris–Frankfurt, 1965).
[3] H. ARAKI, Publ. RIMS *4*, 361 (1968).

[2]) This result has been announced for φ_2^4 by Dobrushin and Minlos in 1973, but no proof has yet appeared.

[4] R. JOST, *The General Theory of Quantized Fields*, Lectures in Applied Mathematics (American Math. Soc., Providence 1965). R. F. STREATER and A. S. WIGHTMAN, *PCT, Spin and Statistics and All That* (W. A. Benjamin, New York–Amsterdam 1964).
[5] J. GINIBRE, J. Math. Phys. *6*, 238, 252, 1432 (1965).
[6] G. VELO and A. S. WIGHTMAN (Eds.), *Constructive Quantum Field Theory*, Lecture Notes in Physics, Vol. 25 (Springer Verlag, Berlin–Heidelberg–New York 1973).
[7] R. HØEGH-KROHN, *Relativistic Quantum Statistical Mechanics in Two Dimensional Space-Time* (University of Oslo, Preprint 1973).
[8] E. NELSON, J. Funct. Anal. *12*, 97 (1973).
[9] K. OSTERWALDER and R. SCHRADER, Comm. Math. Phys. *31*, 83 (1973); contribution in Ref. [6]; Harvard preprint (1974).
[10] J. FRÖHLICH, *Verification of axioms for Euclidean and relativistic fields and Haag's theorem in a class of $P(\varphi)_2$-models*, Harvard University, 1974, to appear in Ann. Inst. H. Poincaré.
[11] J. FRÖHLICH, unpublished report, Princeton, 1974.
[12] J. GLIMM and A. JAFFE, in *Statistical Mechanics and Quantum Field Theory*, Les Houches 1970, edited by C. DE WITT and R. STORA (Gordon and Breach, New York, 1971); contributions in Ref. [6] and references cited there.
[13] S. WEINBERG, *Gauge and Global Symmetries at High Temperature* (Havard University, Preprint 1974).
[14] J. FRÖHLICH, *Schwinger functions and their generating functionals, I*, Helv. Phys. Acta *47*, 265 (1974); *Schwinger functions and their generating functionals, II*, to appear in Adv. Math. 1974, *The Quantized 'Sine-Gordon'* Equation with a Non-Vanishing Mass Term in Two Space-Time Dimensions (Princeton preprint, 1974).
[15] F. GUERRA, L. ROSEN and B. SIMON, '*The $P(\varphi)_2$ Euclidean quantum field theory as classical statistical mechanics*, to appear in Ann. Math.; contributions in Ref. [6].
[16] S. ALBEVERIO and R. HØEGH-KROHN, *The Wightman Axioms and the Mass Gap for Strong Interactions of Exponential Type in Two-Dimensional Space-Time* (University of Oslo, preprint 1973).
[17] D. RUELLE, J. Math. Phys. *12*, 901 (1971) (see also Helv. Phys. Acta *45*, 215 (1972).
[18] J. GLIMM and A. JAFFE, *The φ_2^4 Quantum Field Model in the Single Phase Region: Differentiability of the Mass and Bounds on Critical Exponents* (IHES preprint, 1974). Also J. GLIMM and A. JAFFE, *Critical point dominance in quantum field models*, to appear in Ann. Inst. H. Poincaré.
[19] S. COLEMAN and E. WEINBERG, Phys. Rev. D7, 1888 (1973).