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The stability of force free configurations in type II superconductors

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Abstract. The stability of the vortex structure in a wire carrying a current parallel to an external field is investigated. It is shown that if the structure is fixed at the ends of the specimen an instability will occur when the surface field reaches an angle of about 30° with the axis. This is in reasonable agreement with experimental results at low fields.

Introduction

When a superconductor carries a current parallel to an external magnetic field the critical current is much higher than in the transverse geometry. However, the current distribution is unknown and there is no satisfactory theory for the prediction of critical currents in the parallel geometry. Theoretical work has been limited to situations which have translational or helical symmetry in the direction of the current (the z direction). In this case a flux flow resistivity cannot occur without flux lines cutting across each other. Brandt [1] and Clem [2], have considered this process in some detail, but their models do not provide realistic estimates for the voltages and critical currents observed. Experimentally Cave et al. [3] have shown that the flux does not penetrate in the form of complete helical vortices and Fillion et al. [4] have demonstrated that axial flux lines can cross a ring containing azimuthal trapped flux.

There is however experimental evidence that the flux structure is very inhomogeneous in the direction of the current [5, 6]. If this is the case, it is no longer necessary for vortices to cut inside the superconductor. This applies both to the standard measurements of flux flow voltage and also to the experiment of Fillion et al. in which the current is induced by an external field instead of being supplied by a current source. Instability in an isolated vortex has been demonstrated by Clem [12] and recently Brandt has used the Ginzburg Landau equations to derive an instability in the vortex lattice [13].

In fact it is well known from other systems that a current parallel to an external field tends to be unstable. Fluids show the 'pinch' effect, plasmas form helical instabilities, and a wire carrying a current between the poles of a magnet will bend into an arc.

The form of the instability

This last example seems the closest to the superconductor as the energy terms involved are similar. The source of the instability is that the field on the inside of a curve is greater than that on the outside. The excess magnetic pressure tends to increase the curvature until the pressure is balanced by the tension.

Consider the current shown in Fig. 1a. (This simple distribution probably does not occur in practice but will serve to illustrate the points of the argument.) The current is made up of a helical array of vortices which can move in the superconductor. Suppose it moves into an arc of a circle as shown in Fig. 1b. This can be done without requiring vortices to cut each other. We can find the force per unit length on the centre section from Biot–Savart law. The largest term is that due to the curved section, but this has a logarithmic divergence if the current is treated as a line current. The Biot–Savart law gives a force/length

$$\frac{\mu_0 I^2}{4\pi R}\ln\frac{\tan\phi_1/4}{\tan\phi_0/4}$$

where I is the current, R the radius of curvature and ϕ_1 and ϕ_0 the limits of integration. The lower limit is the point at which the current can no longer be treated as a line, i.e. at a distance from the centre which is of the order of the diameter. The best value can be obtained by choosing it to give the correct force for a full circle. From the variation of self inductance with radius this is known to be

$$\frac{\mu_0 I^2}{4\pi R}\ln\left[\frac{3.8R}{r}\right]$$

where r is the radius of the wire. Hence we put $\tan\phi_0/4 = r/3.8R$ and obtain a force/length

$$\frac{\mu_0 I^2}{4\pi R}\ln\left(\frac{3.8R}{r}\tan\frac{\phi_1}{4}\right) \simeq \frac{\mu_0 I^2}{4\pi R}\ln\left(\frac{l}{r}\right)$$

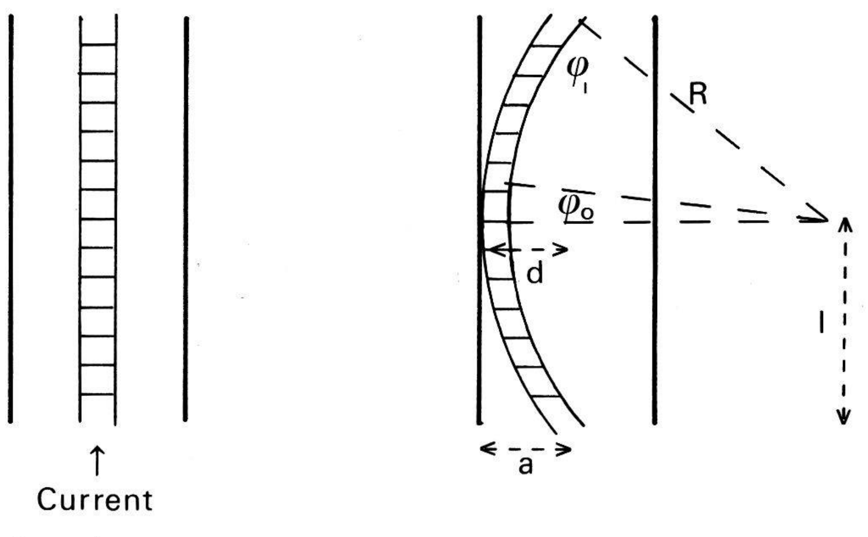

Figure 1
(a) A current restricted to the axis of a wire.
(b) At high currents the vortex structure is more stable in the form of an arc. In this figure the current has been displaced a distance 'd' which is approximately equal to the wire radius, 'a'.

If we take this as the average force, then the total force on a current of length $2l$ is approximately $\mu_0 I_d^2/\pi l \ln(l/r)$ where d is the displacement of the current from the axis, assuming $d \ll R$. The current reaches stable equilibrium when this force is balanced by the line tension of the flux lines. Therefore, for a current flowing in a superconductor of radius a we get stable equilibrium when

$$\frac{4d}{l}\frac{1}{2}\mu_0 H_z^2 \pi a^2 = \frac{\mu_0 I^2}{\pi}\frac{d}{l}\ln\left(\frac{l}{a}\right)$$

$$\frac{d}{l}\left(I^2 - \frac{2\pi^2 a^2 H_z^2}{\ln\left(\frac{l}{a}\right)}\right) = 0$$

$$d = 0 \quad \text{or} \quad I^2 = 2\pi^2 a^2 H_z^2/\ln(l/a) \tag{1}$$

Hence if I is less than this value, the structure is stable with current flowing parallel to the axis of the wire. If I exceeds this value, the structure is unstable and will move into a periodic structure of some kind. The fact that the period of this structure, $4l$, appears only as a logarithm means that at only slightly higher currents structures with much shorter periods can be formed, so that the solutions become essentially independent of the end conditions. If we put $H_\theta = I/2\pi a$ the condition for instability becomes $H_\theta = H_z/\{2\ln(l/a)\}^{1/2}$. If $l = 50a$ this is when the surface field is at about 20° to the axis. For $l = 3a$ it is 34°.

Any pinning will help to stabilize the system. Although very low pinning will stabilize an ideally symmetric starting configuration, quite small perturbations will destabilize it if the pinning forces are less than the magnetic pressure. If we consider the forces on a half period which has been moved one specimen radius, the pinning forces are $\simeq BJ_c l\pi a^2/2$. Hence in equilibrium

$$BJ_c\frac{\pi a^2 l}{2} \simeq \frac{\mu_0 I^2 a}{2\pi l}\ln\left(\frac{l}{a}\right)$$

(Assuming the line tension term is small.) Assuming also that $\ln(l/a) \simeq 1$ we find

$$\frac{H_\theta}{|H|} = \left[\frac{aJ_c}{|H|}\right]^{1/2} \tag{2}$$

So far we have considered the stability of a line of current in the superconductor. In practice the current is unlikely to be distributed in this way, but will flow initially on the surface. The flux in the interior of the wire remains parallel to the axis, and there will be a sharp change in the angle of the vortices to the axis as we pass through the current carrying layer. The thickness of this layer is unknown but must be determined by the fact that current can only penetrate more than one vortex layer if vortices cut each other [9, 10], and this can only occur if there is a large difference in angle between vortices in adjacent layers [11]. Instead of a line of current up the axis the starting configuration will be a thin cylindrical shell on the circumference.

The same arguments can however be applied to this configuration as were used in the case of a line current. The flux lattice will bend into an arc of a circle and in general any current distribution must move with the vortex lattice, provided this does not involve currents being carried outside the superconductor.

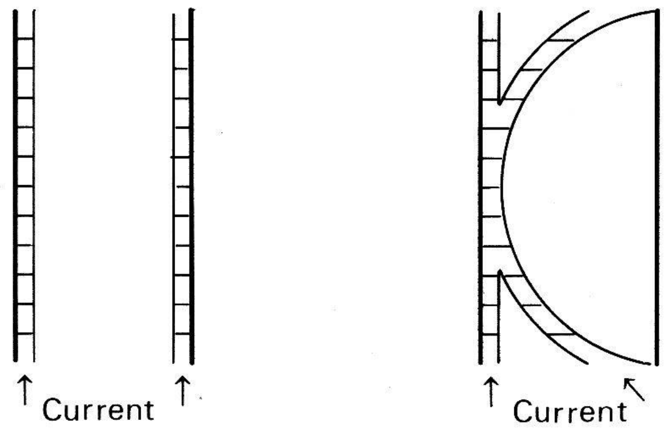

Figure 2
(a) The current distribution in a wire when the current is first turned on.
(b) If the vortex structure is bent the current moves with it, except at the outer surface of the sample. This figure shows the distribution at the critical current. The energies involved will be similar to those in Fig. 1.

Where flux is moving out of the superconductor flux cutting can occur and the current associated with this flux will remain on the surface. Hence as the curvature increases current on one side of the specimen will remain constant while the current on the other side approaches it across the specimen. This is illustrated in Fig. 2.

When the two current sheets meet we will have a region carrying a very high current density with flux lines at large relative angles. A configuration like this can create a flux flow voltage by two possible mechanisms. Firstly the high relative angles of the vortices may lead to flux cutting in this region. Alternatively it can be seen that the local field and current are no longer parallel so that the wire is behaving as if it were at an angle to the applied field. For large enough angles flux flow can occur without difficulty since flux cutting is only necessary when the vortices do not emerge through the sides of the specimen. In either case a configuration such as that in Fig. 2 can lead to a voltage, and therefore an instability which moves the vortex structure across a specimen diameter will mark the critical current of the wire. In practice the current is more likely to move into a helix than the arc considered here [5] and other instabilities could occur, depending on the end conditions. The one considered here is appropriate if the current is fixed at the ends of the specimen by, for example, stronger pinning in this region.

Comparison with experiment

In Fig. 3 the critical current predicted by equation (1) (with $l = 3a$) is compared with the results of Cave [7] on Pb–In. It can be seen that for fields between B_{c_1} and about $0.15B_{c_2}$ the stability condition predicts the right order of magnitude for the critical current density. Near B_{c_1} deviations must be expected since at zero field the critical current is given by the Silsbee rule. At high fields the arguments used to derive equation (1) cannot hold since the axial field, which stabilizes the structure, becomes larger while the critical current goes to zero at B_{c_2}. In this range the critical current density is probably determined by a different instability. It was observed by Cave and Evetts [6] that in this regime the voltage

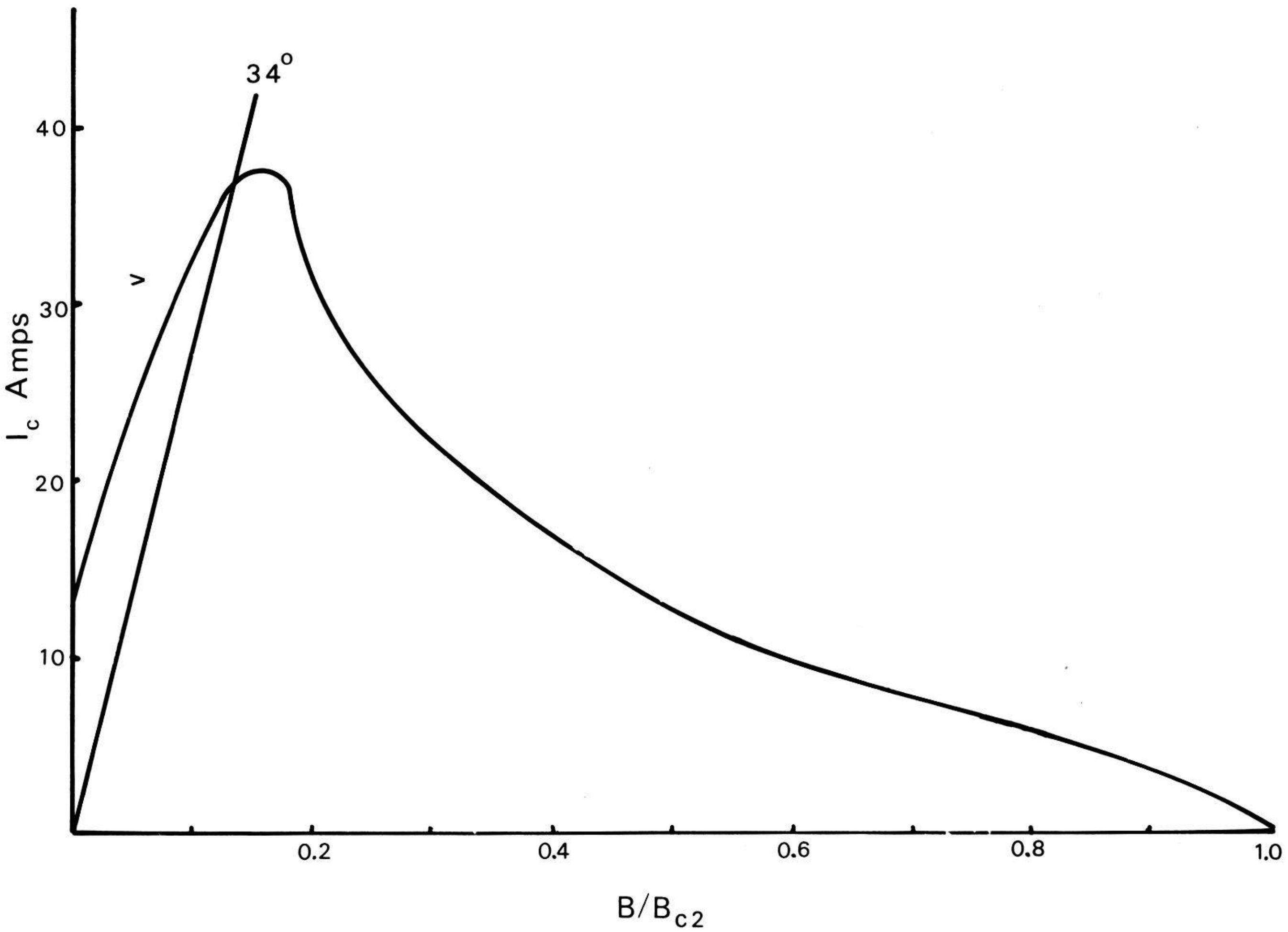

Figure 3
The experimental critical current of a 0.5 mm diameter Pb–In wire from reference 7. The straight line shows the current at which the surface field is at 34 degrees to the axis.

drop occurred almost entirely at the specimen ends. This suggests that the vortex structure may move around near the contacts and the instability is limited to this region. We have assumed here that the current is fixed in position at the ends of the specimen, which is not always realistic.

If we now look at irreversible materials the results of Karasik and Vereshagin [8] show that the longitudinal current certainly varies with J_c, although not in as simple a way as equation (2) predicts. However, if we confine attention to recrystallized specimens in which the pinning is probably isotropic, we again obtain the correct order of magnitude for J_c if $B \ll B_{c_2}/2$. Indeed equation (2) gives a tolerable description up to B_{c_2}, but since the currents above $B = B_{c_2}/2$ fall below those predicted by equation (1), clearly they cannot be compared with the experimental results. It is however interesting to see that it needs a transverse current density as high as that provided by NbTi before the pinning begins to dominate the longitudinal current density.

Conclusion

Theoretical work and experimental evidence both show that the current flow in a superconducting cylinder parallel to an applied field cannot have cylindrical symmetry. It has been shown in this paper that an instability will occur at currents which are of the order of magnitude of the observed values for $B < 0.15 B_{c_2}$. By bringing the current to the surface of the specimen the resulting flux distribution can provide the large relative angles needed for flux cutting, or may allow flux flow of the type which must occur in a wire at an angle to the field. However, the configurations suggested can only be regarded as a first step towards the real solutions which are certainly much more complex.

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