Energy loss of charged particles in a medium of resonant atoms in the presence of an electromagnetic field

Energy Loss of Charged Particles in a Medium of Resonant Atoms in the Presence of an Electromagnetic Field

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Abstract. The process of the energy loss of a massive charged particle in a medium of independent atoms in the presence of a resonant electromagnetic field is investigated. It is shown that the field changes radically the character of the movement of the particle, from elastic to inelastic. The sign of the energetic losses depends on the sign of the difference between the frequency of the field and transition frequency of atoms.

1. The Description of an Atom Interacting with a Resonant Electromagnetic Field in the Language of Compound Systems. The System of Equations

A number of workers have recently considered photon absorption by atoms colliding in a strong electromagnetic field [1–4]. We shall say that an electromagnetic field $\epsilon_0 \cos \omega t$ is strong if it effectively changes the densities of the atomic levels during the time of an atomic collision.

For such a field V. S. Lisitsa and S. I. Yakovlenko have solved the problem of absorption of light due to collisions between resonant atoms and charged particles [1, 2].

S. P. Andreiev and V. S. Lisitsa have solved the same problem for a system of identical resonant atoms [4]. But the question of the energy loss of charged particles (or atoms) in such collisions is still open. This question is especially interesting because:

1. For slow particles the energy loss is zero when the electromagnetic field is absent.

2. It is not evident what the sign of the energy loss will be in the presence of the field.

3. A more complete experimental investigation of the interatomic interaction is possible by measuring the energy loss of massive particles in gases [2].

We shall investigate the energy loss of a massive charged particle moving in a medium of independent identical atoms, excited by an electromagnetic field $\epsilon_0 \cos \omega t$. The hamiltonian describing the interaction between an atom of the medium and a charged particle in the presence of the electromagnetic field is

$$\hat{H} = \hat{V}_{x\epsilon} + V \tag{1.1}$$

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where

$$\hat{V}_{x\epsilon} = -\vec{d}_x \vec{\epsilon}_0 \cos \omega t$$

interaction hamiltonian between the electromagnetic field and the atom (x).

$\hat{\vec{d}}_x$ = dipole moment operator of the atomic electron.

$\hat{V}$ = interaction hamiltonian between the atom and the charged particle.

We shall assume that the distance between the atomic energy levels $(E_2 - E_1)$ is close to the energy of the quanta of the field, i.e.

$$\omega - (E_2 - E_1)/h \equiv \Delta\omega \ll \omega. \tag{1.2}$$

If the effective impact parameter between the particle and the atom ρ_{eff} is larger than the atomic size $d_0(\rho_{\text{eff}} \gg d_0)$ and the velocity of the particle v_0 is small enough

$$v_0/\rho_{\text{eff}} \ll (E_2 - E_1)/h \tag{1.3}$$

then the hamiltonian $\hat{V}$ can be written in the form:

$$\hat{V} = -\frac{eqr^2}{r_0^3(t)}. \tag{1.4}$$

Here e is the charge of the electron of the atom; $\vec{r}$ its radius-vector; q is the charge of the massive particle; $\vec{r}_0(t)$ is the trajectory of the particle, which for a massive particle can be assumed to be of the form

$$\vec{r}_0(t) = \vec{\rho} + \vec{v}_0 t, \quad \vec{\rho}\cdot\vec{v}_0 = 0 \tag{1.5}$$

and ρ is the impact parameter.

In the resonant situation the wave-function of the atomic electron can be written as a combination [5]:

$$\psi = C_1 e^{i(\Delta\omega t/2)}\psi_1 + C_2 e^{-i(\Delta\omega t/2)}\psi_2 \tag{1.6}$$

in which ψ_i is a wave-function with energy level E_i ($i = 1, 2$). It is very simple to obtain a system of equations for the coefficients C_i from the Schrödinger equation. The result is:

$$i\left(\dot{C}_1 + i\frac{\Delta\omega}{2} C_1\right) = -V_\epsilon C_2 + V_{11}C_1, \tag{1.7a}$$

$$i\left(\dot{C}_2 - i\frac{\Delta\omega}{2} C_2\right) = -V_\epsilon^* C_1 + V_{22}C_2 \tag{1.7b}$$

where

$$V_\epsilon = \frac{(\vec{d}_x)_{12}\vec{E}_0}{2h}, \quad V_{kk} = \frac{\langle\psi_k|V|\psi_k\rangle}{h}, \qquad k = 1, 2$$

now we shall make a change of variables:

$$b_{\text{I}} = \left[b_- C_1 + \frac{V_\epsilon}{|V_\epsilon|} b_+ C_2\right] \exp\left[-i\frac{\Omega_0}{2} t + i\int_{-\infty}^{t} U_{\text{I}}(t')\,dt'\right] \tag{1.8a}$$

$$b_{\text{II}} = \left[-b_+ C_1 + \frac{V_\epsilon^*}{|V_\epsilon|} b_- C_2\right] \exp\left[i\frac{\Omega_0 t}{2} + i\int_{-\infty}^{t} U_{\text{II}}(t')\,dt'\right]. \tag{1.8b}$$

Here:

$$b_{\pm} = \frac{1}{\sqrt{2}}\sqrt{1 \pm \frac{\Delta\omega}{\Omega_0}}, \quad \Omega_0 = \sqrt{\Delta\omega^2 + 4|V_\epsilon|^2},$$

$$U_{\mathrm{I}} = b_-^2 V_{11} + b_+^2 V_{22}, \quad U_{\mathrm{II}} = b_+^2 V_{11} + b_-^2 V_{22}.$$

The system of equations for the coefficients b_{I} and b_{II} becomes [2]:

$$\begin{aligned} i\dot{b}_{\mathrm{I}} &= b_{\mathrm{II}}\cdot\varkappa(t)\frac{|V_\epsilon|}{\Omega_0}\cdot\exp\left\{-i\left[\Omega_0 t - \frac{\Delta\omega}{\Omega_0}\int_{-\infty}^{t}\varkappa(t')\,dt'\right]\right\}, \\ i\dot{b}_{\mathrm{II}} &= b_{\mathrm{I}}\cdot\varkappa(t)\frac{|V_\epsilon|}{\Omega_0}\cdot\exp\left\{i\left[\Omega_0 t - \frac{\Delta\omega}{\Omega_0}\int_{-\infty}^{t}\varkappa(t')\,dt'\right]\right\}, \end{aligned} \tag{1.9}$$

$$\varkappa(t) = C[\rho^2 + v_0^2 t^2]^{-3/2}, \quad C = -\frac{qe}{\hbar}(r_2^2 - r_1^2), \quad r_i^2 = \langle\psi_i|r^2|\psi_i\rangle. \tag{1.10}$$

For further calculations it is necessary to know the wave function of the atom in the resonant electromagnetic field. We can use directly the expression for the wave function of the atomic electron from Ref. [5] and supposing that at time t_x the electron occupies the lowest energy level, we can write:

$$\begin{aligned} \psi &= \left[\cos\frac{\Omega_0}{2}(t - t_x) - i\frac{\Delta\omega}{\Omega_0}\sin\frac{\Omega_0}{2}(t - t_x)\right]e^{i(\Delta\omega/2)t}\psi_1 \\ &\quad + i\frac{V_\epsilon^*}{\Omega_0}2\sin\frac{\Omega_0}{2}(t - t_x)e^{-i(\Delta\omega/2)t}\psi_2. \end{aligned} \tag{1.11}$$

Using $b_{\mathrm{I,II}}$-coefficients it is not difficult to rewrite the last formula as:

$$\begin{aligned} \psi &= b_{\mathrm{I}}^0 e^{i(\Omega_0/2)t}\psi_{\mathrm{I}} + b_{\mathrm{II}}^0 e^{-i(\Omega_0/2)t}\psi_{\mathrm{II}}, \\ b_{\mathrm{I}}^0 &= b_- e^{-i(\Omega_0/2)t_x}, \quad b_{\mathrm{II}}^0 = -b_+ e^{i(\Omega_0/2)t_x}. \end{aligned} \tag{1.12}$$

The functions $\psi_{\mathrm{I,II}}$ are normalized, orthogonal and they describe the two-level compound system, with average quantum-mechanical energies $\bar{E}_{\mathrm{I}} = b_-^2 \times E_1 + b_+^2 \times E_2$ and $\bar{E}_{\mathrm{II}} = b_+^2 \times E_1 + b_-^2 \times E_2$, as it is shown below:

$$\psi_{\mathrm{I}} = b_-\psi_1 e^{i(\Delta\omega/2)t} = \frac{V_\epsilon^*}{|V_\epsilon|}b_+\psi_2 e^{-i(\Delta\omega/2)t}, \qquad \bar{E}_{\mathrm{I}} = b_-^2 E_1 + b_+^2 E_2;$$

$$\psi_{\mathrm{II}} = b_+\psi_1 e^{i(\Delta\omega/2)t} - \frac{V_\epsilon^*}{|V_\epsilon|}b_-\psi_2 e^{-i(\Delta\omega/2)t}, \qquad E_{\mathrm{II}} = b_+^2 E_1 + b_-^2 E_2.$$

The distance of the energies $\bar{E}_{\mathrm{I}} - \bar{E}_{\mathrm{II}}$ is

$$E_{\mathrm{I}} - E_{\mathrm{II}} = \frac{\Delta\omega}{\Omega_0}(E_2 - E_1) \tag{1.13}$$

so we can tell from (1.9), (1.12), (1.13) that in the presence of a resonant field we have interaction not between the particle and an atom, but between the compound system (the atom + the field) and the particle. Every collision between the compound system and the charged particle is described by the system of equations (1.9) and leads to an absorption of light proportional to [4]:

$$\Delta E_{\mathrm{abs}} \sim \frac{\Delta\omega}{\Omega_0}(E_2 - E_1) \tag{1.14}$$

but the energy necessary for such a transition is proportional to $\hbar\Omega_0 \ll (E_2 - E_1)$ as can be seen from equations (1.9). From this it is possible to conclude that, in a resonant field, the energy loss of charged particles will take place, though it will not in a non-excited medium. The energy necessary for a transition of the atomic electron in the absence of the field is proportional to $(E_2 - E_1) \gg \Omega_0 h$.

2. Energy Loss of the Charged Particle Interacting with the Compound System

For further calculations it is necessary to obtain a general expression for the energy loss of the particle in terms of the density amplitudes of the compound system states b_{I}, b_{II}.

The atomic potential of every point of space can be written as:

$$\varphi(\vec{r}) = \frac{|e|}{r} + e\int \frac{|\psi(\vec{r}\,', t)|^2}{|\vec{r} - \vec{r}\,'|}\, d^3r'. \tag{2.1}$$

Making the same assumption which has been used to obtain (1.4), and using the expression for the wave-function (1.6) we can rewrite the last formula in the form:

$$\varphi(\vec{r}) = -\frac{e}{r^3}\{|C_1(t)|^2 r_1^2 + |C_2(t)|^2 r_2^2\}. \tag{2.2}$$

Hence for the intensity of the electric field at the point where the charged particle is, we shall have:

$$\vec{E}_0(t) = -\frac{3e\vec{r}_0(t)}{r_0^5(t)}\{|C_1(t)|^2 r_1^2 + |C_2(t)|^2 r_2^2\}. \tag{2.3}$$

The work done by the field in unit time, i.e. the rate of change of energy of the particle is:

$$\frac{dA}{dt} = \frac{dE}{dt} = q\vec{E}_0(t)\vec{v}_0$$

and by integrating this expression in time, we obtain the change of energy of the charged particle:

$$\Delta E = -3eqv_0^2 \int_{-\infty}^{+\infty} \frac{t\,dt}{[\rho^2 + v_0^2 t^2]^{5/2}}\,[|C_1(t)|^2 r_1^2 + |C_2(t)|^2 r_2^2]. \tag{2.4}$$

With the help of expression (1.8), it is not difficult to write the formula for energy loss of the particle in terms of the coefficients b_{I}, b_{II}, which characterize the compound system. But we shall not consider the energy loss of the particle due to the individual act of collision, but the energy loss of the particle in the medium of the resonant atoms. So, multiplying (2.4) by $2\pi n_0 \rho\, d\rho v_0$ and integrating over the impact parameter ρ, we have

$$\frac{dE}{dt} = 6\pi n_0 \hbar C v_0^3 \int_0^{\infty} \rho\, d\rho \int_{-\infty}^{+\infty} \frac{t\,dt}{[\rho^2 + v_0^2 t^2]^{5/2}} \cdot \left\{\frac{1}{2}\frac{\Delta\omega}{\Omega_0} \cdot [|b_{\mathrm{I}}(t)|^2 - |b_{\mathrm{II}}(t)|^2] \right.$$
$$\left. + \frac{2|V_\epsilon|}{\Omega_0}\,\mathrm{Re}\left[b_{\mathrm{I}}(t) b_{\mathrm{II}}^*(t) \exp\left(-2i\frac{\Delta\omega}{\Omega_0}\Delta(t) + i\Omega_0 t\right)\right]\right\}. \tag{2.5}$$

Here:

$$\Delta(t) = \frac{1}{2}\int_{-\infty}^{t} \varkappa(t')\,dt' = \frac{C}{2v_0\rho^2}\left[1 + \frac{v_0 t}{\sqrt{v_0^2 t^2 + \rho^2}}\right] \tag{2.6}$$

and n_0 = density of the resonant atoms.

The system of equations (1.9) and the last formula (2.5) describe completely the process of energy loss of massive charged particles in a medium of resonant atoms in the presence of an electromagnetic field. But in general it is impossible to solve the system of equations (1.9) for the compound system coefficients. However, we can investigate some special cases and in this way obtain a rather full picture of the process. It is evident from (1.9) and (2.5) that in the absence of the electromagnetic field the energy loss of the particle is zero, i.e. the movement has an elastic character.

3. Perturbation Theory

In the second approximation of perturbation theory the coefficients of the compound system have the form:

$$\begin{aligned} b_{\mathrm{I}}(t) = b_{\mathrm{I}}^0 &- ib_{\mathrm{II}}^0 \frac{|V_\epsilon|}{\Omega_0}\int_{-\infty}^{t} \varkappa(t')\exp\left\{-i\left[\Omega_0 t' - \frac{\Delta\omega}{\Omega_0}2\Delta(t')\right]\right\}dt' \\ &- b_{\mathrm{I}}^0\frac{|V_\epsilon|^2}{\Omega_0^2}\int_{-\infty}^{t} dt'\varkappa(t')\exp\left\{-i\left[\Omega_0 t' - \frac{\Delta\omega}{\Omega_0}2\Delta(t')\right]\right\} \\ &\times \int_{-\infty}^{t'} \varkappa(t'')\exp\left\{i\left[\Omega_0 t'' - \frac{\Delta\omega}{\Omega_0}2\Delta(t'')\right]\right\}dt''; \end{aligned} \tag{3.1}$$

$$\begin{aligned} b_{\mathrm{II}}(t) = b_{\mathrm{II}}^0 &- ib_{\mathrm{I}}^0 \frac{|V_\epsilon|}{\Omega_0}\int_{-\infty}^{t} \varkappa(t')\exp\left\{i\left[\Omega_0 t' - \frac{\Delta\omega}{\Omega_0}2\Delta(t')\right]\right\}dt' \\ &- b_{\mathrm{II}}^0\frac{|V_\epsilon|^2}{\Omega_0^2}\int_{-\infty}^{t} dt'\varkappa(t')\exp\left\{i\left[\Omega_0 t' - \frac{\Delta\omega}{\Omega_0}2\Delta(t')\right]\right\} \\ &\times \int_{-\infty}^{t'} \varkappa(t'')\exp\left\{-i\left[\Omega_0 t'' - \frac{\Delta\omega}{\Omega_0}2\Delta(t'')\right]\right\}dt''. \end{aligned} \tag{3.2}$$

The conditions for the validity of the perturbation theory are different for the cases of high and low frequencies $\Delta\omega$. They can be written in the form [2]:

$$|V_\epsilon| \ll |\Delta\omega| \qquad \text{if } |\Delta\omega| \ll \Omega_W \tag{3.3}$$

and

$$|V_\epsilon| \ll (\Omega_W \Delta\omega^2)^{1/3} \ll |\Delta\omega| \qquad \text{if } |\Delta\omega| \gg \Omega_W \tag{3.4}$$

$\Omega_W \equiv v_0^{3/2}/|C|^{1/2} = v_0/\rho_W$ is the characteristic frequency scale.

It is necessary to mention here that although we have developed the perturbation theory for the compound system the inequalities (3.3–3.4) automatically lead us to the perturbation theory for the atom when $|V_\epsilon| \ll |\Delta\omega|$, i.e. the resonant electromagnetic field must be small enough so that the probability of finding the atomic electron on the high energy level will be small compared with one. But the inequality (3.1) is absent in the usual atomic perturbation theory. By substituting (3.1–3.2) in the expression (2.5) for the energy loss of the particle, and by extending this expression to all possible times t_x, and by introducing new integration variables $v_0 t = \rho x$; $v_0 t' = \rho x'$, $\rho/\rho_W = y$: we readily obtain:

$$\frac{dE}{dt} = 6\pi n_0 \hbar C v_0 \frac{|V_\epsilon|^2}{\Delta\omega^2} \frac{1}{\rho_W} \int_0^\infty \frac{dy}{y^4}$$
$$\times \left\{ \frac{1}{y^2} \frac{2}{3} \int_0^\infty \frac{dx}{(x^2+1)^3} \cdot \cos\left[\beta y x - \eta_{c\Delta\omega} \cdot \frac{1}{y^2} \frac{x}{\sqrt{x^2+1}}\right] \right.$$
$$\left. + 2\eta_{c\Delta\omega} \cdot \int_0^\infty \frac{x\,dx}{(x^2+1)^{5/2}} \cdot \sin\left[\beta y x - \eta_{c\Delta\omega} \frac{1}{y^2} \frac{x}{\sqrt{x^2+1}}\right] \right\}$$
$$\times \int_{-\infty}^{+\infty} \frac{dx'}{(x'^2+1)^{3/2}} \cos\left[\beta y x - \eta_{c\Delta\omega} \frac{1}{y^2} \frac{x'}{\sqrt{x'^2+1}}\right]. \tag{3.5}$$

Here

$$\beta \equiv |\Delta\omega| \rho_W / v_0; \quad \eta_{c\Delta\omega} \equiv \operatorname{sgn}(C\Delta\omega); \quad \rho_W \equiv \left|\frac{C}{v_0}\right|^{1/2}. \tag{3.6}$$

From the expression (3.5) it follows that if conditions (3.3) and (3.4) are satisfied, then the energy loss of the particles is proportional to the square of the resonant field.

Let us investigate the expression (3.5) for high and low values of the parameter β. If the inequality

$$|\Delta\omega| \ll v_0/\rho_W, \qquad \text{i.e. } \beta \ll 1 \tag{3.7}$$

is satisfied, the first terms in the arguments of the sine and cosine in (3.5) are small and we immediately obtain:

$$\frac{dE}{dt} \simeq \operatorname{sgn}(\Delta\omega) 8\beta\pi n_0 \hbar |C| v_0 \frac{|V_\epsilon|^2}{\Delta\omega^2} \frac{1}{\rho_W} \sim \Delta\omega |C| \frac{|V_\epsilon|^2}{\Delta\omega^2}. \tag{3.8}$$

Now we shall study the expression (3.5) for large values of β:

$$|\Delta\omega| \gg v_0/\rho_W, \qquad \text{i.e. } \beta \gg 1. \tag{3.9}$$

This limit is determined by the relative signs of $\Delta\omega$ and C, which determine the presence or absence of a stationary phase point $x = \Delta\omega$ in the arguments of sine and cosine in (3.5). In the first case, if we evaluate the integrals in (3.5) by the method of steepest descent on the real axis [3] we have:

$$\frac{dE}{dt} = \frac{8\pi}{3} n_0 \hbar |C| v_0 \frac{|V_\epsilon|^2}{\Delta\omega^2} \beta \frac{1}{\rho_W} \operatorname{sgn}(\Delta\omega) \sim \Delta\omega |C| \frac{|V_\epsilon|^2}{\Delta\omega^2}. \qquad (C\Delta\omega) > 0; \tag{3.10}$$

and in the second case (no stationary phase point) the integrals in (3.5) are determined by the pole of the function $(X^2 + 1)^{-3/2}$, in the upper half plane. The result of integration is

$$\frac{dE}{dt} = -\frac{3}{4}\pi^2 n_0 \hbar |C| v_0 \frac{|V_\epsilon|^2}{\Delta\omega^2} \frac{\beta^{5/3}}{\rho_W} e^{-3\beta 2/3} \operatorname{sgn}(\Delta\omega) \sim \exp(-\Delta\omega^{2/3}\rho_W^{2/3}/v_0^{2/3}). \tag{3.11}$$

4. Resonant Situation ($\Delta\omega = 0$)

In the case of exact resonance $\Delta\omega = 0$, the formula (2.5) for the energy loss of the particle can be rewritten in the form:

$$\frac{dE}{dt} = 12\pi n_0 \hbar C v_0^3 \int_0^{\infty} \rho\, d\rho \int_{-\infty}^{+\infty} \frac{t\, dt}{[\rho^2 + v_0^2 t^2]^{5/2}} \operatorname{Re}[b_{\mathrm{I}} b_{\mathrm{II}}^* e^{i\Omega_0 t}] \tag{4.1}$$

and from the system of equations (1.9) in the case of exact resonance we can obtain very easily that the average value of the integrated function (4.1) is equal to zero (we have in mind averaging over all the possible times t_x). So in the exact resonant situation we have:

$$\frac{dE}{dt} = 0. \tag{4.2}$$

5. Impact Approximation

If the velocity of the particle is sufficiently high

$$v_0 \gg (\Omega_0 \Delta\omega C)^{1/3} \tag{5.1}$$

the resonant electromagnetic field changes the b-coefficients of the compound system very little during a collision between the particle and the compound system. In such a situation we can obtain the solution of the system of equations (1.9) in the form of a perturbation series with a small parameter

$$\Omega_0 \cdot \tilde{\rho}_W / v_0 \ll 1. \tag{5.2}$$

Here $\tilde{\rho}_W$ is a modified Weiskopff radius [2] $\tilde{\rho}_W = \rho_W(\Delta\omega/\Omega_0)^{1/2}$ characterizing the collisions in a strong electromagnetic field.

The conditions (5.1) or (5.2) do not mean that the electromagnetic field is small in the sense of the inequalities (3.3–3.4), i.e. the coefficients of the compound system levels can have the same order of magnitude $b_{\mathrm{I}} \sim b_{\mathrm{II}}$.

The first two terms of the perturbation series for the solution of (1.9) in powers of $\Omega_0 \tilde{\rho}_W / v_0$ is:

$$\begin{aligned} b_{\mathrm{I}}(t) = b_{\mathrm{I}}^0(t) + |V_\epsilon| e^{i(\Delta\omega/\Omega_0)\Delta} \int_{-\infty}^{t} dt' t' \varkappa(t') \\ \times \Big[-b_{\mathrm{II}}^0(t') e^{i(\Delta\omega/\Omega_0)\Delta} \cos(\Delta - \Delta') - i \sin(\Delta - \Delta') \\ \times \left(\frac{2|V_\epsilon|}{\Omega_0} b_{\mathrm{I}}^0(t') e^{-i(\Delta\omega/\Omega_0)\Delta'} - \frac{\Delta\omega}{\Omega_0} b_{\mathrm{II}}^0(t') \cdot e^{i(\Delta\omega/\Omega_0)\Delta'} \right) \Big]; \end{aligned} \tag{5.3}$$

$$\begin{aligned} b_{\mathrm{II}}(t) = b_{\mathrm{II}}^0(t) + |V_\epsilon| \cdot e^{-i(\Delta\omega/\Omega_0)\Delta} \int_{-\infty}^{t} dt' \cdot t' \cdot \varkappa(t') \\ \times \Big[b_{\mathrm{I}}^0(t') \cdot e^{-i(\Delta\omega/\Omega_0)\Delta'} \cdot \cos(\Delta - \Delta') + i \sin(\Delta - \Delta') \\ \times \left(\frac{2|V_\epsilon|}{\Omega_0} b_{\mathrm{II}}^0(t') \cdot e^{i(\Delta\omega/\Omega_0)\Delta'} + \frac{\Delta\omega}{\Omega_0} b_{\mathrm{I}}^0(t') \cdot e^{-i(\Delta\omega/\Omega_0)\Delta'} \right) \Big] \end{aligned} \tag{5.4}$$

$$b_{\mathrm{I}}^{0}(t) = e^{i(\Delta\omega/\Omega_0)\Delta}\left\{b_{\mathrm{I}}^{0}\cos\Delta - i\left[b_{\mathrm{I}}^{0}\frac{\Delta\omega}{\Omega_0} + 2b_{\mathrm{II}}^{0}\frac{|V_\epsilon|}{\Omega_0}\right]\sin\Delta\right\}, \tag{5.5}$$

$$b_{\mathrm{II}}^{0}(t) = e^{-i(\Delta\omega/\Omega_0)\Delta}\left\{b_{\mathrm{II}}^{0}\cos\Delta + i\left[b_{\mathrm{II}}^{0}\frac{\Delta\omega}{\Omega_0} - 2b_{\mathrm{I}}^{0}\frac{|V_\epsilon|}{\Omega_0}\right]\sin\Delta\right\}. \tag{5.6}$$

$$\Delta \equiv \Delta(t), \quad \Delta(t') \equiv \Delta'.$$

By substituting these expressions in the general formula (2.5) we get after some calculations the following formula for the energy loss of the fast (5.1) particles:

$$\frac{dE}{dt} = \mathrm{sgn}(\Delta\omega)\cdot 8\pi n_0\hbar|C|\frac{|V_\epsilon|^2}{\Omega_0^2}\left|\frac{\Delta\omega}{\Omega_0}\right|\Omega_0. \tag{5.7}$$

In the limiting case $|\Delta\omega| \gg 2|V_\epsilon|$ this expression becomes (3.8). The right-hand side of (5.7) has a maximum when $|\Delta\omega| = 2|V_\epsilon|$. For this point we shall have:

$$\frac{dE}{dt} = \mathrm{sgn}(\Delta\omega)2\pi n_0\hbar|C|\cdot|V_\epsilon|. \tag{5.8}$$

6. Static Approximation

We can solve the system of equations (1.9) in the other case when the velocity of the particle is rather small (i.e. when the electromagnetic field influences very strongly the individual collisions between the particle and the compound system):

$$v_0 \ll (\Omega_0\Delta\omega C)^{1/3}. \tag{6.1}$$

This is the so-called static region [1–4].

We have used in this case the method of an approximate solution proposed by L. Vainshtein, L. Presnyakov and I. Sobel'man [6].

Let us introduce the new variables β_{I} and β_{II} as:

$$\beta_{\mathrm{I}}(t) = b_{\mathrm{I}}\cdot e^{-(i\Delta\omega/\Omega_0)\Delta}; \quad \beta_{\mathrm{II}}(t) = b_{\mathrm{II}}\cdot e^{i(\Delta\omega/\Omega_0)\Delta}. \tag{6.2}$$

The system of equations for these coefficients is:

$$\begin{aligned} i\dot{\beta}_{\mathrm{I}} &= \frac{\Delta\omega}{2\Omega_0}\varkappa(t)\beta_{\mathrm{I}} + \frac{|V_\epsilon|}{\Omega_0}\varkappa(t)\beta_{\mathrm{II}}\cdot e^{-i\Omega_0 t}, \\ i\dot{\beta}_{\mathrm{II}} &= -\frac{\Delta\omega}{2\Omega_0}\varkappa(t)\beta_{\mathrm{II}} + \frac{|V_\epsilon|}{\Omega_0}\varkappa(t)\beta_{\mathrm{I}}\cdot e^{i\Omega_0 t}. \end{aligned} \tag{6.3}$$

This system of equations was solved in [6] for the special case of initial conditions $\beta_{\mathrm{I}}(-\infty) = 1$; $\beta_{\mathrm{II}}(-\infty) = 0$. We have other initial conditions

$$\beta_{\mathrm{I}}(-\infty) = b_-\cdot e^{-i\Omega_0 t_x/2}; \quad -\beta_{\mathrm{II}}(-\infty) = b_+\cdot e^{i\Omega_0 t_x/2}. \tag{6.4}$$

In this case the solution of (6.3) for slow particles has the form:

$$\begin{aligned} \beta_{\mathrm{I}}(t) &= \{b_-\cdot e^{-i(\Omega_0 t_x/2)}\cdot\cos\gamma(t) - b_+\cdot e^{i(\Omega_0 t_x/2)}\cdot\sin\gamma(t)\}, \\ \beta_{\mathrm{II}}(t) &= ie^{i\int_0^t d(\tau)d\tau}\cdot\{-b_+\cdot e^{i(\Omega_0 t_x/2)}\cdot\cos\gamma(t) - b_-\cdot e^{-i(\Omega_0 t_x/2)}\cdot\sin\gamma(t)\}. \end{aligned} \tag{6.5}$$

Here

$$\gamma(t) = \frac{2|V_\epsilon|}{\Omega_0} \int_{-\infty}^{t} \varkappa(t')\, dt' \cos \int_0^{t'} \sqrt{\left(\Omega_0 - \frac{\Delta\omega}{\Omega_0}\varkappa(\tau)\right)^2 + \frac{4|V_\epsilon|^2}{\Omega_0^2}\cdot\varkappa^2(\tau)}\, d\tau; \quad (6.6)$$

$$d(\tau) = \Omega_0 - \sqrt{\left(\Omega_0 - \frac{\Delta\omega}{\Omega_0}\varkappa(\tau)\right)^2 + \frac{4|V_\epsilon|^2}{\Omega_0^2}\varkappa^2(\tau)}. \quad (6.7)$$

As has been shown in Ref. [6], the expressions (6.5) should be accurately expanded to second order in $|V_\epsilon|$. It is more convenient, however, to carry out the expansion at a later stage. By substituting (6.5) into (2.5) we shall have after some transformations:

$$\frac{dE}{dt} = 24\pi n_0 \hbar C v_0^3 \frac{|V_\epsilon|^2}{\Omega_0^2} \int_0^{\infty} \rho\, d\rho \int_{-\infty}^{+\infty} dt$$
$$\times \left\{ \frac{2}{3v_0^2}\frac{\Delta\omega}{\Omega_0}\frac{\varkappa(t)}{(\rho^2 + v_0^2 t^2)^{3/2}}\cdot\cos\int_0^t (\Omega_0 - d(\tau))\, d\tau \right.$$
$$\left. + \sin\int_0^t (\Omega_0 - d(\tau))\, d\tau \frac{t}{(\rho^2 + v_0^2 t^2)^{5/2}} \right\} \int_0^t \varkappa(t')\, dt' \cdot \cos\int_0^{t'} (\Omega_0 - d(\tau))\, d\tau. \quad (6.8)$$

The second term in the brackets in (6.8), divided by the first, is of the order of $(v_0/(\Omega_0\Delta\omega C)^{1/3})^2 \ll 1$, and can be neglected. After that, by introducing the variables $v_0 t = \rho x$; $v_0 t' = \rho x'$; $v_0 t'' = \rho x''$; $\rho = \tilde{\rho}_W \cdot y$, we can rewrite (6.8) in the form:

$$\frac{dE}{dt} = 8\pi n_0 \hbar C^3 v_0^{-1} \frac{|V_\epsilon|^2}{\Omega_0}\frac{1}{\tilde{\rho}_W^5} \int_0^{\infty} \frac{dy}{y^6} \int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)^3}$$
$$\times \cos\alpha(x; y; \beta) \int_{-\infty}^{+\infty} \frac{dx'}{(x'^2+1)^{3/2}} \cos\alpha(x'; y; \beta), \quad (6.9)$$

$$\alpha(x; y; \beta) = \int_0^x \sqrt{\left(\beta - \eta_{C\Delta\omega}\cdot y^{-2}\frac{1}{(x^2+1)^{3/2}}\right)^2 + \frac{4|V_\epsilon|^2}{\Omega_0^2}\frac{1}{y^6}\cdot\frac{1}{(x^2+1)^3}}\, y\, dx;$$
$$\eta_{C\Delta\omega} = \mathrm{sgn}(C\Delta\omega). \quad (6.10)$$

In (6.10) when $\eta_{\Delta\omega C} > 0$, the main contribution to the integrals (6.9) occurs in the region where $\beta - [y^{-2}/(x^2+1)^{3/2}]$ is small. (This is the so-called case of Landau and Zener [7].) For this case, we have:

$$\frac{dE}{dt} = \frac{\pi^2}{4}\frac{n_0\hbar|C|^3}{v_0}\cdot\frac{|V_\epsilon|^2}{\Delta\omega^2}\cdot\frac{|\Delta\omega\rho_W \mid v_0|^{2/3}}{\rho_W^5}\cdot\exp -\frac{2}{3}\cdot\frac{4|V_\epsilon|^2}{\Delta\omega^2}\cdot|\Delta\omega\rho_W \mid v_0|\cdot\mathrm{sgn}(\Delta\omega). \quad (6.11)$$

Here

$$\beta = |\Delta\omega\rho_W \mid v_0| \gg 1; \quad |\Delta\omega| \gg |V_\epsilon|.$$

In the second case, when $\eta_{C\Delta\omega} < 0$ (this case was considered by Stueckelberg [7]) the estimate of the integrals in (6.10) gives us:

$$\frac{dE}{dt} = -\frac{2}{3} n_0 \hbar v_0^2 \rho_W \cdot \exp -\left(\frac{|V_\epsilon|\cdot|C|\cdot\Delta\omega}{v_0^3}\right)^{1/3}\cdot\mathrm{sgn}(\Delta\omega). \quad (6.12)$$

Here

$$|V_\epsilon| \gg |\Delta\omega|. \quad \beta = \frac{\rho_W}{v_0}(|V_\epsilon|\Delta\omega)^{1/2} \gg 1.$$

So we can conclude that in the case $(\Delta\omega \cdot C) > 0$ the sign of the energy loss of a charged particle in the medium of resonant atoms depends only on the sign of $\Delta\omega$. If $\Delta\omega > 0$, the particle moving in the medium gains the energy of the field through interaction with the compound system. If $\Delta\omega < 0$, the particle loses its energy. For the fast particles the same conclusions can be drawn if $(C\Delta\omega)$ is negative.

In both cases $(C\Delta\omega \lessgtr 0)$ the change of energy is exponentially small for slow particles. In the exact resonant situation $(\Delta\omega = 0)$ the movement of the particle has an elastic character.

The above effects are of direct interest because they can be seen even in fields ϵ_0 much lower than the characteristic atomic field $\epsilon_{at} = 0.5 \cdot 10^{10}$ v/cm. We estimate the order of magnitude for the characteristic parameters used above for the point of maximum energy loss $|V_\epsilon|\rho_W/v_0 \sim 1$. For $v_0 \approx 10^5$ cm/sec, $C \sim 10$ at. units (1.16) $d_x/h \sim 2$ at. units, we obtain $\rho_W \sim 10^{-7}$ cm, $\epsilon_0 \approx 10^4$ V/cm $\ll \epsilon_{at}$ [1].

Variable-frequency laser beams with a power per pulse of ~ 1 MW are now available. By focusing this beam into a spot of radius 10^{-2} cm it is possible to achieve $\epsilon_0 \approx 10^4$ V/cm [8]. Such an experiment would provide a test for the theory presented in this paper.

We have not investigated in this paper the dependence of the excited atomic level on the different projections of the angular momentum. In the case of fast particles, it leads to a negligible change of the results, but for slow particles such dependence can be more important.

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