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Optical approach to hadron-nucleus scattering

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To Professor Jean ROSSEL for his 60th birthday

Abstract. An optical treatment of hadron-nucleus scattering originally proposed by Watson is reformulated in term of scattering amplitudes instead of Möller operators. This multiple scattering formalism although equivalent to the Kerman, McManus and Thaler approach is found to be less involved and can easily be extended to inelastic scattering. The convergence of the optical potential expansion as well as the role of mesonic absorption in the elastic channel are also discussed in this framework.

Introduction

Because hadron-nucleus scattering is a complicated many-body problem no explicit solution can be found in general except perhaps in the deuteron case. Consequently, well-defined schemes of approximations have to be developed to describe the multiple scattering of the hadron within the nucleus. One widely-used scheme is the optical approach developed by Watson and collaborators [1, 2] and in a slightly different formulation by Kerman, McManus and Thaler [3] (referred later on as KMT). It consists in constructing a one-body optical potential which when inserted in a Schrödinger equation gives the desired scattering amplitude. Of course all the complexity of the many-body problem is now hidden in the expression of this optical potential. However, it can be approximately expanded in term of the elementary hadron-nucleon scattering amplitude, the first order depending essentially on the target nuclear density, the second order on the nuclear pair correlation function, etc. Numerous calculations have been done along these lines to explain experimental data on pion and proton elastic scattering off nuclei. These analyses are generally restricted to the first- and second-order terms in the expansion of the optical potential, and the overall agreement with the data is fairly good. This agreement is however not good enough to yield precise information about the nuclear structure. We do not believe that it can be improved by just taking into account a few higher-order terms in the expansion of the optical potential. Arguments supporting this view are developed at the end of Section 2 and based on two examples. In the first one (Section 5) the higher-order terms of the optical potential happen to be meaningless. In the second one (Section 6) a huge cancellation between them is exhibited. It is thus natural to believe that further information would already result from an improved treatment of the first- and second-order terms. The case of meson-nucleus elastic scattering

should receive special attention because mesons can undergo true absorption within the target, a process which modifies the elastic channel as well. The counterpart of this phenomena is hidden in specific higher-order terms of the optical potential as discussed in Section 4 for pions.

In Section 1 we briefly recall the main expressions used in the optical treatment of hadron-nucleus scattering. The formalism has been generalized in such a way that it contains the approach of either Watson or KMT. Comparison between these two formulations then becomes more immediate. In Section 2 the multiple scattering expansion of the optical operators is reviewed following Watson [2]. We exhume an alternative version proposed by this author which has been later used by KMT but in a much less transparent form. In Section 3 the inelastic scattering leaving the target in a bound state is formulated also in terms of optical potentials. The scattering amplitude to an excited nuclear state is expressed as the matrix element of a transition operator between physical wave functions generated by elastic optical potentials as opposed to the KMT formalism. Approximation of this transition operator by its first-order in t leads to the usual distorted wave impulse approximation (DWIA). The coupled-channels approach is also considered. But if the excited states are not strongly coupled to the ground state the expansion of the multichannel optical potential cannot be truncated after a few terms without introducing important double counting effects. On the contrary our distorted wave formalism is free of such problems.

In this paper non relativistic scattering theory is used throughout. Coulomb effects are neglected although they should be incorporated in any quantitative calculation.

1. The optical treatment of nuclear scattering

An incoming hadron of mass m interacts with a target-nucleus containing $A=N+Z$ nucleons. The projectile-target interaction V is assumed to be the sum of two-body potentials

$$V = \sum_{i=1}^{A} v_i .$$

The total hamiltonian in the center-of-mass system is

$$H = H_A + K + V, \tag{1.1}$$

where H_A denotes the intrinsic nuclear hamiltonian and K the projectile-target relative kinetic energy. The projectile is assumed to be distinguishable from the target nucleons.

The scattering amplitudes for reaction channels in which the target is in a bound state are given by matrix elements of the scattering operator T:

$$f_{\beta\alpha}(\mathbf{p}', \mathbf{p}) = -\frac{1}{(2\pi)^2 m} \langle \mathbf{p}'\beta | T | \alpha\mathbf{p} \rangle, \tag{1.2}$$

where T satisfies the equation

$$T = V + V\frac{1}{D}V \tag{1.3}$$

with

$$D = E + i0 - H.$$

In case the projectile can be absorbed inside the target (as in pion-nuclear scattering for example) the interaction V have a non-vanishing anti-hermitean part.

The free states $|\mathbf{p}\alpha\rangle = |\mathbf{p}\rangle|\alpha)$ and $|\mathbf{p}\rangle$ are eigenstates of $H_A + K$ and K respectively, and $|\alpha)$ is a bound eigenstate of H_A. The scattering operator T satisfies the Lippman–Schwinger equation (L.-S.)

$$T = V + V\frac{1}{d}T \tag{1.4}$$

with $d = E + i0 - H_A - K$.

Let p be a subset of orthonormal target states and

$$P = \sum_{\alpha \in p} |\alpha)(\alpha| \tag{1.5}$$

the corresponding projector. Then

$$T = V_p + V_p\frac{P}{d}T \tag{1.6}$$

replaces equation (1.4) if the new interaction V_p satisfies

$$V_p = V + V\frac{1-P}{d}V_p. \tag{1.7}$$

The optical treatment defined by equations (1.6) and (1.7) can also be rewritten in term of an operator

$$T' = \lambda T \tag{1.8}$$

which is just a multiple of the scattering operator T. This operator satisfies an optical equation similar to equation (1.6), namely

$$T' = V'_q + V'_q\frac{Q}{d}T'. \tag{1.6'}$$

It can easily be shown that the pseudo-optical interaction V'_q is related to the optical interaction V_p by the general formula

$$V'_q = \lambda V_p + V'_q\frac{P-\lambda Q}{d}V_p. \tag{1.9}$$

The arbitrary projectors P and Q appearing in equations (1.6) to (1.9) do not need to be of the type (1.5), they only have to commute with H_A. Thus, we can choose in particular $Q=\mathbb{1}$ and $\lambda=1$, i.a. $T' = T$ and $V'_q = V$. In this case equation (1.9) reduces exactly to equation (1.7).

The correct symmetry under the permutation of target nucleons does not appear in the equations containing T but in equation (1.2) where the pair of target states $|\alpha)$ and $|\beta)$ extract the relevant part from T. It is well known that this restriction can also be achieved from the beginning by means of the projector

$$\mathscr{A} = \sum_{\alpha} |\alpha)(\alpha| \tag{1.10}$$

onto the sub-space of anti-symmetrized target wave functions. Due to the properties $\mathscr{A}^2 = \mathscr{A}^+ = \mathscr{A}$, $\mathscr{A}P = P$ (P defined by equation (1.5)), and $[\mathscr{A}, V] = 0$, the set of equations (1.4), (1.6), (1.7), (1.8), (1.6′) and (1.9) holds for the quantities

$$X^{\mathscr{A}} = \mathscr{A}X = X\mathscr{A}, \qquad X = V, V_p, V'_q, T \text{ or } T'. \tag{1.11}$$

So far the difficulty of solving equation (1.4) has just been replaced by that of computing V_p, since equation (1.6) is tractable for a small set p of states.

For hadron-nuclei scattering equation (1.7) is not believed to yield a useful expansion of V_p. Multiple scattering expansions in term of an effective interaction closely related to the elementary hadron free-nucleon t-matrix seems to be much more preferable. This type of approach mainly due to Watson is revisited in the next section.

2. The multiple scattering approaches

Watson gave a multiple scattering expansion of the T-operator which we quote here for comparison:

$$T = \sum_{i=1}^{A} \hat{t}_i + \sum_{i \neq j} \hat{t}_i \frac{1}{d} \hat{t}_j + \sum_{i \neq j \neq k} \hat{t}_i \frac{1}{d} \hat{t}_j \frac{1}{d} \hat{t}_k + \cdots . \tag{2.1}$$

The operators $\hat{t}_i$ satisfy the L.-S. equation

$$\hat{t}_i = v_i + v_i \frac{1}{d} \hat{t}_i, \tag{2.2}$$

which is the same as the equation satisfied by the hadron free-nucleon scattering operator t_i,

$$t_i = v_i + v_i \frac{1}{d_0} t_i, \tag{2.3}$$

except for the propagator $d_0^{-1} = (E + i0 - K_{\text{hadron-nucleon}})^{-1}$.

Various approximations use in first order $\hat{t}_i \approx t_i$ assuming that the expansion quantities $\hat{t}_i$ are close to the free scattering operators.

An expansion similar to equation (2.1) was also given by Watson for the optical operator V_p:

$$V_p = \sum_{i=1}^{A} t_p(i) + \sum_{i \neq j} t_p(i) \frac{1 - P}{d} t_p(j) + \sum_{i \neq j \neq k} t_p(i) \frac{1 - P}{d} t_p(j) \frac{1 - P}{d} t_p(k) + \cdots . \tag{2.4}$$

Each $(A + 1)$–body operator $t_p(i)$ is implicitly defined as a function of the elementary interaction v_i by the equation

$$t_p(i) = v_i + v_i \frac{1 - P}{d} t_p(i), \tag{2.5}$$

or as a function of the $\hat{t}_i$ introduced above by

$$t_p(i) = \hat{t}_i - \hat{t}_i \frac{P}{d} t_p(i), \tag{2.6}$$

which can be formally solved to yield

$$t_p(i) = \hat{t}_i - \hat{t}_i \frac{P}{d + P\hat{t}_i P} \hat{t}_i. \tag{2.7}$$

Note that the expansion (2.4) of V_p is given in terms of t_p and not $\hat{t}_i$ as sometimes used [4]. In fact t_p is not a good expansion quantity, being neither close to v nor to t, two available pieces of information about the hadron-nucleon interaction. Therefore the expression (2.7) for t_p has to be used in connection with expansion (2.4) to compute V_p to a given order in $\hat{t}$. The evident complexity of this method can be avoided by slightly modifying the optical treatment, the new optical operator being exactly given by equation (2.4) in which $\hat{t}$ replaces the unwanted t_p. This method was mentioned in a short appendix by Watson [2]. In order to express this result in our notations we introduce the following related scattering operator

$$T'^{\mathscr{A}} = \frac{A-1}{A} \mathscr{A} T. \tag{2.8}$$

Using equation (1.6′) and (1.9) with $\lambda=(A-1)/A$ and $P=Q$, it is seen that $T'^{\mathscr{A}}$ satisfies the optical equation

$$T'^{\mathscr{A}} = V_p'^{\mathscr{A}} + V_p'^{\mathscr{A}} \frac{P}{d} T'^{\mathscr{A}}, \tag{2.9}$$

the new optical interaction $V_p'^{\mathscr{A}}$ being related to $V_p^{\mathscr{A}}$ by [5]

$$V_p'^{\mathscr{A}} = \frac{A-1}{A} V_p^{\mathscr{A}} + \frac{1}{A-1}\left(\frac{A-1}{A} V_p^{\mathscr{A}}\right)\frac{P}{d} V_p'^{\mathscr{A}}. \tag{2.10}$$

The main advantage of $V_p'^{\mathscr{A}}$ over $V_p^{\mathscr{A}}$ is that its expansion is exactly

$$V_p'^{\mathscr{A}} = \frac{A-1}{A} \mathscr{A}\left\{\sum_{i=1}^{A} \hat{t}_i + \sum_{i\neq j} \hat{t}_i \frac{1-P}{d} \hat{t}_j + \sum_{i\neq j\neq k} \hat{t}_i \frac{1-P}{d} \hat{t}_j \frac{1-P}{d} \hat{t}_k + \cdots\right\}. \tag{2.11}$$

It has the simple form of equation (2.4) but contains the operators $\hat{t}$ instead of t_p. This expression has not been mentioned by K.M.T. [3] who have expanded $V_p'^{\mathscr{A}}$ in terms of complicated operators T_i. Feshbach and coworkers [6] have computed the first few order of $V_p'^{\mathscr{A}}$ by expanding T_i and found an expression close to equation (2.11) at the same order in $\hat{t}$, the difference resulting probably from a subsequent approximation made by these authors.

The convergence of expansion (2.11) has never been established even in schematic models but many calculations taking the lowest orders into account yield valuable results. Let us have a look at this question in the case of an optical potential

$$U' = (0|V_{p_0}'^{\mathscr{A}}|0), \quad P_0 = |0)(0|,$$

for elastic scattering onto a target in the non-degenerate ground-state $|0)$. By using the properties $\mathscr{A}|0) = |0)$ and

$$\mathscr{A} \sum_{i \neq j} \hat{t}_i \frac{1 - P_0}{d} \hat{t}_j = A(A-1)\mathscr{A}\hat{t}_1 \frac{1 - P}{d} \hat{t}_2 \mathscr{A}, \ldots$$

equation (2.11) becomes

$$\begin{aligned} U' = \sum_{n=1}^{\infty} \mathscr{U}'_{(n)} &= (A-1)(0|\hat{t}|0) + (A-1)^2 (0|\hat{t}_1 \frac{1 - P_0}{d} \hat{t}_2 |0) \\ &+ (A-1)^2 \Big[(A-2)(0|\hat{t}_1 \frac{1 - P_0}{d} \hat{t}_2 \frac{1 - P_0}{d} \hat{t}_3 |0) \\ &+ (0|\hat{t}_1 \frac{1 - P_0}{d} \hat{t}_2 \frac{1 - P_0}{d} \hat{t}_1 |0) \Big] + \mathscr{U}'_{(4)} + \cdots . \end{aligned} \tag{2.12}$$

The nth order term $\mathscr{U}'_{(n)}$ contains two types of contributions:

- the full-scattering (f.s.) term $(12 \ldots n)$
- the re-scattering (res.) terms $(12 \ldots n\text{-}11), \ldots .$

The symbol $(ij \ldots k)$ is a short notation for $(0|\hat{t}_i \frac{1 - P_0}{d} \hat{t}_j \cdots \frac{1 - P_0}{d} \hat{t}_k|0)$. The f.s. terms cannot entail the convergence because they vanish for $n > A$. The res. terms are non-zero for $3 \leq n \leq \infty$ and responsible for a possible divergence of the sum (2.12).

To go further we now assume that the operators $\hat{t}_i$ can be approximated by a two-body operator t_i, for instance the t-matrix for scattering of a pion on a nucleon fixed at position $\mathbf{x}_i$. This is not always a good approximation as pointed out by Lenz [7] who discussed the first-order optical potential $\mathscr{U}'_{(0)}$. However this assumption should not invalidate the two points we want to make about higher orders:

(i) The convergence of the res. part $U'_{\text{res.}} = \sum_{n=3}^{\infty} \mathscr{U}'_{(n)\text{res.}} = (A-1)^2[(121) + \cdots]$ of U' depends not only on the strength of t but also on its range in close relation with the radius of nuclear correlation.

For example it is shown in Section 5 that if t is a zero range operator in configuration space then all terms of $U'_{\text{res.}}$ diverge individually when the correlation radius r_0 vanishes. This means that truncated expansions of $U'_{\text{res.}}$ can have a critical and meaningless dependence on r_0. Consequently this radius becomes an 'ad hoc' cut-off. On the other hand a partial summation over all two-nucleon res. terms yields in the same case a well behaved contribution.

(ii) Important cancellations between res. terms of all orders occur and reduce $U'_{\text{res.}}$ to a large amount.

This property proved in Section 5 on the basis of the eikonal approximation is the counterpart of the so-called Harrington cancellation in the expansion (2.1) of the on-shell scattering amplitude [8]. This is of course a typical feature of the Glauber theory [9].

In summary the calculation of an optical potential should first of all be correctly done up to the second order in $\hat{t}$ as in ref. [7] for pions. Higher order f.s. terms are non-'dangerous' and are believed to play a minor role in most situations. The res.

terms should be completely ignored as proposed in ref. [10]. They are individually 'dangerous' but their global effect is small.

Notice that this statement should be revised whenever a strong absorption channel of the projectile exists as in the case of pion-nucleus scattering. In fact the absorption channel reacts on the elastic one mostly via the two-nucleons res. terms as shown in Section 4.

3. Elastic and inelastic scattering

We adopt the notations

$$T_{\beta\alpha} = (\beta|T|\alpha)$$

and

$$d_\alpha = (\alpha|d|\alpha)$$

for matrix elements of T and d (which is diagonal) between target bound states. The upperscript $\mathscr{A}$ will be omitted since $(\beta|T^{\mathscr{A}}|\alpha) = (\beta|T|\alpha)$ and $(\beta|V_p^{\mathscr{A}}|\alpha) = (\beta|V_p|\alpha)$. The quantity $T_{\alpha\alpha}$ describes the elastic scattering from a target in the bound state $|\alpha)$ and $T_{\beta\alpha}$ the inelastic scattering with the target going from the initial state $|\alpha)$ to the final state $|\beta)$. Equation (1.6′) for $T' = T\cdot(A-1)/A$ takes the explicit form

$$T'_{\beta\alpha} = (\beta|V'_p|\alpha) + \sum_{\gamma\in p} (\beta|V'_p|\gamma)\frac{1}{d_\gamma} T_{\gamma\alpha}, \tag{3.1}$$

with $Q = P = \sum_{\alpha\in p} |\alpha)(\alpha|$.

There are essentially two different cases:

(i) The scattering process involves non-degenerate target states.
(ii) The scattering process involves degenerate target states or especially strongly coupled channels (as for instance in deformed nuclei).

For elastic scattering of the type (i) one clearly has to take $P = |\alpha)(\alpha| = P_\alpha$, the projector onto the target state. The equation to be solved is then an ordinary L.-S. equation

$$T'_{\alpha\alpha} = (\alpha|V'_\alpha|\alpha)\left(1 + \frac{1}{d_\alpha} T'_{\alpha\alpha}\right), \tag{3.2}$$

where V'_α denotes V'_p for the particular choice $p = \alpha$. To compute inelastic scattering of type (i) from the same target state α the natural choice is also $P = P_\alpha$. Equation (3.1) reads in this case

$$T'_{\beta\alpha} = (\beta|V'_\alpha|\alpha)\left(1 + \frac{1}{d_\alpha} T'_{\alpha\alpha}\right). \tag{3.3}$$

Since the factor

$$\left(1 + \frac{1}{d_\alpha} T'_{\alpha\alpha}\right) = \Omega'^{(+)}_\alpha \tag{3.4}$$

is the Möller operator for elastic scattering in channel α,

$$\Omega'^{(+)}_\alpha |\mathbf{k}\rangle = |\chi'^{(+)}_{\alpha,\mathbf{k}}\rangle, \tag{3.5}$$

the inelastic amplitude $T'_{\beta\alpha}$ is given by a straightforward integration. However, the computation of $(\beta|V'_\alpha|\alpha)$ is complicated by the fact that it contains all orders of scattering in the final state $|\beta)$, this state being present as an intermediate state in the expansion (2.11). In order to extract from $(\beta|V'_\alpha|\alpha)$ the final state interaction part we define the *transition operator* from channel α to β by

$$W'_{\beta\alpha} = V'_\alpha - V'_\beta \frac{P_\beta}{d_\beta} V'_\alpha = V'_\beta - V'_\beta \frac{P_\alpha}{d_\alpha} V'_\alpha. \tag{3.6}$$

One obviously has

$$(\beta|W'_{\beta\alpha}|\alpha) = \left(1 - (\beta|V'_\beta|\beta)\frac{1}{d_\beta}\right)(\beta|V'_\alpha|\alpha),$$

where the inverse of the Möller operator for the channel β

$$\Omega'^{(-)}_\beta = \left(1 - (\beta|V'_\beta|\beta)\frac{1}{d_\beta}\right)^{-1} \tag{3.7}$$

can be identified. Thus the factorization reads

$$(\beta|V'_\alpha|\alpha) = \Omega'^{(-)}_\beta(\beta|W'_{\beta\alpha}|\alpha). \tag{3.8}$$

By means of equations (3.8) and (3.4) the operator $T'_{\beta\alpha}$ given by equation (3.3) becomes

$$T'_{\beta\alpha} = \Omega'^{(-)}_\beta(\beta|W'_{\beta\alpha}|\alpha)\Omega'^{(+)}_\alpha. \tag{3.9}$$

It has a completely symmetrical form with respect to initial *and* final states. The scattering amplitude is exactly proportional to the matrix element of the transition operator $(\beta|W'_{\beta\alpha}|\alpha)$ between *physical* optical wave functions $\chi'^{(\pm)}_{\alpha\mathbf{k}}$:

$$f_{\beta\alpha}(\mathbf{k}', \mathbf{k}) = -\frac{1}{(2\pi)^2 m}\frac{A}{A-1}\langle \mathbf{k}'|T'_{\beta\alpha}|\mathbf{k}\rangle = -\frac{1}{(2\pi)^2 m}\frac{A}{A-1}\langle \chi'^{(-)}_{\beta\mathbf{k}'}|(\beta|W_{\beta\alpha}|\alpha)|\chi'^{(+)}_{\alpha\mathbf{k}}\rangle. \tag{3.10}$$

This exact expression is a sound basis for DWIA–type calculations based on various approximations of $W'_{\beta\alpha}$. The initial wave function $\chi'^{(+)}_{\alpha\mathbf{k}}$ can be tested by computing the elastic scattering since $|\alpha)$ is always the target ground state. This test is also useful for $\chi'^{(-)}_{\beta\mathbf{k}}$ (which is directly related to $\chi'^{(+)}_{\beta\mathbf{k}}$ as shown in ref. [11]) when the elastic scattering on the non-degenerate state $|\beta)$ only differ by small corrections from the scattering on the ground state. This control is not possible in the K.M.T. formulation in which the expression equivalent to equation (3.10) is asymmetrical and contains the wave function of an hypothetical final-state scattering in a state $|\beta)$ 'that would be completely disconnected from the ground state'.

Scattering processes of type (ii) require a specific optical treatment in which the particular set of strongly coupled or degenerate channels plays the same role as the previous elastic channel. In this way all interactions connecting these channels are individually present in the set of coupled channel equations. Consider for instance the case of elastic scattering on a target in a ground state $|\alpha_0) = |JM_0)$. For this state the single equation (3.2) is no longer appropriate because the spin-flip part of the optical interaction is uncontrollably hidden within the matrix element

$(JM_0|V'_{JM_0}|JM_0)$. In such a case it is preferable to use the projector $P_J = \sum_M |JM)(JM|$ instead of $P_{JM_0} = |JM_0)(JM_0|$, and solve the coupled system

$$T'_{M'M} = (M'|V'_J|M) + \sum_{M''} (M'|V'_J|M'') \frac{1}{d_J} T'_{M''M}. \tag{3.11}$$

The same assertion holds in presence of particularly strongly coupled channels. For instance if back and forth virtual transitions between the ground state and a particular set of excited states yield large contributions to the elastic scattering the interaction responsible for these transitions ought to appear explicitly in the equations. This can be done by choosing P as the projector onto the strongly coupled set of states. As a simple illustration we write down the case of two states denoted by 0 and 1 (for instance, single charge exchange via analog states in π-nucleus scattering):

$$\begin{aligned} P &= |0)(0| + |1)(1| = P_0 + P_1 \\ V'^{\mathscr{A}}_p &= \mathscr{A} V'_{0+1} = \mathscr{A} \frac{A-1}{A} \left\{ \sum_{i=1}^{A} \hat{t}_i + \sum_{i \neq j} \hat{t}_i \frac{1 - P_0 - P_1}{d} \hat{t}_j + \cdots \right\} \end{aligned} \tag{3.12}$$

$$\begin{aligned} T'_{00} &= (0|V'_{0+1}|0)\left(1 + \frac{1}{d_0} T'_{00}\right) + (0|V'_{0+1}|1) \frac{1}{d_1} T'_{10} \\ T'_{10} &= (1|V'_{0+1}|0)\left(1 + \frac{1}{d_0} T'_{00}\right) + (1|V'_{0+1}|1) \frac{1}{d_1} T'_{11}. \end{aligned} \tag{3.13}$$

The strong coupling potential $(0|V'_{0+1}|1)$ appears explicitly in the equations (3.13) and its first order term can easily be computed if something is known about the overlap of the wave functions of $|1)$ and $|0)$.

The above arguments fail if all channel couplings are comparable in magnitude. It is clearly unjustified to solve coupled channels optical equations involving the first few lower states [12], because the matrix element $(0|V'_{0+1}|0)$ is not equal to the optical interaction $(0|V'_0|0)$ for elastic scattering alone. They only coincide to first order in $\hat{t}$ and to approximate $(0|V'_{0+1}|0)$ by $(0|V'_0|0)$ in equation (3.13) introduces non negligible effects even in the forward direction [13]. In practical calculations it is easy to approximate the optical potential $(0|V'_0|0)$ because it can be tested using elastic scattering only [14]. But it is non-trivial to calculate $(0|V'_{0+1}|0)$ by substracting the necessary quantity. This could be done in principle by using the relation (1.9) which reads

$$V'_{0+1} = V'_0 - V'_0 \frac{P_1}{d_1} V'_{0+1}. \tag{3.14}$$

4. Pion absorption reaction onto the elastic π-nucleus scattering

Pions are a typical example of projectiles which can disappear inside the target during the scattering process. This absorption can be measured directly, but it also manifests its existence in the other opened channels making their S-matrix sub-unitary. The calculation of pion absorption is still at a lower level of refinement than the calculation of elastic optical potential because it needs both the π-nucleon t-matrix and the elementary interaction. No satisfactory calculation method free of double-counting is known. The situation looks better with respect to the 'shadow' of

absorption in elastic amplitudes. Ch. Schmit and J.-P. Maillet [15] have recently studied an idealized π-two nucleons scattering process described by the Lee model. They show that the multiple scattering expansion (2.1) is still valid in this case and that the shadow of absorption is present in the elastic channel whenever the π-nucleon t-matrix has a single nucleon pole term. This pole gives rise to a pole of the π-two nucleons T-matrix which depends on the nuclear correlation. It is found above the π-absorption threshold for small nuclear distances. Hopefully one will soon be able to calculate the additional inelasticity due to absorption.

We know that pions are to a large extent absorbed by systems of two target nucleons whose binding and correlation are large enough. This means that the part of the optical potential U' (equation (2.12)) which is responsible for the shadow effect contains at least the sum of all two-body contributions, namely the 2nd order f.s. term

$$\mathcal{U}'_{(2)} = (A-1)^2(12) \tag{4.1}$$

and the sum of all 2-body res. terms

$$R^{(2)} = (A-1)^2[(121) + (1212) + \cdots]. \tag{4.2}$$

Our argument of Section 2 for dropping the res. terms no longer holds because absorption depends on correlation and on a specific off-shell behaviour of t. The pion may go back and forth between two nucleons until it becomes absorbed and the contributions add coherently.

The summation in equation (4.2) for $R^{(2)}$ can be formally done because by definition

$$R^{(2)} = (A-1)^2\left[(0|V'^{(2)}|0) - (0|\hat{t}_1 + \hat{t}_1 \frac{1-P_0}{d} \hat{t}_2|0)\right]. \tag{4.3}$$

Here $(0|V'^{(2)}|0)$ denotes the optical potential of the sub T-matrix for scattering of a pion on a pair of target nucleons (for $A = 2$, $(A-1)/A = \frac{1}{2}$):

$$(0|T'^{(2)}|0) = \tfrac{1}{2}(0|T^{(2)}|0) = \tfrac{1}{2}(0|\hat{t}_1 + \hat{t}_2 + \hat{t}_1 \frac{1}{d} \hat{t}_2 + \hat{t}_2 \frac{1}{d} \hat{t}_1 + \cdots|0). \tag{4.4}$$

Note that d^{-1} is the complete propagator. The L.-S. equation giving $(0|T'^{(2)}|0)$

$$(0|T'^{(2)}|0) = (0|V'^{(2)}|0) + (0|V'^{(2)}|0) \frac{1}{d_0} (0|T'^{(2)}|0) \tag{4.5}$$

can be used backwards to evaluate $(0|V'^{(2)}|0)$ once $(0|T'^{(2)}|0)$ has been estimated on the basis of a simplified model. This program is not so unrealistic since one has first to solve a pion-'deuteron' problem. Even an oversimplified result could help to write down a reasonable phenomenological absorption correction, as a function of the range and strength of the π-nucleon t-matrix and of the nuclear correlation radius. One must however be careful to perform exactly the substraction of equation (4.3) because the two last terms have to cancel the first- and second-order terms of $(0|V'^{(2)}|0)$. An incomplete cancellation would produce an unphysical correction proportional to $(A-1)^2$, to the optical potential U'. In practice, if $V'^{(2)}$ is given as a

function of $\hat{t}$ one just have to multiply $\hat{t}$ with a factor λ and compute $R^{(2)}$ using the formula

$$R'^{(2)} = (A-1)^2\left[(0|V'^{(2)}|0) - \left(\frac{d}{d\lambda} + \frac{1}{2}\frac{d^2}{d\lambda^2}\right)(0|V'^{(2)}|0)|_{\lambda=0}\right]. \tag{4.6}$$

Whereas individual res. terms can happen to diverge, the partial sum $(0|T'^{(2)}|0)$ as well as $(0|V'^{(2)}|0)$ are well behaved functions of the nuclear correlation radius as exemplified at the end of Section 5.

5. Pion-nucleus optical potential in the limit of zero range effective interaction

At energies around the (3.3) resonance the π-nucleon scattering amplitude is dominated by the p-wave contribution. The on-shell behaviour of the isoscalar t-matrix is fairly well reproduced by the form

$$\langle \mathbf{p}'|t|\mathbf{p}\rangle = -\frac{2\pi}{m} C_0(E)\mathbf{p}'\cdot\mathbf{p}. \tag{5.1}$$

We shall approximate the effective interactions $\hat{t}_i$ by this expression assuming that the i-th nucleon is fixed at position $\mathbf{x}_i$:

$$\langle \mathbf{p}'|t(\mathbf{x}_i)|\mathbf{p}\rangle = -\frac{2\pi}{m} C_0\mathbf{p}'\cdot\mathbf{p}\exp\left[i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{x}_i\right]. \tag{5.1'}$$

This is a limiting case of zero-range interaction in configuration space. Within this assumption the first-order optical potential (2.12) is of the Kisslinger type [16]:

$$\mathscr{U}'_{(1)}(\mathbf{r},\mathbf{r}') = \frac{2\pi}{m}(A-1)C_0(E)\nabla\cdot\rho(\mathbf{r})\nabla\delta(\mathbf{r}-\mathbf{r}'), \tag{5.2}$$

where ρ is the target nuclear density normalized to unity. To calculate the higher-order terms we approximate the n-point nuclear density by

$$\rho^{(n)}(\mathbf{x}_1\ldots\mathbf{x}_A) = \prod_{i=1}^{A}\rho(\mathbf{x}_i)\prod_{j<k}\Theta(|\mathbf{x}_j-\mathbf{x}_k|-r_0), \tag{5.3}$$

where r_0 is the correlation radius and Θ the step-function. Then in the limit of vanishing r_0 the n-th order f.s. term of U' is given by

$$\lambda_n(A-1)\frac{2\pi}{m}C_0\nabla\cdot\rho(\mathbf{r})\left[(A-1)\cdots(A-n+1)\left(\frac{4\pi}{3}C_0\rho(\mathbf{r})\right)^{n-1}\right]\nabla\delta(\mathbf{r}-\mathbf{r}'), \tag{5.4}$$

the numerical factors λ_n taking the value 1 for $n = 2$, 31/16 for $n = 3$ and being close to 1 for all $n \leq A$. Thus, the f.s. part of U' reads

$$U'_{\text{f.s.}} = \frac{2\pi}{m}(A-1)C_0\nabla\cdot\rho(\mathbf{r})\left[1+\sum_{n=2}^{A}\lambda_n\frac{(A-1)!}{(A-n)!}\left(\frac{4\pi}{3}C_0\rho(\mathbf{r})\right)^{n-1}\right] \times\nabla\delta(\mathbf{r}-\mathbf{r}') \tag{5.5}$$

which looks like the Kisslinger potential [16] modified by the Lorenz–Lorentz correction [17] $(1-(A-1)4\pi C_0\rho/3)^{-1}$ expanded in power of $(A-1)4\pi C_0\rho/3$.

On the other hand the res. terms all diverge in the limit $r_0 = 0$. The third-order res. term can be written as

$$\mathcal{U}'_{(3)\text{res.}} = (A-1)^2 \int d^3x_1\, d^3x_2 \rho^{(2)}(\mathbf{x}_1, \mathbf{x}_2) t(\mathbf{x}_1) \frac{1}{d_0} t(\mathbf{x}_2) \frac{1}{d_0} t(\mathbf{x}_1) - \frac{1}{A-1}\left[\mathcal{U}'_{(1)} \frac{1}{d_0} \mathcal{U}'_{(2)} + \mathcal{U}'_{(2)} \frac{1}{d_0} \mathcal{U}'_{(1)} + \mathcal{U}'_{(1)} \frac{1}{d_0} \mathcal{U}'_{(1)} \frac{1}{d_0} \mathcal{U}'_{(1)}\right], \tag{5.6}$$

where $t(\mathbf{x}_i)$ is defined by equation (5.1′).

Note that the last bracket which contains only the first- and second-orders of U' is finite for any r_0. But the integral giving the first term diverge when $r_0 \to 0$ because t appears twice as a function of the same variable $\mathbf{x}_1$. This property is common to all the res. terms.

Of course all the $\mathcal{U}'_{(n)\text{res.}}$ are finite for $r_0 \neq 0$ but their dependence on the radius r_0 is so critical that they lose any individual significance. On the other hand their sum has a non-singular behaviour. For example, the sum of the two-nucleon res. terms $R^{(2)}$ defined in equation (4.2) can be written as

$$R^{(2)} = (A-1)^2 (0|V'^{(2)}|0) - (A-1)\mathcal{U}'_{(1)} - \mathcal{U}'_{(2)}, \tag{5.7}$$

where the last two quantities are explicitly given by equations (5.2) and (5.4) and the first one is $(A-1)^2$ times the optical potential of the sub T-matrix $(0|T'^{(2)}|0)$ as defined by equation (4.4). Because of the projector $1 - P_0$ it is hard to calculate the optical potential $(0|V'^{(2)}|0)$ whereas it is easy to do it for $(0|T'^{(2)}|0)$. Summing up the expansion (4.4) leads to

$$\begin{aligned}\langle \mathbf{p}'0|T'^{(2)}(E)|0\mathbf{p}\rangle = {} & -\frac{2\pi}{m} C_0(E) \int\int d^3a\, d^3x\, \rho^{(2)}(\mathbf{x} + \tfrac{1}{2}\mathbf{a}, \mathbf{x} - \tfrac{1}{2}\mathbf{a})\, e^{i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{x}} \\ & \times \Big\{ (\chi_1(a) \cos[\tfrac{1}{2}(\mathbf{p}'-\mathbf{p})\cdot\mathbf{a}] + \chi_3(a) \cos[\tfrac{1}{2}(\mathbf{p}+\mathbf{p})\cdot\mathbf{a}])\mathbf{p}'\cdot\mathbf{p} \\ & + (\chi_2(a) \cos[\tfrac{1}{2}(\mathbf{p}'-\mathbf{p})\cdot\mathbf{a}] + \chi_4(a) \cos[\tfrac{1}{2}(\mathbf{p}'+\mathbf{p})\cdot\mathbf{a}]) \\ & \times \left(3\frac{\mathbf{p}'\cdot\mathbf{a}\mathbf{p}\cdot\mathbf{a}}{a^2} - \mathbf{p}'\cdot\mathbf{p}\right)\Big\},\end{aligned} \tag{5.8}$$

where the functions χ_i depend on the internucleons distance a in the following way:

$$\begin{aligned}\chi_1 &= \frac{1}{3}\left(\frac{2}{1 - C_0^2U^2} + \frac{1}{1 - C_0^2V^2}\right) \qquad \chi_3 = \frac{1}{3}\left(\frac{2C_0U}{1 - C_0^2U^2} - \frac{C_0V}{1 - C_0^2V^2}\right) \\ \chi_2 &= \frac{1}{3}\left(-\frac{1}{1 - C_0^2U^2} + \frac{1}{1 - C_0^2V^2}\right) \qquad \chi_4 = -\frac{1}{3}\left(\frac{C_0U}{1 - C_0^2U^2} + \frac{C_0V}{1 - C_0^2V^2}\right),\end{aligned} \tag{5.9}$$

with

$$U = (1 - ika)\frac{e^{ika}}{a^3}, \qquad V = 2(1 - ika - \tfrac{1}{2}k^2a^2)\frac{e^{ika}}{a^3} \quad \text{and} \quad k = (2mE)^{1/2}. \tag{5.10}$$

In particular all the χ_i vanish when a tends to zero or infinity except χ_1 which is finite for $a = 0$. Consequently $T'^{(2)}$ is a smooth function of this radius a. This is not the case however for each term of the expansion (4.4) which can be recovered from equation (5.8) by expanding all χ_i in power of C_0. Because of the singular behaviour of U and V when a tends to zero the term proportional to $(C_0)^n$ diverges for $n \geq 3$ if $\int d^3x \rho^{(2)}(\mathbf{x} + \frac{1}{2}\mathbf{a}, \mathbf{x} - \frac{1}{2}\mathbf{a}) \exp(i\mathbf{q}\cdot\mathbf{x})$ vanishes less rapidly than a^{n-2}, which holds for a two-point nuclear density of the form (5.3) in the limit $r_0 = 0$. Since $(0|V'^{(2)}|0)$ is related to $(0|T'^{(2)}|0)$ by equation (4.5) it has the same special behaviour in function of r_0 and so does $R^{(2)}$.

This calculation can also be performed with other forms of the π-nucleon t-matrix containing some additional cut-off factors. Then the dependence on r_0 seems less critical but it is hard to give any signification to it if a truncated expansion (4.2) is used.

6. The optical potentials in eikonal approximation

Unlike the singular interaction used in the last section the projectile-nucleon t-matrix is now assumed to have a smooth behaviour. It results from a local interaction potential

$$\langle \mathbf{r}'|v|\mathbf{r}\rangle = v(\mathbf{r})\,\delta(\mathbf{r} - \mathbf{r}'), \tag{6.1}$$

and therefore cannot contain any remnants of true projectile absorption, if ever. The eikonal approximation can be justified in this case and will be used to get further information about the structure of the optical potential in the so-called fixed-scatterers approximation. Under these assumptions it is easier to calculate T than U. The optical potential U is then indirectly derived from T averaged over the ground state density. This procedure is justified since the propagator d^{-1} can be consistently replaced throughout the first two sections by the fixed-scatterers, eikonal propagator

$$\langle \mathbf{p}'|d_e^{-1}(\mathbf{k})|\mathbf{p}\rangle = m\frac{\delta(\mathbf{p}' - \mathbf{p})}{\mathbf{k}\cdot(\mathbf{k} - \mathbf{p}) + i0}, \tag{6.2}$$

where the direction of $\mathbf{k}$ is kept fixed and $\mathbf{k}^2 = 2mE$.

For instance the projectile-nucleon eikonal t-matrix t_e satisfies the equation

$$t_e = v + v\frac{1}{d_e}t_e \tag{6.3}$$

where the potential v is given by equation (6.1) if the nucleon is fixed at the origin. The solution of equation (6.3) in configuration space reads

$$\langle \mathbf{r}'|t_e(\mathbf{k})|\mathbf{r}\rangle = \frac{ik}{m}\,\delta(\mathbf{b}' - \mathbf{b})\,e^{ik(z'-z)}\frac{\partial}{\partial z}\,\theta(z' - z)\frac{\partial}{\partial z'}\,e^{i[\Delta(\mathbf{b},z') - \Delta(\mathbf{b},z)]} \tag{6.4}$$

where

$$\Delta(\mathbf{b}, z) = -\frac{m}{k}\int_{-\infty}^{z} du\, v(\mathbf{b}, u), \tag{6.5}$$

and becomes in momentum space

$$\langle \mathbf{p}'|t_e(\mathbf{k})|\mathbf{p}\rangle = -\frac{ik}{m}\int d^2b\, e^{-i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{b}} \times \int\int_{-\infty}^{\infty} dz'\, dz\, e^{i[(k-p_z')z'-(k-p_z)z]}\frac{\partial}{\partial z}\theta(z'-z)\frac{\partial}{\partial z}e^{i[\Delta(\mathbf{b},z')-\Delta(\mathbf{b},z)]}. \tag{6.6}$$

As usual **b** denotes the impact vector and the z-axis is chosen along the direction of **k**; in the same way p_z is the projection of **p** along **k**.

Equation (6.6) defines the 'off-shell' behaviour of the projectile-nucleon t-matrix as prescribed by the eikonal approximation. However the usual energy shell $p^2 = p'^2 = 2mE$ is replaced by the **k**-plane $p_z = p_z' = |\mathbf{k}| = (2mE)^{1/2}$ [18]. The on-**k**-plane t-matrix t_e has the well-known form

$$\langle \mathbf{p}'|t_e(\mathbf{k})|\mathbf{p}\rangle = -\frac{ik}{m}\int d^2b\, e^{-i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{b}}(1-e^{i\chi(\mathbf{b})}),$$

with

$$\chi(\mathbf{b}) = \Delta(\mathbf{b},\infty) = -\frac{m}{k}\int_{-\infty}^{\infty} dz\, v(\mathbf{b},z). \tag{6.7}$$

This result can easily be generalized to the case of A scatterers fixed in $\mathbf{x}_i = (\mathbf{s}_i, \zeta_i)$. The two-body interaction $v(\mathbf{r})$ is replaced by the sum

$$V(\mathbf{r}) = \sum_{i=1}^{A} v(\mathbf{r}-\mathbf{x}_i) \tag{6.8}$$

in the definition (6.5) of the phase Δ. The eikonal A-body scattering operator T_e and its phase Δ_A are given by

$$\Delta_A(\mathbf{b},z) = \sum_{i=1}^{A}\Delta(\mathbf{b}-\mathbf{s}_i, z-\zeta_i), \tag{6.9}$$

$$\langle \mathbf{p}'|T_e(\mathbf{k};\mathbf{x}_1\ldots\mathbf{x}_A)|\mathbf{p}\rangle = -\frac{ik}{m}\int d^2b\, e^{-i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{b}}\int\int_{-\infty}^{\infty} dz'\, dz \exp\{i[(k-p_z')z'-(k-p_z)z]\} \times\frac{\partial}{\partial z}\theta(z'-z)\frac{\partial}{\partial z'}\prod_{j=1}^{A}\exp\{i[\Delta(\mathbf{b}-\mathbf{s}_j, z'-\zeta_j)-\Delta(\mathbf{b}-\mathbf{s}_j, z-\zeta_j)]\}. \tag{6.10}$$

On **k**-plane the operator T_e is given by the Glauber formula [9]:

$$T_e = -\frac{ik}{m}\int d^2b\, e^{-i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{b}}\prod_{j=1}^{A}(1-e^{i\chi(\mathbf{b}-\mathbf{s}_j)}). \tag{6.11}$$

The hadron-nucleus elastic scattering matrix is obtained from equation (6.10) by averaging over the A nucleons coordinates:

$$\langle T_e\rangle = \langle \mathbf{p}'0|T_e|0\mathbf{p}\rangle = \int d^3x_1 \ldots d^3x_A \rho^{(A)}(\mathbf{x}_1 \ldots \mathbf{x}_A)\langle \mathbf{p}'|T_e(\mathbf{u}; \mathbf{x}_1 \ldots \mathbf{x}_A)|\mathbf{p}\rangle$$
$$= -\frac{ik}{m}\int d^2b\, e^{-i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{b}}$$
$$\times \int\int_{-\infty}^{\infty} dz'\, dz \exp\{i[(k - p_z')z' - (k - p_z)z]\}$$
$$\times \frac{\partial}{dz}\theta(z' - z)\frac{\partial}{dz'} e^{i\Phi(\mathbf{b}, z', z)} \tag{6.12}$$

with

$$i\Phi(\mathbf{b}, z', z) = \ln \int d^3x_1 \ldots d^3x_A \rho^{(A)}(\mathbf{x}_1 \ldots \mathbf{x}_A) \prod_{j=1}^{A} \exp\{i[\Delta(\mathbf{b} - \mathbf{s}_j, z' - \zeta_j) - \Delta(\mathbf{b} - \mathbf{s}_j, z - \zeta_j)]\}.$$

The phase $\Phi(\mathbf{b}, z', z)$ has no longer the additivity property with respect to z and z' as the phase of the non-averaged T_e. This means that the average $\langle T_e\rangle$ of T_e defined in equation (6.12) is not the eikonal approximation of a scattering matrix produced by a local potential. This is also true for the average

$$\langle t_e\rangle = \int d^3x\, \rho(\mathbf{x})\langle \mathbf{p}'|t_e(\mathbf{k}, \mathbf{x})|\mathbf{p}\rangle$$

of t_e which is given by equation (6.12) with $i\Phi$ replaced by

$$i\varphi(\mathbf{b}, z', z) = \ln \int d^3x_1\, \rho(\mathbf{x}_1) \exp\{i[\Delta(\mathbf{b} - \mathbf{s}_1, z' - \zeta_1) - \Delta(\mathbf{b} - \mathbf{s}_1, z - \zeta_1)]\}. \tag{6.13}$$

But all we need is love and the on-**k**-plane part of $\langle T_e\rangle$. Indeed, the half-**k**-plane part

$$\langle T_e\rangle|_{p_z = k} = \frac{ik}{m}\int d^2b\, e^{-i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{b}} \int_{-\infty}^{\infty} dz\, e^{i(k - p_z')z}\frac{\partial}{\partial z} e^{i\Phi(\mathbf{b}, z, -\infty)} \tag{6.14}$$

can be obtained using a local effective potential

$$U_e(\mathbf{r}) = -\frac{k}{m}\frac{\partial\Phi}{\partial z}(\mathbf{b}, z, -\infty), \tag{6.15}$$

and the same holds for $\langle t_e\rangle$ using the potential

$$u_e(\mathbf{r}) = -\frac{k}{m}\frac{\partial\varphi}{\partial z}(\mathbf{b}, z, -\infty). \tag{6.16}$$

In summary the t-matrices T and τ produced in eikonal approximation by the potentials U_e and u_e coincide on- and half-k-plane with $\langle T_e\rangle$, resp. $\langle t_e\rangle$. In addition the potential U_e is equivalent to the eikonal approximation of U which is obtained through substitution of d by d_e in equations (2.4) and (2.5), since they both lead to the same scattering amplitude.

In order to study the res. terms it is easier to consider a case where the f.s. terms

vanish. This can be achieved apart from first order by neglecting all the nuclear correlations inside the target. If we set

$$\rho^{(A)} \simeq \prod_{j=1}^{A} \rho(\mathbf{x}_j)$$

in equation (6.12) we obtain

$$\Phi = A\varphi$$

which implies using equations (6.14) and (6.15)

$$U_e(\text{no cor.}) = A\, u_e. \tag{6.17}$$

This result is equivalent for the primed optical potential to

$$U'_e\,(\text{no cor.}) = (A - 1)\tau \tag{6.18}$$

and half **k**-plane to

$$U'_e\,(\text{no cor.}) = (A - 1)\langle t_e \rangle, \tag{6.19}$$

as it is easy to prove using equations (2.10) and (6.17).

This means that the half-**k**-plane part of the (non-local!) optical potential U'_e of $\langle T'_e \rangle$ reduces to its first order in absence of target correlation. This result is unfortunately not complete since it tells us nothing about the full-off-**k**-plane U'_e (no cor.). Nevertheless it shows that a huge cancellation between res. terms occurs in eikonal approximation. This conclusion is in complete agreement with the Glauber formula which tells that no target nucleon can be hit twice by the projectile.

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