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Light cone sum rules in nonabelian gauge field theory[1])

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Abstract. We examine, in the context of nonabelian gauge field theory, the derivation of the light cone sum rules which were obtained earlier on the assumption of dominance of canonical singularity in the current commutator on the light cone. The retarded scaling functions appearing in the sum rules are numbers known in terms of the charges of the quarks and the number of quarks and gluons in the theory. Possible applications of the sum rules are suggested.

1. Introduction

It is well-known that asymptotic freedom [1] enjoyed by colour gauge theory of strong interactions [2] modifies the predictions for short distance and light cone phenomena based on canonical (free field) singularities or parton model at most by powers of logarithm. To leading order, the sum rules related to conserved or partially conserved quantities are not modified at all. Yet, for other quantities like structure functions, the logarithmic deviations can lead to a picture qualitatively different from that given by the canonical formalism.

In this work we examine the derivation of the light cone sum rules [3, 4] in the framework of gauge field theory of strong interactions. Besides causality, the only other input required for the sum rules is the asymptotic behaviour of the retarded amplitude in the Bjorken limit (called retarded scaling function in the following). They were obtained earlier on the assumption that the current commutators on the light cone are dominated by canonical singularities. The predictions of gauge field theory concern typically the behaviour of moments of structure functions (absorptive parts of amplitudes), which indicate a rather singular behaviour of the structure functions themselves. The behaviour of the retarded scaling function in this theory is therefore not immediately clear.

The method of operator product expansion (OPE) together with the renormalization group equation for the Wilson coefficients give the retarded amplitude formally as an infinite series diverging in the physical region and thus appears *a priori* not to be useful for our purpose. On the basis of the shrinking behaviour of the absorptive part in the scaling variable, we show that the retarded scaling function can be simply extracted from the fixed mass dispersion relation for the retarded amplitude. The retarded scaling function is not affected by the question of subtractions in the dispersion representation. Our analysis provides, in effect, a

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justification for representing the retarded scaling function by the leading term in the formal series for the retarded amplitudes.

In Section 2 we illustrate our result with the help of a simple model example of 'scattering' of quark densities. It is then easy to write down sum rules for the practical case of Compton amplitudes involving electromagnetic currents. This is considered in Section 3. Different aspects of the sum rules and its possible applications are discussed in Section 4.

2. Retarded scaling function

To illustrate the method of obtaining the retarded scaling function in a simple context, we consider the forward retarded amplitude for 'scattering' of quark density of momentum q off nucleon target of momentum p,

$$T(q^2,\nu)=i\int dz e^{iq\cdot z}\langle p|\,\theta(z_0)[J(z/2),J(-z/2)]\,|p\rangle, \tag{2.1}$$

the matrix element being averaged over the target spin. Here $\nu=p\cdot q$. In terms of the scaling variable $x=-q^2/2\nu$, the Bjorken limit is given by $-q^2$, $\nu\to\infty$, while x is held fixed. Consider further the flavour singlet amplitude,[2]) so that the OPE of the retarded commutator for light like distances is

$$i\theta(z_0)[J(z/2),J(-z/2)]=\frac{1}{(z^2)^2}\sum_{n=2}^{\infty}\sum_{i=1,2} i^n C_{n,i}(z^2)O_{n,i}^{\mu_1\cdots\mu_n}(0)z_{\mu_1}\cdots z_{\mu_n} \tag{2.2}$$
$$+\text{terms with operators of higher twist}$$

where $O_{n,1}$ and $O_{n,2}$ denote the usual two strings of twist 2 gauge invariant local operators bilinear in quark and gluon fields respectively [1]. With

$$\langle p|\,O_{n,i}^{\mu_1\cdots\mu_n}\,|p\rangle=O_{n,i}(p^{\mu_1}\cdots p^{\mu_n}-\text{traces}), \tag{2.3}$$

we have

$$T(q^2,\nu)\to\sum_{n=2}^{\infty}\frac{1}{x^n}\sum_{i=1,2}\tilde{C}_{n,i}O_{n,i}+O\left(\frac{1}{q^2}\right), \tag{2.4}$$

where

$$\tilde{C}_{n,i}(q^2)=(q^2)^n\left(\frac{\partial}{\partial q^2}\right)^n\int dz e^{iq\cdot z}\frac{C_{n,i}(z^2)}{(z^2)^2} \tag{2.5}$$

As $T(\nu,q^2)$ is symmetric under crossing ($\nu\to-\nu$), the summation over n in equation (2.4) runs only over even integers.

The behaviour of the Wilson coefficients are determined by the eigenvalues of the matrix of anomalous dimensions of $O_{n,i}$ [1]. The eigenvalues are all greater than zero, except for $n=2$, for which the smaller of the two eigenvalues is zero,

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[2]) The reason for considering the flavour singlet rather than the simpler case of flavour octet amplitude (where only quark bilinear operators contribute) is that we wish to have an example where the lowest moment of the structure function contributing to $T(\nu,q^2)$ is nonvanishing in the Bjorken limit.

the corresponding eigenvector being the energy momentum tensor. In this case, one has

$$\sum_{i=1,2} \tilde{C}_{2,i} O_{2,i} \to 2\langle Q^2\rangle r + O([\log(-q^2)^{-\alpha}), \qquad r = \frac{n_f}{2n_g + n_f}, \qquad \alpha > 0, \tag{2.6}$$

$\langle Q^2\rangle$ being the average of the squared quark charge and n_f and n_g denoting the number of quarks (counting flavour and colour) and gluons respectively in the theory. For $n>2$, we have

$$\sum_{i=1,2} \tilde{C}_{n,i} O_{n,i} \to A_n[\log(-q^2)]^{-a_n} + O([\log(-q^2)]^{-b_n}), \qquad b_n > a_n, \tag{2.7}$$

where A_n depends on n only[3]) and a_n grows as $\log n$ for large n.

The representation (2.4) does not converge inside the physical region of x, $0 \leq x \leq 1$. Ignoring, for the moment, the problem of convergence, it is tempting to conclude that the Bjorken limit of $T(\nu, q^2)$ is given by the first term in the series:

$$T(x, q^2) \to \frac{1}{x^2} 2\langle Q^2\rangle r + O([\log(-q^2)]^{-\beta}), \tag{2.8}$$

as $\nu, -q^2 \to \infty$, with x kept fixed. Here $\beta > 0$ and is given by either the leading correction to (2.6) or the second term in (2.4), whichever, is smaller. In the following we justify this result with the help of fixed q^2 dispersion relation for $T(\nu, q^2)$.

We now follow the usual procedure of extracting the behaviour of moments of the structure function $W(x, q^2)$, defined as

$$W(x, q^2) = \frac{1}{\pi} \text{Abs}\, T(x, q^2), \tag{2.9}$$

by comparing (2.4) with a similar expansion obtained from fixed q^2 dispersion relation for T. Regge asymptotics would suggest $T(\nu, q^2) \sim \nu^{\alpha(0)}$, for ν large and q^2 fixed, where $\alpha(0)$ is the $t=0$ intercept of the Pomeranchuk trajectory, $\alpha(t)$. Let us, however, allow an arbitrary number m of subtractions in the variable ν^2, so that we get

$$T(\nu, q^2) = \nu^{2m} \int_{(q^2/2)^2}^{\infty} \frac{d\nu'^2 W(\nu'^2, q^2)}{\nu'^{2m}(\nu'^2 - \nu^2)} + \sum_{j=0}^{m-1} a_j(q^2)\nu^{2j}, \tag{2.10}$$

where the subtraction constants are, at this point, unknown functions of q^2. Changing the integration variable ν' to $x' = -q^2/2\nu'$,

$$T(x, q^2) = \left(\frac{1}{x^2}\right)^{m-1} 2\int_0^1 dx' \frac{x'^{2m-1} W(x', q^2)}{x^2 - x'^2} + \sum_{j=0}^{m-1} b_j(q^2)\left(\frac{1}{x^2}\right)^j, \tag{2.11}$$

where $b_j(q^2)$ are again unknown functions related to $a_j(q^2)$. For large q^2 we must, however, exclude any power behaviour of $b_i(q^2)$ in q^2, since $W(q^2, x)$ and $T(q^2, x)$ cannot have such a behaviour in the Bjorken region. Keeping $|x| > 1$, the

[3]) The n-dependence of A_n is not predicted by the theory. However, the fact that equation (2.4) has to converge outside $|x| < 1$ does constrain it for large n.

denominator of (2.11) can be expanded to get

$$T(x, q^2) = \sum_{j=0}^{m-1} b_j(q^2)\left(\frac{1}{x^2}\right)^j + \sum_{j=m}^{\infty}\left(\frac{1}{x^2}\right)^j 2\int_0^1 dx' x'^{2j-1} W(q^2, x'). \tag{2.12}$$

Comparing (2.4) and (2.12) we get for large q^2,

$$b_0(q^2) = O\left(\frac{1}{q^2}\right), \qquad b_j(q^2) = \sum_{i=1,2} \tilde{C}_{2j,i} O_{2j,i} + O\left(\frac{1}{q^2}\right), \qquad j = 1, \ldots, m-1, \tag{2.13a}$$

$$2\int_o^1 dx x^{2j-1} W(x, q^2) = \sum_{i=1,2} \tilde{C}_{2j,i} O_{2j,i} + O\left(\frac{1}{q^2}\right), \qquad j \geqslant m \tag{2.13b}$$

Equation (2.13b) can be continued analytically to hold for all j. We then get a rough picture of the behaviour of the structure function itself [1]: At any point $x(0<x\leqslant 1)$ the structure function vanishes faster than any power of $\log(-q^2)$, provided $\log(-q^2)$ is large enough. We can then terminate the upper limit of the integral in (2.11) by ϵ, $0<\epsilon<x$, neglecting terms vanishing faster than any power of $\log(-q^2)$. The denominator can then be expanded again to obtain (2.12) with the upper limit of the integral being replaced by ϵ, the variable x now being in the physical region. Then the series for $x^2 T(x, q^2)$ converges uniformly in q^2 at fixed x, since

$$\left(\frac{1}{x^2}\right)^{j-1}\int_0^{\epsilon} dx' x'^{2j-1} W(x', q^2) < \left(\frac{\epsilon}{x}\right)^{2(j-1)}\int_0^1 dx' x' W(x', q^2) \simeq \left(\frac{\epsilon}{x}\right)^{2(j-1)} \langle Q^2\rangle r, \tag{2.14}$$

thereby proving the validity of the limit given by (2.8). Note that the number of subtractions does not play any role in the argument: the limit is reproduced either by one of the subtraction terms or by the leading integral in (2.12), depending on the number of subtractions.

The limit (2.8) for $T(x, q^2)$ is obtained in the physical region of deep inelastic scattering. But, since, unlike the absorptive part W, the retarded amplitude T is an analytic function of ν with x held fixed, the limit is, in fact, valid in all directions of the complex plane of the variable ν. (This argument of course excludes an exponential growth for T.)

We are now in a position to write down the light cone dispersion relation for T, which follow from the observation [2] that, since causality requires the commutator in (2.1) to be non-zero only for $z^2 = z_0^2 - z^2 \geqslant 0$, $\theta(z_0)$ may be replaced by $\theta(nz)$ where n_μ is a lightlike vector, $n^2 = 0$. The path of integration in the dispersion integral is a straight line given by $q^2 = -2x\nu + y$, where x is the scaling variable $(x\neq 0)$ and y is a constant. Knowledge of the Bjorken limit settles the question of subtractions. With one subtraction at infinity, the light cone dispersion relation for T reads

$$T(\nu, q^2 = 2x\nu + y) = \frac{2\langle Q^2\rangle}{x^2} r + \int_{-\infty}^{+\infty} \frac{d\nu'}{\nu' - \nu} \text{Abs}\, T(\nu', q^{2'} = -2x\nu' + y). \tag{2.15}$$

In this model example, we do not have any superconvergence type sum rule. Below we derive such sum rules for the interesting practical case of inclusive electroproduction.

3. Sum rule

The standard decomposition of the forward, spin-averaged nucleon Compton scattering into invariant amplitudes is

$$i\int dze^{iq\cdot z}\langle p|\,\theta(z_0)[J_\mu^{em}(z/2), J_\nu^{em}(-z/2)]\,|p\rangle$$
$$=\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)T_1(\nu, q^2)+\frac{1}{m^2}\left(p_\mu-q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu-q_\nu\frac{p\cdot q}{q^2}\right)T_2(\nu, q^2), \quad (3.1)$$

where a, b are flavour symmetry ($SU(N)$) indices and m is the mass of the nucleon. The dispersion relations[4] for $T_{1,2}$ give

$$T_1(x, q^2)=\sum_{j=2,4,\dots}\left(\frac{1}{x}\right)^j 2\int_0^1 dx'x'^{j-2}x'W_1(x', q^2),$$
$$T_2(x, q^2)=\frac{1}{\nu}\sum_{j=2,4,\dots}\left(\frac{1}{x}\right)^{j-1} 2\int_0^1 dx'x'^{j-2}\nu W_2(x', q^2), \quad (3.2)$$

where the structure functions W_i are again related to the absorptive part of T_i as in equation (2.9).

The large q^2 behaviour of the $(j-2)$th moments of xW_1 and νW_2 are governed by the anomalous dimensions of the operators of spin j (the number of symmetric Lorentz indices) appearing in the OPE of the retarded commutator in (3.1). The symmetry property of T_i under flavour transformation determines the singlet or non-singlet nature of the contributing operator.

The amplitudes T_1, T_2 have kinematical zeroes at $q^2=0$. An alternative causal set T_1', T_2', free from these zeroes and thus possessing better asymptotic behaviour for the purpose of writing down sum rules, are related to the former set by

$$T_1'=\frac{1}{q^2}\left(T_1+\frac{1}{m^2}\cdot\frac{\nu^2}{q^2}T_2\right), \qquad T_2'=-\frac{1}{q^2}T_2. \quad (3.3)$$

As explained earlier, the retarded scaling functions are given by the first term of the formally divergent series (3.2); it is the contribution of operator(s) of lowest spin in OPE allowed by crossing and flavours symmetry of T_i ($i=1,2$). The values of the lowest contributing moments of the structure functions are [1],

$$\int_0^1 dxxW_1^{eN}(x, q^2)\to\langle Q^2\rangle r,$$
$$\int_0^1 dx\nu W_2^{eN}(x, q^2)\to 2m^2\langle Q^2\rangle r, \quad (3.4)$$

giving the Bjorken limit of the retarded amplitudes T_1', T_2' as

$$\nu T_1'^{eN}(x, \nu)\to 0, \qquad \nu^2 T_2'^{eN}(x, \nu)\to\frac{2m^2}{x^2}\langle Q^2\rangle r. \quad (3.5)$$

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[4]) We have assumed no subtractions for T_i, though, according to standard Regge asymptotics, this is not true for T_1. But, as already discussed, the question of subtractions is irrelevant for our purpose.

Here N stands for neutron or proton. In each case the next non-leading term in the limit behaves as $(\log q^2)^{-a}$, $a > 0$.

Clearly none of the light cone dispersion relations require subtractions. In addition, we get the following sum rules,

$$\int_{-\infty}^{+\infty} d\nu W_{1,2}^{\prime eN}(\nu, q^2 = -2x\nu + y) = 0 \tag{3.6}$$

and

$$\int_{-\infty}^{+\infty} d\nu \nu W_2^{\prime eN}(\nu, q^2 = -2x\nu + y) = \frac{2m^2}{x^2} \langle Q^2 \rangle r. \tag{3.7}$$

Convergence of the integrals in (3.6–7) is assured, since $\nu W_1'$, $\nu^2 W_2'$ vanish faster than any power of ($\log \nu$) as the two limits of integration are approached. In the canonical case [2], however, the integrals converge only on symmetrical integration. Note that for the octet combination (ep–en), the righthand side of (3.7) is zero.

What distinguishes basically these light cone sum rules (3.6–7) from the lowest moment sum rules (3.4) is that while in the former case q^2 varies with integration over ν, it is held fixed (at a sufficiently large value) in the latter case. Similar sum rules can also be written down for charged current (neutrino) processes, the only complication being that T_2' may not be free from kinematic singularities at $q^2 = \nu = 0$ and one must isolate it before writing the dispersion relations [5, 2].

4. Discussion

The retarded scaling function entering these sum rules are known numbers. This is in contrast to the canonical case where the scaling function as well as the retarded scaling functions are not known in its details. This simplicity in the case of asymptotically free theory is deceptive, however. While in the canonical case, it is easy to fnd causal interpolations of the scaling function, no one, to our knowledge has so far been able to find such scaling functions in an analytic form satisfying the moment conditions for it in the asumptotically free theory.[5]) Had we been able to do so, we could have written down finite energy form [6] of these sum rules also in asymptotically free theory, thereby greatly enchancing the practical usefulness of these sum rules.

The behaviour of the structure functions near $x = 0$ is not known with certainty. On the basis of certain analyticity considerations in the Mellin transform variable of the moments of the structure functions, it has been argued [7] that the structure functions grow faster than any power of $\log \nu$ for large ν, compared to that given by the simple Regge pole model. This behaviour also contradicts the prediction of Regge on field theory [8], which predicts a growth given by a finite power of $\log \nu$. Settling this question, we can subtract out the

[5]) Such scaling functions in the form of integral representations has been worked out by J. Gasser (unpublished). They are, however, not simple enough to deal with in practical applications. I thank him for informative discussions on this point.

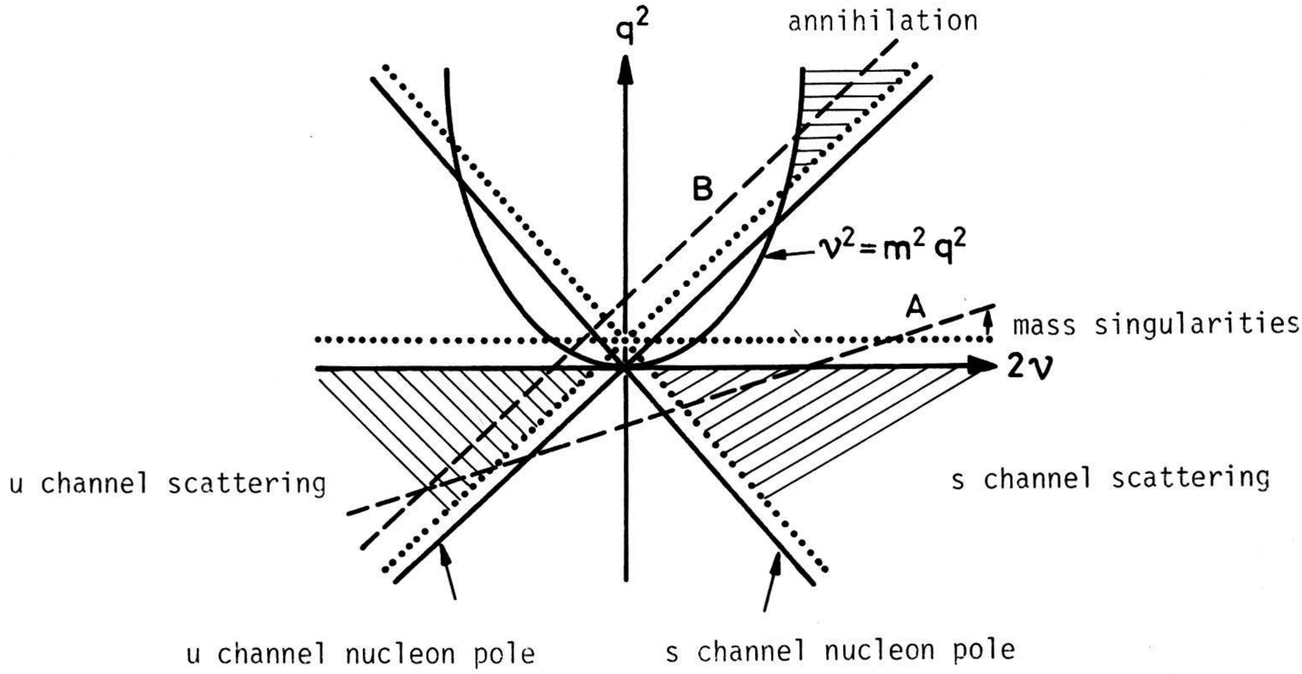

Figure 1
The shaded regions correspond to physical domains of deep inelastic scattering and one particle inclusive e^+e^- annihilation. Lines like A, B along which the integrals in the sum rules may be integrated meet unphysical regions.

Regge piece in equation (2.11) and evaluate the limit $x \rightarrow 0$ to know about the nature of the fixed poles, where present, in the amplitudes.

An attempt to evaluate the dispersion integrals in the sum rules, as such, is fraught with difficulties of estimating the absorptive parts in the unphysical region. This may be seen from Fig. 1, where the physical region of the amplitude corresponding to scattering as well as that for one particle inclusive e^+e^- annihilation are shown. It would be interesting to evaluate a smeared version [3] of the sum rules which emphasises the physical regions in the dispersion integrals.

These sum rules can have an important application to determine the mass ratio of up(down) and strange quarks. Earlier, working in the canonical formalism, Leutwyler et al. [9] applied these sum rules to K_{l_3} decay, where, for zero momentum transfer, they could evaluate the dispersion integral in terms of the commutator of the axial divergence and strangeness changing charge. However, because of the nonforward matrix element (between vacuum and K-meson state) encountered here, the OPE is complicated by the presence of external derivatives of operators. Thus the present evaluation of the retarded scaling function does not apply immediately. This problem is being investigated.

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