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Objekttyp: **Article**

Zeitschrift: **Helvetica Physica Acta**

Band (Jahr): **49 (1976)**

Heft 4

PDF erstellt am: **12.07.2024**

Persistenter Link: https://doi.org/10.5169/seals-114781

Propagation of Ion Acoustic Solitons in a Warm Ion Plasma

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(3. II. 1976)

Abstract. We study the propagation of ion acoustic solitons in a large collisionless plasma with hot ion ($T_e/T_i = 9$). Mach number and soliton widths were measured and the experimental results are well described by Sakanaka's theory. The number of solitons is in good agreement with the theoretical predictions of Gardner et al. when ion pressure is considered in the Korteweg de Vries equation.

1. Introduction

The nonlinear development of a wave propagating in a dispersive medium which satisfies the dispersion relation

$$\omega = kc_s - \beta k^3$$

can be described by the well known Korteweg de Vries (KdV) equation.

$$\frac{\partial n}{\partial \eta} + n\frac{\partial n}{\partial \xi} + \frac{1}{2}\frac{\partial^3 n}{\partial \xi^3} = 0 \tag{1}$$

where $\xi = x - c_s t$, $\eta = x$, n is the perturbed density normalized to the unperturbed plasma density n_0. The spatial coordinate x is measured in units of the electron Debye length λ_{De}, the time t in units of the inverse ion plasma frequency ω_{pi}^{-1} and $c_s = \sqrt{kT_e/m_i}$ is the ion acoustic speed.

Using a reductive perturbation method, Washimi and Taniuti [1] have shown that small but finite amplitude ion-acoustic perturbations in a cold ion plasma evolves following the KdV equation (1). In a frame moving with velocities v greater than c_s, i.e. with Mach number $M = v/c_s$ greater than unity, stationary solutions of the KdV equation or solitons are given by

$$\delta n = n\,\mathrm{sech}^2[(x - vt)/D]. \tag{2}$$

From equations (1) and (2) the following relationship between the Mach number M, the soliton width D and the soliton amplitude can be written

$$M = 1 + \frac{1}{3}\frac{\delta n}{n}, \tag{3}$$

$$D/\lambda_{De} = \sqrt{6\frac{n}{\delta n}}. \tag{4}$$

The effect of ion temperature on the KdV equation and its stationary solutions have been studied by Tappert [2], who gives the following relations for M, D and the temperature ratio $\theta = T_e/T_i$

$$M = 1 + \frac{1}{3}\frac{\delta n}{n}\frac{(1 + 6/\theta)}{(1 + 3/\theta)^{3/2}}, \tag{5}$$

$$D/\lambda_{De} = \sqrt{6\frac{n}{\delta n}\frac{(1 + 3/\theta)^{1/4}}{(1 + 6/\theta)^{1/2}}}. \tag{6}$$

Using Moiseev and Sagdeev's approach, namely looking for stationary solutions of the whole set of non-linear fluid equations, Sakanaka [3] derives an expression for the solitary waves in a warm ion plasma with

$$M = 1 + \left(\frac{\theta/3 + 3}{\theta + 3}\right)\frac{\delta n}{n} \tag{7}$$

and

$$D/\lambda_{De} = \left[\frac{4\{M^2\theta + 3(M^2 - 1)\}}{(M^2 - 1)(\theta + 3)}\right]^{1/2}. \tag{8}$$

Both of the preceding two sets of formulas (5, 6) and (7, 8) give the same well known limit when $\theta \to \infty$ but as θ decreases, the Mach number given by (5) is smaller than the one computed from (7). On the other hand the soliton width obtained from the modified KdV equation is larger than the one predicted by Sakanaka's theory.

Solitons have been observed by Ikezi et al. [4, 5]. The temperature ratio θ in these experiments was rather high ($\theta = 30$), however a deviation from the predictions of the KdV equation has been reported. Recently Watanabe [6] studied the

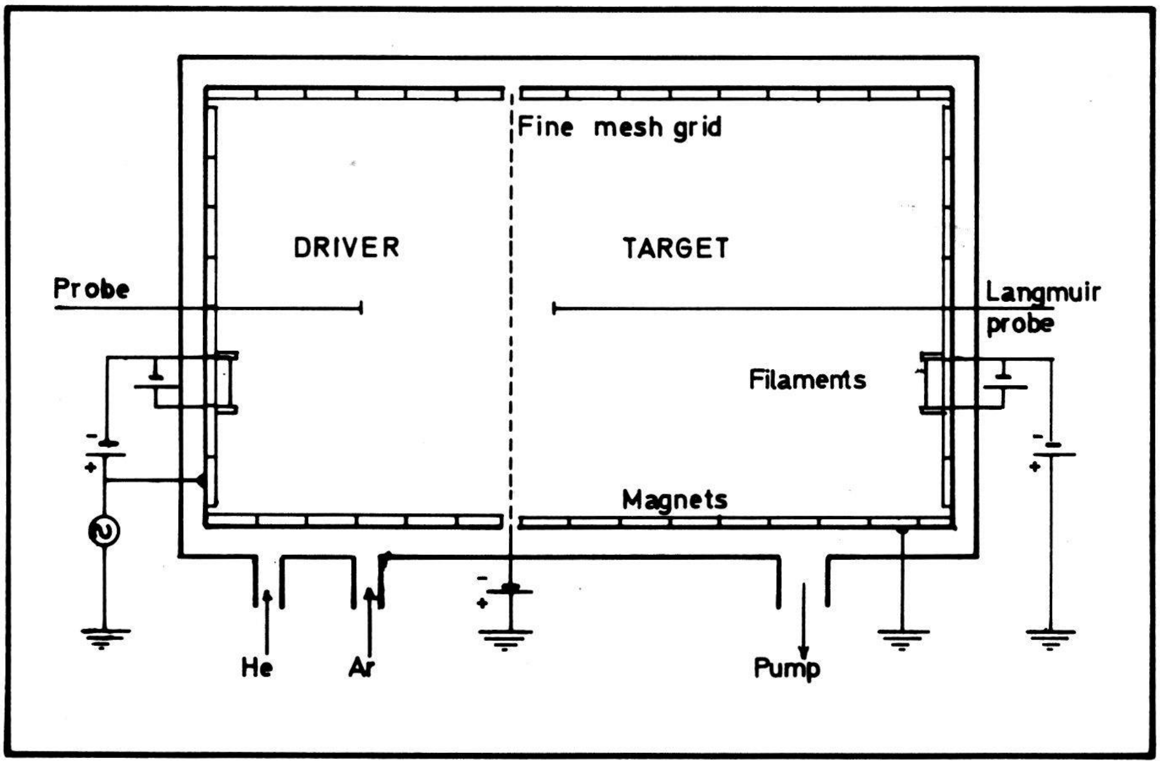

Figure 1
Experimental set up.

propagation of solitary waves in a plasma with a temperature ratio of $\theta = 16$. His experimental results fit more closely to Sakanaka's prediction than to those obtained from the modified KdV equation.

In our note, we present some experimental investigations on the propagation of solitary waves in a relatively hot ion plasma. Working at a temperature ratio $\theta = 9$ we are able to distinguish clearly between the different theories. Our experimental data confirm Watanabe's results.

2. Experimental Results

The experiments were carried out in a multipole Double Plasma (DP) device [7, 8], schematically shown in Figure 1. The driver and the target plasmas, separated by a negatively biased grid, are created by collision of energetic electrons with atoms of a noble gas, in our case Argon and Helium, and are confined by arrays of permanent magnets. Comparing the wavelength of ion acoustic waves propagating in the driver and the target plasma, the plasma potentials in both sides were adjusted to avoid any ion beam.

As diagnostics, Langmuir probes and the propagation of ion acoustic waves were used. Linear ion acoustic waves ($\delta n/n < 1\%$) were excited with the DP mechanism and detected by an interferometric method [9]. By an appropriate choice of ω_{pi} and λ_{De}, the experimental points were then fitted to the theoretical curve (Fig. 2) computed from the well known dispersion equation:

$$\epsilon(\omega, k) = 1 + \frac{1}{k^2\lambda_{\mathrm{De}}^2} + \frac{\theta}{2k^2\lambda_{\mathrm{De}}^2} Z'\left(\frac{\omega}{k}\sqrt{\frac{\theta m_i}{m_e}}\right) = 0.$$

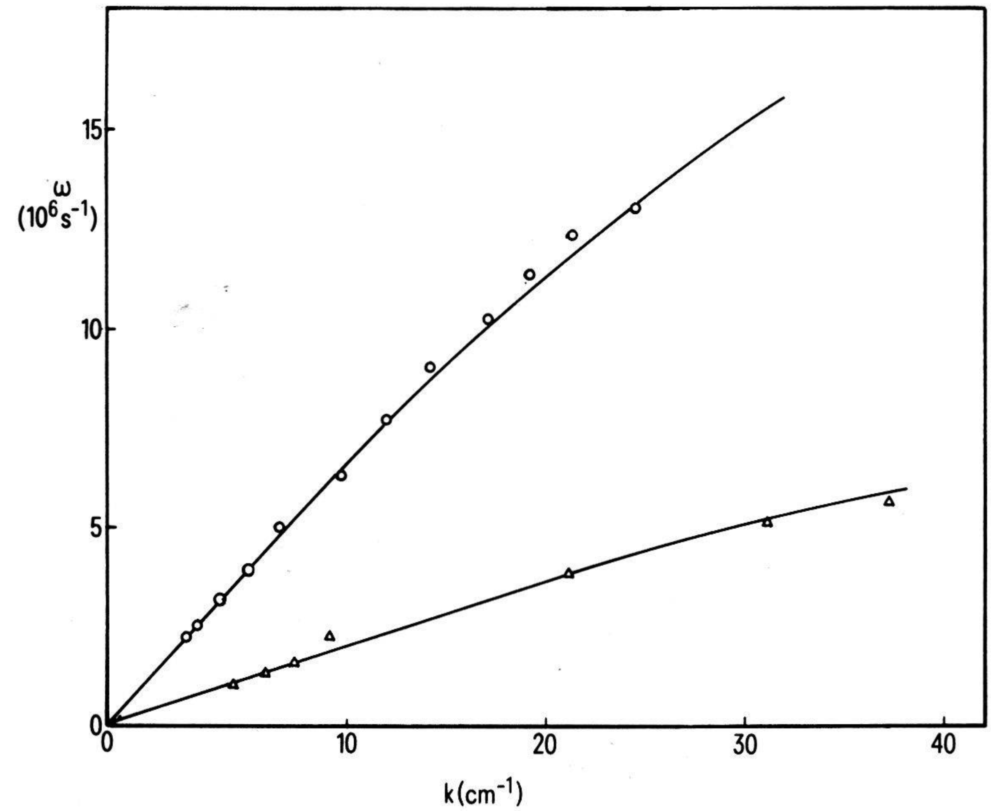

Figure 2
Dispersion relation of small-amplitude ion acoustic wave. (—— Theoretical, △, ○ Experimental values for Argon and Helium, respectively.

θ was measured from the damping rate k_i/k_r and was found to be 9. Using the value $T_e = 1$ eV, we estimate λ_{De} to 0.017 cm for the Argon plasma, and to 0.028 cm for the Helium plasma. The phase velocities obtained are 1780 m/s, and 5700 m/s, respectively, in agreement with time of flight measurements.

The solitons are created by applying a sinusoidal pulse to the driver chamber. Typical pulse widths are 5–12 μs and pulse heights between 0.3 and 1.6 V in the case of the Argon plasma. For the Helium plasma the pulse widths were smaller (2–5 μs), but the excitation pulse height had to be larger, up to 4.5 V.

A movable and positively biased Langmuir probe (diam. 6 mm) in connection with a Boxcar integrator, to eliminate noise, was used to detect the density perturbations in the target chamber. The spatial evolution of the perturbation is shown in Figures 3 and 4 for an Argon and a Helium plasma, respectively.

Linear pulse travels through the plasma without any change in its structure. By increasing the amplitude the perturbation steepens. As the whole structure propagates, the amplitude is reduced by damping. A precursor consisting of reflected ions can be clearly observed. Time of flight measurements show that the observed structure is supersonic ($M > 1$). However it cannot be confused with pseudowaves, especially in the case of the Helium plasma, where relatively high voltage (up to 4.5 V) is applied to the driver. The measured velocity is within the range $1 < M < 1.1$ which is much less than the pseudowave propagation speed $\sqrt{2e\Phi/m_i}$.

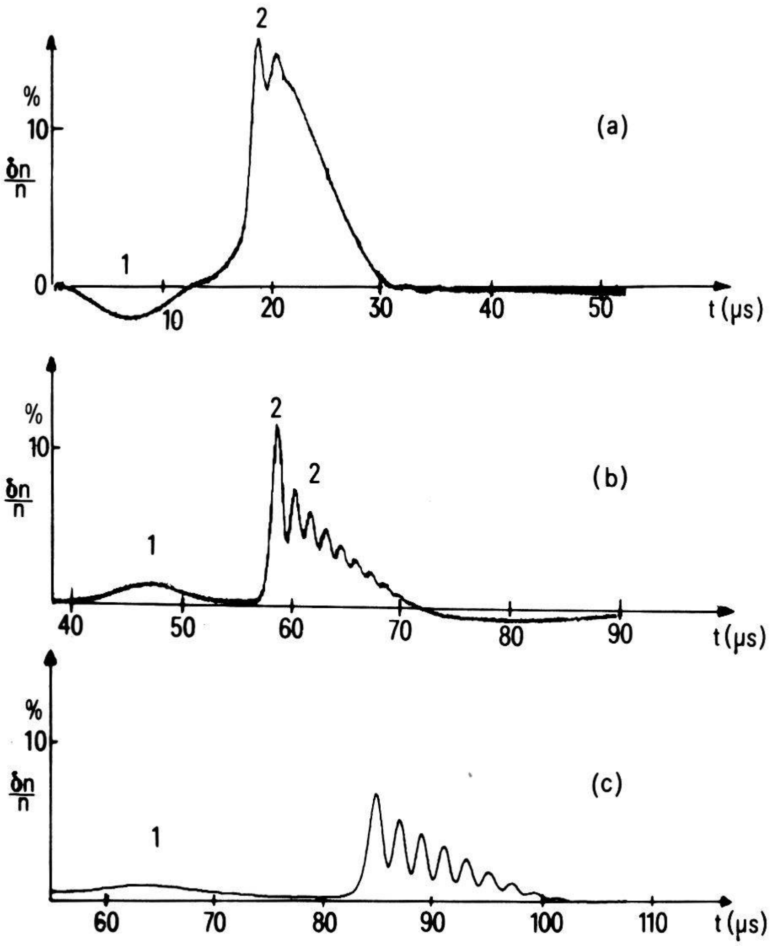

Figure 3
The electron density perturbation $\delta n/n$ versus time for different distances from the wave excitation point, in the case of the Argon plasma. (a) 34.3 mm, 1 pick up, 2 first soliton; (b) 103.8 mm, 1 reflected ions, 2 solitons; (c) 154.9 mm, 1 reflected ions, 2 solitons.

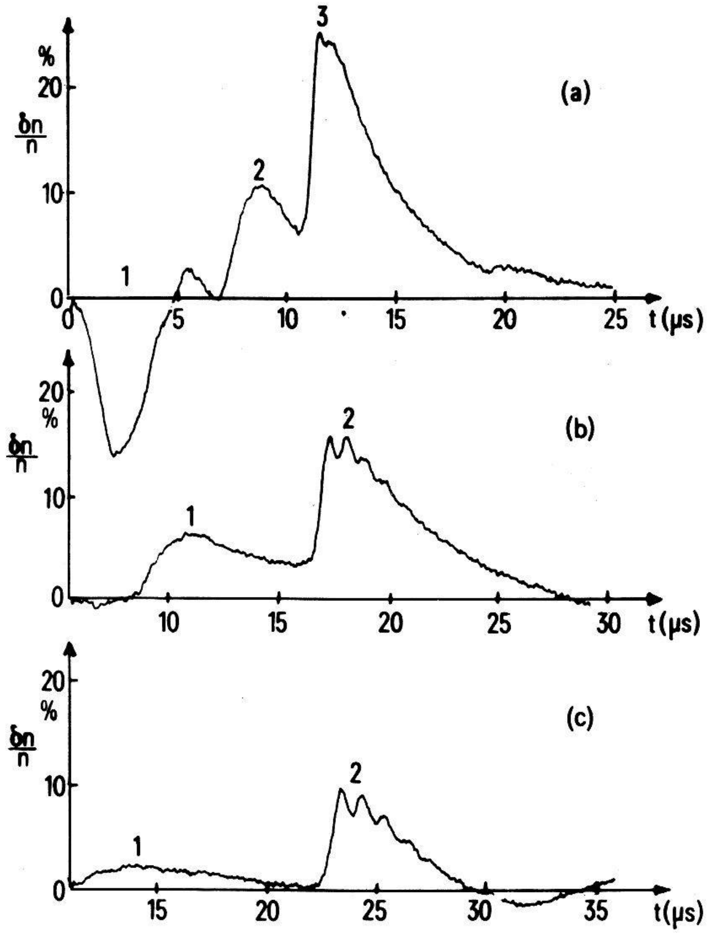

Figure 4
The electron density perturbation $\delta n/n$ versus time for different distances from the wave excitation point, in the case of the Helium plasma. (a) 53.2 mm, 1 pick up, 2 reflected ions, 3 solitons; (b) 85 mm, 1 reflected ion, 2 solitons; (c) 113.6 mm, 1 reflected ion, 2 solitons.

The velocity of the first peak increases linearly with the amplitude whereas the width is inversely proportional to it. These two features clearly identify the first peak as a soliton. Such a clearcut statement cannot be made in regard to the other peaks since they are not separated well enough to allow precise measurements. As seen from Figures 3 and 4, they are compressional pulses and not an oscillatory structure. They also exhibit qualitatively the soliton properties and therefore can be considered as solitons. The characteristic soliton relationship between the velocity, the width and the amplitude are also qualitatively satisfied.

3. Discussion

a. Dependence of M and D/λ_{De} versus $\delta n/n$

The dependence of the Mach number of the first soliton versus its amplitude is reported in Figures 5 and 6 for Ar and He, respectively. The measured values are systematically greater than those predicted by a cold ion KdV equation. The introduction of an ion temperature in the KdV equation slightly increases the Mach number and does not account for the experimental results. As can be seen in Figures 5 and 6 the observed values closely fit Sakanaka's theory [3]. A similar remark can be made

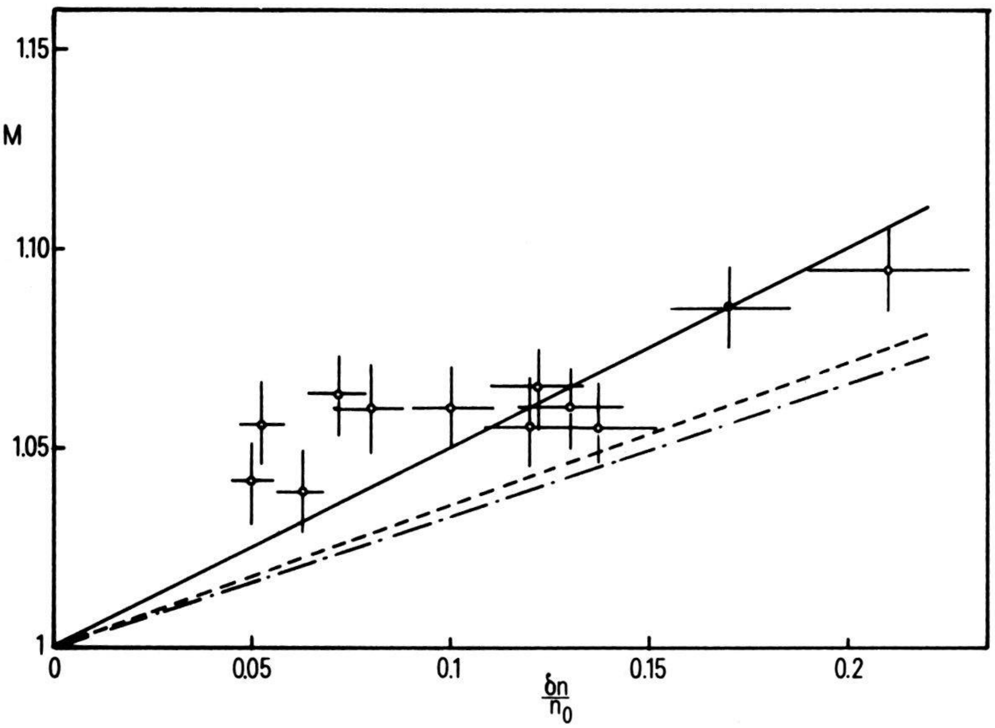

Figure 5
Velocity of soliton in an Argon plasma ($\theta = 9$). (–·–·– cold ion theory, – – – Tappert's theory, —— Sakanaka's theory.)

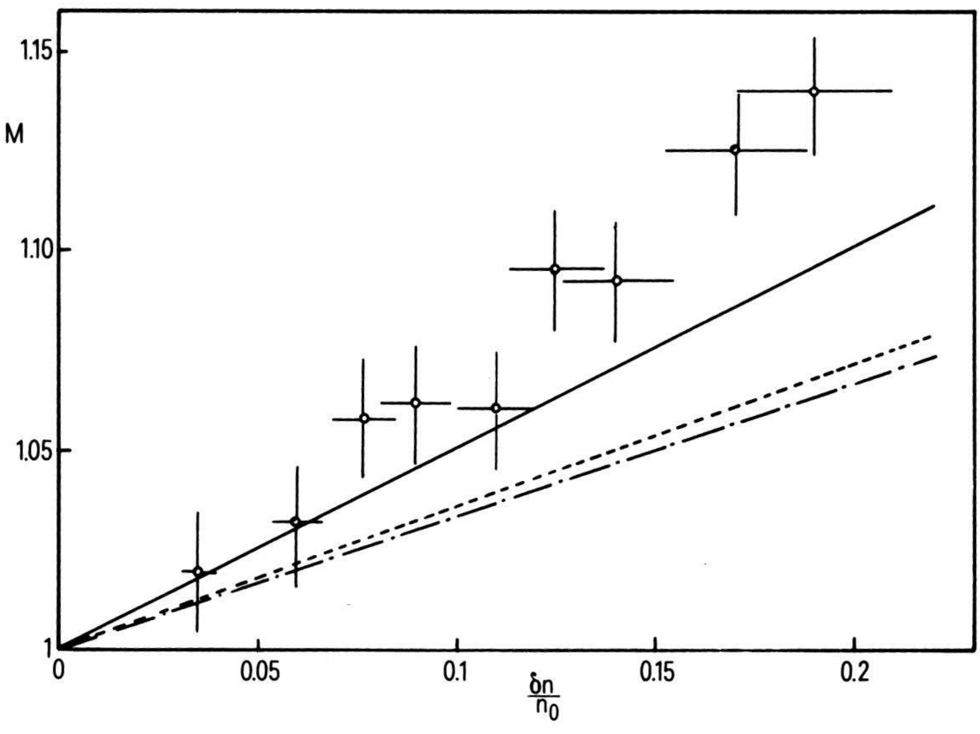

Figure 6
Velocity of soliton in an Helium plasma ($\theta = 9$).

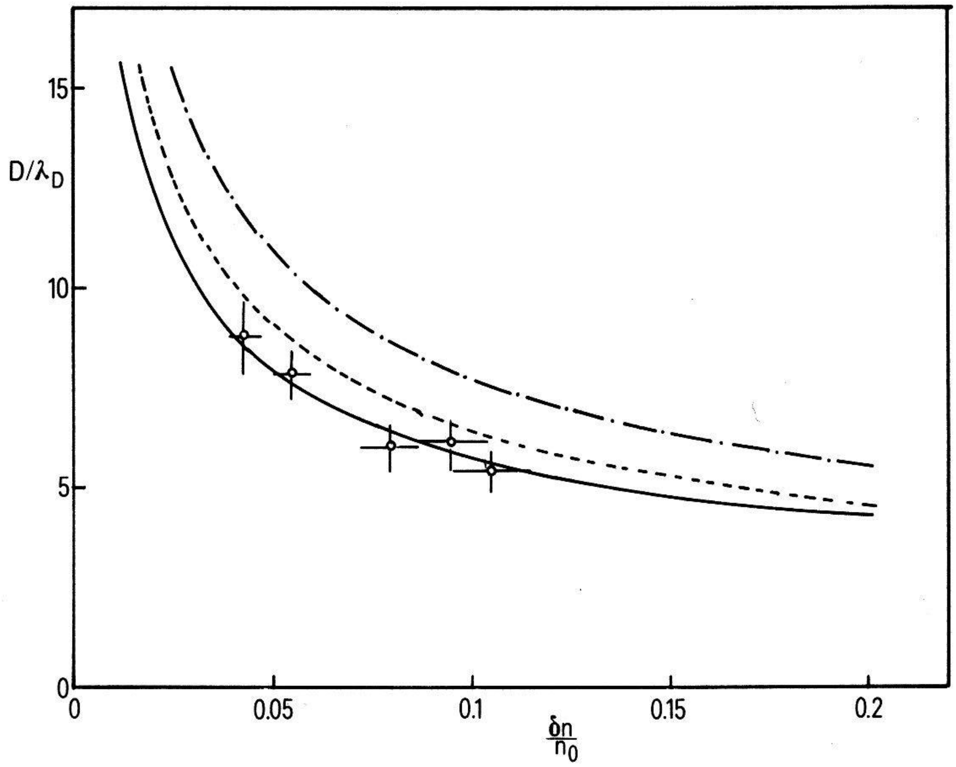

Figure 7
Width of soliton as function of its amplitude for an Argon plasma with $\theta = 9$. (–·–·– cold ion theory, – – – Tappert's theory, —— Sakanaka's theory.)

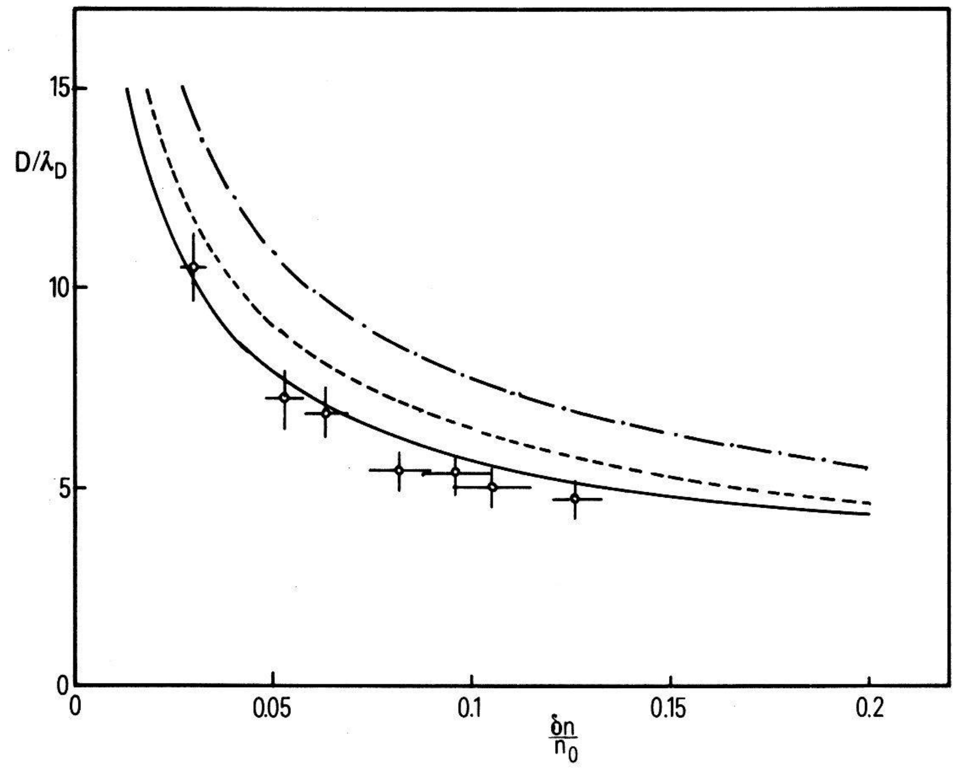

Figure 8
Width of soliton as function of its amplitude for an Helium plasma with $\theta = 9$.

for the soliton's width D/λ_{De}. D/λ_{De} is smaller than the values predicted by equations (4) and (6) and is better described by Sakanaka's equation (8). This conclusion has also been reported by Watanabe [6] for an ion temperature ratio of 16.

b. Number of solitons

Starting from the KdV equation, the theory of Gardner et al. [10] gives the spatial and temporal evolution of a pulsed perturbation. In particular one can predict the number of solitons into which the initial perturbation will break. In a warm ion plasma, the KdV equation (1) is modified to take into account the ion pressure term:

$$\frac{\partial n}{\partial \eta} + \alpha n \frac{\partial n}{\partial \xi} + \beta \frac{\partial^3 n}{\partial \xi^3} = 0 \tag{9}$$

where

$$\alpha = \frac{1 + 6/\theta}{1 + 3/\theta}, \quad \beta = \frac{1}{2(1 + 3/\theta)}, \quad \eta = x, \quad \xi = x - \lambda t, \quad \text{and} \quad \lambda = \sqrt{1 + 3/\theta}.$$

Equation (9) has been derived from the fluid equations with an adiabatic law for the ion. Note that (9) is different from the modified KdV equation obtained by Tagare [11] since in our case η is a spatial coordinate and not a temporal variable: this choice is much more relevant to an experimental situation where a perturbation $n(t)$ is applied at $x = 0$.

Equation (9) can be put into a canonical form

$$\frac{\partial V}{\partial \eta} - 6V \frac{\partial V}{\partial \tilde{\xi}} + \frac{\partial^3 V}{\partial \tilde{\xi}^3} = 0$$

with the following change of coordinate

$$V = -\tfrac{1}{6} n \alpha \beta^{-1/3}, \quad \tilde{\xi} = \xi \beta^{-1/3}.$$

The number of solitons is then given by the number of bound state of the Schroedinger equation

$$\frac{\partial^2 \psi}{\partial \tilde{\xi}^2} + [E - V(\eta = 0, \tilde{\xi})]\psi = 0 \tag{10}$$

where $V(\eta = 0, \tilde{\xi})$ is the initial perturbation

$$V(\eta = 0, \tilde{\xi}) = \begin{cases} \dfrac{V_0}{2}\left[1 + \cos \dfrac{\pi \tilde{\xi}}{\tilde{\Delta}}\right] & -1 < \tilde{\xi} < 1, \\ 0 \quad \text{elsewhere} & \end{cases} \tag{11}$$

The quantities V_0 and $\tilde{\Delta}$ are written in the $(V, \tilde{\xi})$ system of coordinate and are related to the experimental values measured in laboratory units (n normalized to n_0 and time t to ω_{pi}^{-1}) by:

$$V_0 = n(x = 0)\alpha\beta^{-1/3}, \quad \tilde{\Delta} = \Delta\beta^{-1/3}\lambda.$$

Equation (10) can then be reduced to a Mathieu equation

$$\frac{\partial^2\psi}{\partial z^2} + (a - 2q\cos(2z))\psi = 0$$

by introducing the new variables:

$$z = \frac{\pi\tilde{\xi}}{\tilde{\Delta}} \qquad a = \begin{cases} \dfrac{-k\tilde{\Delta}^2 E}{\pi^2}, & z < -\pi, z > \pi \\ \dfrac{k\tilde{\Delta}^2}{\pi^2}\left[\dfrac{V_0}{2} - E\right], & -\pi < z < \pi \end{cases}$$

$$q = \begin{cases} 0 & z < -\pi, z > \pi, \\ \dfrac{\Delta^2 V_0}{\pi^2} & -\pi < z < \pi. \end{cases}$$

It has been shown by Ikezi [5] that for this pulse shape one soliton will merge from the initial perturbation for $0 < q < q_1$, two solitons for $q_1 < q < q_2$ and so on. The q_j are the real positive roots of the equation $2a = a_j(q)$, where a_j are the characteristic values associated with even periodic solutions of Mathieu's equation [12].

In laboratory units, one has one soliton if

$$0 < \Delta n^{1/2} < q_1^{1/2}[3\pi^2(1 + 6/\theta)/(1 + 3/\theta)]^{1/2},$$

two solitons if

$$q_1^{1/2}\left[3\pi^2\frac{(1 + 6/\theta)}{(1 + 3/\theta)}\right]^{1/2} < \Delta n^{1/2} < q_2^{1/2}\left[3\pi^2\frac{(1 + 6/\theta)}{(1 + 3/\theta)}\right]^{1/2},$$

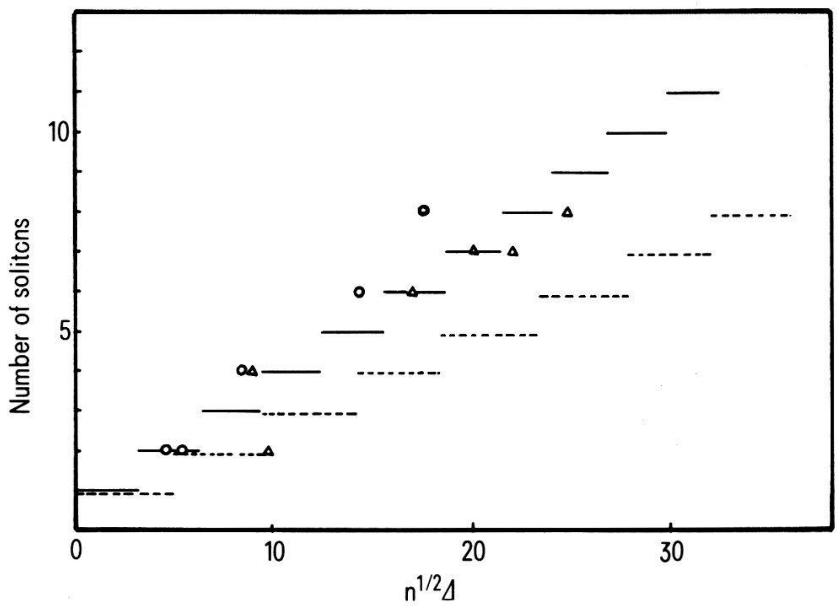

Figure 9
Number of solitons as a function $\Delta n^{1/2}$. n is the initial perturbation normalized to the unperturbed plasma density and Δ is the width of the pulse in unit of ω_{pi}^{-1}. (—— hot ion ($\theta = 9$) theory; ···· cold ion theory.) ○, △ are the experimental values for Ar and He, respectively.

and so on. The same initial perturbation ($\Delta n^{1/2}$ = constant), will break into more solitons in a warm ion than in a cold ion plasma. The theoretical as well as the experimental results are reported in Figure 9. One can note the good agreement between theory and experiments although we have remarked that the soliton's characteristics are better fitted by Sakanaka's theory than by a warm ion KdV equation.

4. Conclusion

Our experimental work has pointed out some effects of finite ion temperature in the propagation of ion wave soliton. (i) Even at high $\theta = T_e/T_i$ ($\theta = 9$), and therefore in the presence of a non-negligible amount of ions reflected by the potential hump, solitons have been observed. (ii) Their width D/λ_{De} and their Mach number M are well described by Sakanaka's theory. A modified KdV (MKdV) equation, taking into account the ion pressure term gives only slight change in D/λ_{De} and M does not correspond to the experimental results. (iii) Applying the theory of Gardner et al. to the MKdV equation, one can explain the number of solitons observed experimentally.

5. Acknowledgment

We would like to acknowledge many discussions with Doctor M. Bitter, and the technical assistance of Mr. Ripper and his team in the maintenance of the device. This work was supported by the Swiss National Science Foundation.

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