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Projectile fragmentation and stripping to unbound states: an important reaction mechanism for peripheral nucleus–nucleus collisions

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Abstract. In this paper we apply the theory of elastic and inelastic fragmentation to study the properties of the light projectile fragmentation in the energy range from 10 to 40 MeV/A. It is shown that the inelastic fragmentation contains the mechanism for stripping to unbound states (resonances). The continuous transition from bound to unbound state stripping is shown to follow naturally from the formulation. The detailed calculation for the probability of fragmentation for deuteron, ^{3}He and α-particles are presented. These probabilities are strongly localized around the grazing angular momentum of the collision and depend in a simple way on the gross properties of the target and projectile nuclei and the energy of the projectile. The simple parametrization introduced for the fragmentation probability leads to the total break-up cross section showing the factorization property observed in the heavy ion fragmentation.

1. Introduction

Many different reaction modes can be distinguished in the collision of nuclei with nuclei. They reach from the very slow compound (fusion) reaction over the pre-equilibrium (deep inelastic) collisions to the direct reactions which take place in a very short time scale. For grazing collisions, the main effects are associated with these direct (fast, one-step) processes which are rather well understood by now [1]. However, in addition to the commonly studied direct processes leading to bound final states (two-body final states: stripping to bound states, inelastic scattering, pick-up) there are many more direct processes with three- (or more) body final states. These processes give rise to continuum spectra. The direct mechanism (knock-out) that leads into the continuum of inelastically scattered projectiles (like p, d or α-particles) has recently been studied theoretically [2]. It is generally agreed that one-step processes are not sufficient to explain the experimentally observed cross-sections, yet they form a substantial part of it.

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It is the purpose of this paper to study the fragmentation process, where projectile a disintegrates into particle b and the rest $x(a=b+x)$. We are especially interested in the calculation of inclusive spectra; if only particle b is observed we have to integrate over all the possible channels which the system $(A+x)$ can have, where A is the target nucleus. The inclusive spectra of particle b are found to be characterized experimentally by a peak at the energy $E_b = (m_b/m_a)E_a$, where E_a is the energy of the incident particle. The theory to be described in this paper corresponds to the spectator mechanism: Particle b passes the target nucleus essentially undisturbed, thus keeping its initial velocity, which consists of the beam velocity smeared out by its Fermi motion in the projectile a. This will essentially account for the width of the break-up peak. Particle x, on the other hand, will interact in all possible ways with the target nucleus. It must be mentioned at this point that there is also another simple mechanism which can contribute to the inclusive spectra: Particle a is inelastically scattered to some excited state above the particle threshold, this state will then decay (say into $b+x$) after the projectile nucleus has left the region of interaction with the target nucleus (see e.g. Refs. 3, 4). The theoretical description of such a mechanism will involve rather detailed nuclear structure properties, like collective excitation strength and particle decay properties of those states [3].

In Chapter 2 we recall the theory of elastic [5] and inelastic break-up [3, 6, 7] modes. Then it will be shown explicitly how the inelastic mode accounts for the continuous transition from the stripping to bound states to the unbound region.

In order to study completely the effect of the break-up channel back to the elastic one, it would be necessary to set up a coupled channel theory. In principle, this is possible, but we want to treat here only a simpler problem: We study the absorption due to break-up on the elastic channel. This becomes possible in our first order theory by the use of unitarity. This is done in Chapter 3. We study the probability for break-up as a function of the l-value of the incoming particle, which is directly related to the semiclassical concept of the impact parameter. In our numerical studies we find simple laws like scaling and factorization properties for the break-up probability. Thus the situation at our rather low energies, typically 10–40 MeV/A, is very much reminiscent of the surprisingly simple concepts that emerged from the heavy ion fragmentation process at relativistic energies (for a review see e.g. Ref. 8). We present results for (d, p), (d, n), $(^3\text{He}, d)$, $(^3\text{He}, p)$, (α, t) and $(\alpha, ^3\text{He})$ processes at various incident energies for a variety of target masses ranging from $A=27$ to 197. Our conclusions are summarized in Chapter 4.

2. Review of the theory

(a) *Qualitative introduction*

Direct reactions are fast one-step processes, therefore, theories have been formulated mainly within a first order approach (DWBA). Higher order effects are considered only as far as they are included in the optical model wave function in the entrance and exit channels. The interaction which causes these transitions is treated in first order. Under special conditions also higher order effects can become important. This can occur when first order transitions are weak (hindered) due to some kind of nuclear structure effect. Then second order effects become important; however, such calculations are difficult because the choice of intermediate states is generally not obvious and it is hard to include very many states. We are dealing here with a different situation: As we shall see, the break-up channel will exhaust up to about 30% of the possible total reaction cross section for grazing partial waves. In such a situation coupling effects can become important. The influence of the break-up channels on the elastic channel was studied e.g. in Ref. 9. Similar coupled channel equations could be set up in our present formulation, yet the long range nature of the coupling potential would make the solution of such a system, e.g. by discretization methods, rather cumbersome.

(b) *Theory of elastic and inelastic break-up*

Let us first consider the reaction

$$A+a\rightarrow A+b+x, \tag{1}$$

where the projectile a disintegrates into the constituents b and x ($a=b+x$) in the Coulomb and nuclear fields of the target nucleus A. Both particles b and x are supposed to be detected; because nucleus A stays in the ground state we call this mode the elastic break-up. We assume that the system is described by a Hamiltonian

$$H=T+V_{Ax}+V_{Ab}+V_{bx}, \tag{2}$$

where T denotes the kinetic energy, the interaction between the 3 particles of the system is denoted by V_{Ax}, V_{Ab} and V_{bx}, respectively. The interactions V_{Ax} and V_{Ab} are supposed to be given by phenomenological (complex) optical model potentials. The (real) potential V_{bx} is responsible for the bound state of the projectile $a=b+x$. It is our aim to calculate, in a good approximation, the triple differential (coincidence) cross section for the reaction (1). We introduce two decompositions of the total Hamiltonian (equation (2)), corresponding to the initial and final channel. For the initial channel we introduce the phenomenological optical model potential V_{Aa} which describes the c.m. motion of projectile a in the field of the target A.

$$H_i=T+V_{Aa}+V_{bx}, \qquad V_i=H-H_i=V_{Ax}+V_{Ab}-V_{Aa}. \tag{3a}$$

For the final channel we use

$$H_f=T+V_{Ab}+V_{Ax}, \qquad V_f=V_{bx}. \tag{3b}$$

Then we write the T-matrix element for process (equation (1)) in the post-interaction form of the DWBA as (for details see Ref. 5)

$$T_{\mathbf{q}_a\rightarrow\mathbf{q}_b\mathbf{q}_x}=\iint d^3r_{bx}d^3R_a\chi_b^{(-)}(\mathbf{R}_{bA})^*\chi_x^{(-)}(\mathbf{r}_x)^*V_{bx}(\mathbf{r}_{bx})\psi_{bx}(\mathbf{r}_{bx})\chi_a(\mathbf{R}_a), \tag{4}$$

where $\mathbf{q}_a$, $\mathbf{q}_b$ and $\mathbf{q}_x$ denote the momenta of a, b and x in the initial and final state. The internal ground state wave function of projectile a is denoted by ψ_{bx}, the χ's denote the scattering wave functions of a, b and x generated by the appropriate optical potentials. The vector between b and x is denoted by $\mathbf{r}_{bx}$, $\mathbf{R}_a$

denotes the vector between the centers of mass of A and a. In order to write down the final state as a product wave function, we assume that the mass of A is much larger than that of a. Then we neglect the difference between $\mathbf{R}_{Ax}$ and $\mathbf{r}_x = \mathbf{R}_{x-Ab}$ and separate the final state as is done in equation (4). (Obviously, such an approximation should be symmetric with the interchange of b and x.) It is important to note that a different decomposition of Hamiltonian (3b) for the final state is appropriate if the break-up proceeds via the excitation of a resonant state a^* which decays subsequently into $b+x$. Then it is appropriate to use for the final state again the decomposition (3a). In our choice the physics is different. The break-up occurs in a 'spectator mechanism', part of the projectile, either b or x, interact strongly with the target and cause the break-up. The final state interaction V_{bx} is supposed to be negligible. Of course, in a complete 3-body theory all the processes mentioned here would be taken into account automatically, yet such a theory seems not to be feasible at present. Therefore, we have to rely on some kind of approximations which are physically reasonable.

For light particle induced reactions we evaluate expression (4) in the zero-range approximation, introducing the usual zero range constant D_0 appropriate for the vertex $a \to b+x$. Examples for the calculation of such coincidence cross sections and comparison with experiment can be found in Refs. 5, 10 and 11. Up to now our theoretical development has been symmetrical with respect to particles b and x. Now we want to treat that kind of cross section where only particle b is detected ('inclusive cross section'). In this case the interaction of x with A can be of any kind, elastic or inelastic. It will be seen below that it is important to treat all these inelastic processes. It would be a hopeless task to try to calculate all kinds of inelastic processes individually and sum them up. However, by using unitarity we can calculate very simply all these inelastic processes, as will be seen below.

First, the contribution of the elastic break-up to the inclusive (a, b) cross section can be obtained by an integration over the angle of the unobserved particle x. In order to do this analytically, we introduce a partial wave expansion in the matrix element equation (4). (We use the zero range approximation with the 'standard' LEA (local energy approximation) corrections for the finite range effects throughout, see e.g. Ref. 11.)

$$T_{\mathbf{q}_a \to \mathbf{q}_b \mathbf{q}_x} = (4\pi)^2 D_0 \sum_{l_a l_b l_x} i^{l_a+l_b+l_x} e^{i(\sigma_{l_a}+\sigma_{l_b}+\sigma_{l_x})} \hat{l}_a \hat{l}_b \hat{l}_x \begin{pmatrix} l_a l_b l_x \\ 000 \end{pmatrix} R_{l_a l_b l_x} \sum_{m_x} (l_b - m_x l_x m_x \mid l_a 0) Y_{l_b - m_x}(\theta_b, 0) Y_{l_x m_x}(\theta_x, \varphi_x), \tag{5}$$

where σ_l denote the Coulomb phases. The radial integral $R_{l_a l_b l_x}$ is defined by

$$R_{l_a l_b l_x} = \frac{1}{q_a q_b q_x} \int_0^\infty \frac{dr}{r} \chi_{l_a}(q_a, r) \chi_{l_b}(q_b, r) \chi_{l_x}(q_x, r), \tag{6}$$

where the radial part of the optical model wave function is denoted by $\chi_l(q, r)$. By virtue of the orthogonality of the spherical harmonics the integration over the angle of the unobserved particle x leads to the following expression for the elastic part of the double differential cross section

$$\frac{d^2\sigma(\mathrm{el})}{d\Omega_b\, dE_b} = \frac{m_a m_b m_x}{4(\pi\hbar^2)^3} \frac{q_b q_x}{q_a} \sum_{l_x m_x} |T_{l_x m_x}(\theta_b)|^2, \tag{7}$$

where we have introduced a 'reduced' T-matrix [6] $T_{l_x m_x}$ which is quite analogous to the usual T-matrix for bound state stripping. Note that the sum over the partial waves l_x has become incoherent.

To the inclusive spectrum, also all those processes will contribute where the interaction of particle x with the target nucleus is inelastic. This is called inelastic break-up. Within rather well fulfilled approximations the inelastic break-up cross section can be calculated with the matrix elements already needed for the elastic break-up.

The starting point of our formulation for the inelastic breakup process is the DWBA expression for the break-up reaction $A+a \to b+c$, where c is some specific two-body final state of the system $B=A+x$. It is given by [12, 13]

$$T_{a,bc} = \langle \phi_{Bc}^{(-)} \chi_b^{(-)} | V_{bx} | \phi_A \chi_a^{(+)} \rangle, \tag{8}$$

where ϕ_A denotes the ground state wave function of nucleus A. $\phi_{Bc}^{(-)}$ denotes a complete scattering state of the system B with the boundary condition c. The transition amplitude (8) is evaluated by first integrating over the internal coordinates ξ_A of ϕ_A. This leads to a generalization of the radial form factor ('wave function of the transferred particle') to inelastic processes:

$$\int d\xi_A \phi_{Bc}^{(-)*} \phi_A = 4\pi \sum_{l_x m_x} i^{l_x} \chi_{l_x}^c(r) Y_{l_x m_x}(\hat{r}) Y_{l_x m_x}(\hat{q}_c)^*. \tag{9}$$

In principle, it would be possible to calculate this form factor with the help of a model wave function for $\phi_{Bc}^{(-)}$. However, this would be very difficult and impracticable if there are many open channels. But, fortunately, there is an approximation procedure which allows us to make use of the unitarity of the S-matrix (for the system $B=A+x$). This simplifies the whole calculation enormously. We note (see Ref. 11) that the main contribution to the DWBA integral comes from the region outside the nuclear interaction $r>R_0$. There we can express the radial form factor $\chi_{l_x}^c$ entirely in terms of the scattering matrix element $S_{l_x,c}$, which connects the elastic channel l_x and the inelastic channel c:

$$\chi_{l_x}^c(r) = \delta_{l_x,c} j_{l_x}(q_x r) + \sqrt{\frac{m_x q_x}{m_c q_c}} \tfrac{1}{2}(S_{l_x,c} - \delta_{l_x,c}) h_{l_x}^{(+)}(q_x r) \qquad (r>R_0). \tag{10}$$

It is worth noticing that in equation (10), $q_x r$ appears as argument of the Hankel function and not some wave number q_c, which would correspond to channel c. That part of the wave function vanishes because of the orthogonality of the ground state ϕ_A of the target nucleus with the excited state. In complete analogy to the situation for the elastic break-up we can carry out the integration over the angle of $\mathbf{q}_c$ in order to obtain from the triple differential cross section a double differential cross section for the (a, b) reaction. With the help of the form factor equation (10) we can introduce a 'reduced' T-matrix (where the integration over $\hat{q}_c$ has already been taken into account) for the process $a+A \to b+c$:

$$T_{a,bc}^{red} = \sqrt{\frac{m_x q_x}{m_c q_c}} \frac{S_{l_x,c}}{S_{l_x,l_x}-1} D_0 \int d^3r \chi_{\mathbf{q}_b}^{(-)}(\mathbf{r})^* [\chi_{l_x}(q_x, r) - j_{l_x}(q_x r)] Y_{l_x m_x}(\hat{r}) \chi_{\mathbf{q}_a}^{(+)}(\mathbf{r}). \tag{11}$$

Somehow arbitrarily we have extended the form factor χ_{l_x} also into the interior region $r < R_0$. As this region contributes only very little to the whole DWBA integral, this is not expected to be a serious approximation. The entire dependence on the channel index c rests now in the S-matrix element $S_{l_x,c}$. This tremendous simplification allows us to sum over all $c \neq l_x$. With the unitarity of the S-matrix we obtain

$$\sum_{c \neq l_x} |S_{l_x,c}|^2 = 1 - |S_{l_x,l_x}|^2. \tag{12}$$

With the usual definitions of the elastic and total reaction cross section $\sigma_{l_x}^{\mathrm{el}}$ and $\sigma_{l_x}^{\mathrm{reaction}}$ we can write the inelastic break-up cross section in the following rather compact form [6, 7].

$$\frac{d^2\sigma(\mathrm{inel})}{d\Omega_b\, dE_b} = \frac{m_a m_b m_x}{4(\pi\hbar^2)^3} \frac{q_b q_x}{q_a} \sum_{l_x m_x} \frac{\sigma_{l_x}^{\mathrm{reaction}}}{\sigma_{l_x}^{\mathrm{el}}} |T_{l_x m_x} - T_{l_x m_x}^0|^2, \tag{13}$$

where the T-matrix has been split into two parts according to equation (11). For the (a, b) double differential cross section, the elastic and inelastic contributions, equations (7) and (13) have to be added up. Our approach shows some similarities to the work of Lipperheide [14]. In this approach, a total 'off-shell reaction cross section' appears, whereas in our equation (13) only on-shell $x - A$ cross sections occur. The reason lies in the different assumption: Whereas Lipperheide uses a plane wave theory, we make use of a suppression of contributions from the interior to the DWBA matrix element. The reason for this suppression is Coulomb repulsion and strong absorption. In the plane wave approaches, the relation of the break-up cross section to the momentum wave function of projectile a can be directly exhibited, see e.g. Refs. 15 and 16.

(c) *The continuous transition from stripping to bound states to unbound states*

A brief account of this paragraph has already been given in Refs. 17 and 18. At the high energy end of the (a, b) spectrum, the transitions to the discrete low lying levels of the residual nucleus $(A + x)$ are observed. As the excitation energy goes up, or in other words, the energy E_b of the emitted particle b goes down, the level density in the residual nucleus will increase. It may be neither of any more interest nor possible to resolve those levels any more. This region was called 'continuum' in Ref. 19, yet these levels can still be discrete bound states. (If there are no other decay channels than the emission of particle x, the width of these states is only due to electromagnetic decays.) Above the threshold for the emission of particle x, there will be a population of isolated resonances. It is seen experimentally that the transition from the bound to the unbound region is continuous, this experimental fact should emerge in a natural way from our theory. In Ref. 20 a somewhat idealized situation was treated. The depth of the (real) potential well, which binds the transferred particle x to the target was decreased, so that this state becomes unbound. It was found that in this pure single particle situation the stripping cross section for the bound state joins smoothly to that for the resonant state. (We define the stripping cross section to the resonance as the energy integral of the double differential cross section over the resonance region.) This property was established by means of the relation

$$\Gamma_{\mathrm{s.p.}} = \frac{\hbar^2}{m_x q_x} N_{\mathrm{s.p.}}^2. \tag{14}$$

Here, $\Gamma_{\text{s.p.}}$ denotes the width of the single particle resonance and $N_{\text{s.p.}}$ denotes the asymptotic normalization of the single particle (Gamow) state. The stripping cross section to the unbound state is proportional to $\Gamma_{\text{s.p.}}$, the corresponding stripping cross section to the bound state is proportional to $N^2_{\text{s.p.}}$; with the help of equation (14) we can relate both of these cross sections to each other in the limit $E_x \to 0$, as is shown in detail in Ref. 20. This pure single particle description is, of course, generally rather unrealistic at the excitation energies of the nuclei at the corresponding particle threshold.

Let us now deal with the more realistic situation, where the single particle strength is spread out over very many (compound) states (see e.g. Ref. 21). We deal now with a specific partial wave lj of the transferred particle (we omit the index j in the following). Then we can define an energy averaged double differential cross section for stripping to the bound states by

$$\frac{d^2\sigma_l}{d\Omega_b\, dE_b} = \frac{1}{2}\frac{m_a m_b}{(2\pi\hbar^2)}\frac{q_b}{q_a} D_0^2 \frac{S \cdot N^2_{\text{s.p.}}}{D} \sum_m |T^+_{lm}|^2, \tag{15}$$

where $1/D$ denotes the number of levels per energy interval (of the given l-value), S denotes the average spectroscopic factor of these states. The matrix element T^+_{lm} is defined by

$$T^+_{lm} = \int d^3r \chi_b^{(-)}(\mathbf{r})^* h_l^{(+)}(i\alpha r) Y_{lm}(\hat{r}) \chi_a^{(+)}(\mathbf{r}), \tag{16}$$

where α is related to the binding energy E_{bind} of particle x in the nucleus $(A+x)$ by $\alpha = \sqrt{2mE_{\text{bind}}/\hbar^2}$. Here, we treat for simplicity of presentation only the transfer of a neutron. The generalizations necessary for the transfer of a charged particle are given in Appendix A.

Let us now establish the connection of equation (15) with the unbound region. The inclusive cross sections consist of the sum of the elastic and inelastic modes. Because of the phase space factor, the elastic break-up cross section tends to zero at the threshold, therefore, we will only have to consider the inelastic break-up. It tends to a limit different from zero in the presence of absorption in the neutron channel at zero energy [6]. We introduce now the well-known relation between the energy averaged total neutron cross section $\langle\sigma_l\rangle$ and the strength function $(\Gamma/D)_l$, in the low energy region:

$$\langle\sigma_l\rangle = \sigma_l^{\text{reaction}} = \frac{2\pi^2}{q_x^2}(2l+1)\left(\frac{\Gamma}{D}\right)_l. \tag{17}$$

With the help of equation (17) we can rewrite the inelastic breakup cross section in the following way

$$\frac{d^2\sigma_l}{d\Omega_b\, dE_b} = \frac{1}{2}\frac{m_a m_b}{(2\pi\hbar^2)^2}\frac{q_b}{q_a} D_0^2 \frac{q_x m_x \Gamma}{D\hbar^2} \sum_m |T^+_{lm}|^2. \tag{18}$$

With the help of relation (14) the continuous transition to the bound region (see equation 15) is immediately established. Hereby we have introduced a natural definition [22] of a spectroscopic factor for resonant states

$$\Gamma = S \cdot \Gamma_{\text{s.p.}} \tag{19}$$

It is gratifying to see how the apparently unrelated formulations of stripping to bound and unbound states do have indeed a common origin. This should be the case, the experimental results demanding such a relation.

In this paragraph we have dealt with a situation which is novel in the study of direct reactions. We are not interested in the cross section for the population of a specific level but in the average cross section for transfer reactions to states (resonances) which contain some fraction of single particle strength.

3. Results

(a) *Numerical results for stripping to unbound states*

In this section we study the region corresponding to the transition from stripping to the bound levels to the unbound levels (resonances). The quantity that governs the cross section in this region is the strength function (Γ/D). If one is mainly interested in gross features one can use the optical model to determine this quantity (cf. equation (17)). The optical potential parameters are usually phenomenologically fitted to the elastic scattering. For the low energies, relevant in our present context, the low partial waves (especially s-waves) determine the cross section. Due to the l-enhancement, discussed e.g. in Ref. 20, different partial waves enter into the double differential cross section. Thus, in this kind of experiment, oné is sensitive to a quantity which is different from what is usually determined from elastic scattering. This point is illustrated in Fig. 1. In the upper part of this figure we show the total elastic and reaction cross section of neutron scattering on ^{62}Ni, calculated with two different sets of potential parameters. We use the optical model potentials given by Becchetti and Greenlees [23] and by Wilmore and Hodgson [24] (excluding the spin orbit term). It is seen that these cross sections are rather similar for the two different potentials. However, the situation changes dramatically when one looks at the corresponding $(\alpha, {}^3\text{He})$ reaction on ^{62}Ni in the bottom part of Fig. 1. There we can see a factor of about two difference for both potential sets. The reason is that mainly the $l = 4$ neutron partial wave determines the cross section, which is not determined by the low energy scattering of neutrons on nuclei. Thus, the transfer reaction is a means to overcome the centrifugal barrier. Similarly, possibly even more dramatic effects can be expected for charged particle transfer.

An even simpler estimate of the strength function (Γ/D) can be made with the black nucleus model (see e.g. Ref. 25). We found that these black nucleus estimates are in qualitative agreement with the optical model results, in which the 'geometry' of the nucleus (finite surface) is treated in a more realistic way. In the optical model description, specific nuclear structure effects, like particle-phonon-coupling, are still not taken into account. These effects may become important in future detailed studies and they are expected to vary strongly from nucleus to nucleus.

Concluding this paragraph, transfer reactions into the continuum are a unique tool to study strength functions, which cannot for example, be studied by elastic scattering. A careful experimental and theoretical study may reveal the longsought 'giant resonance structure' [21], which has up to now withstood its verification.

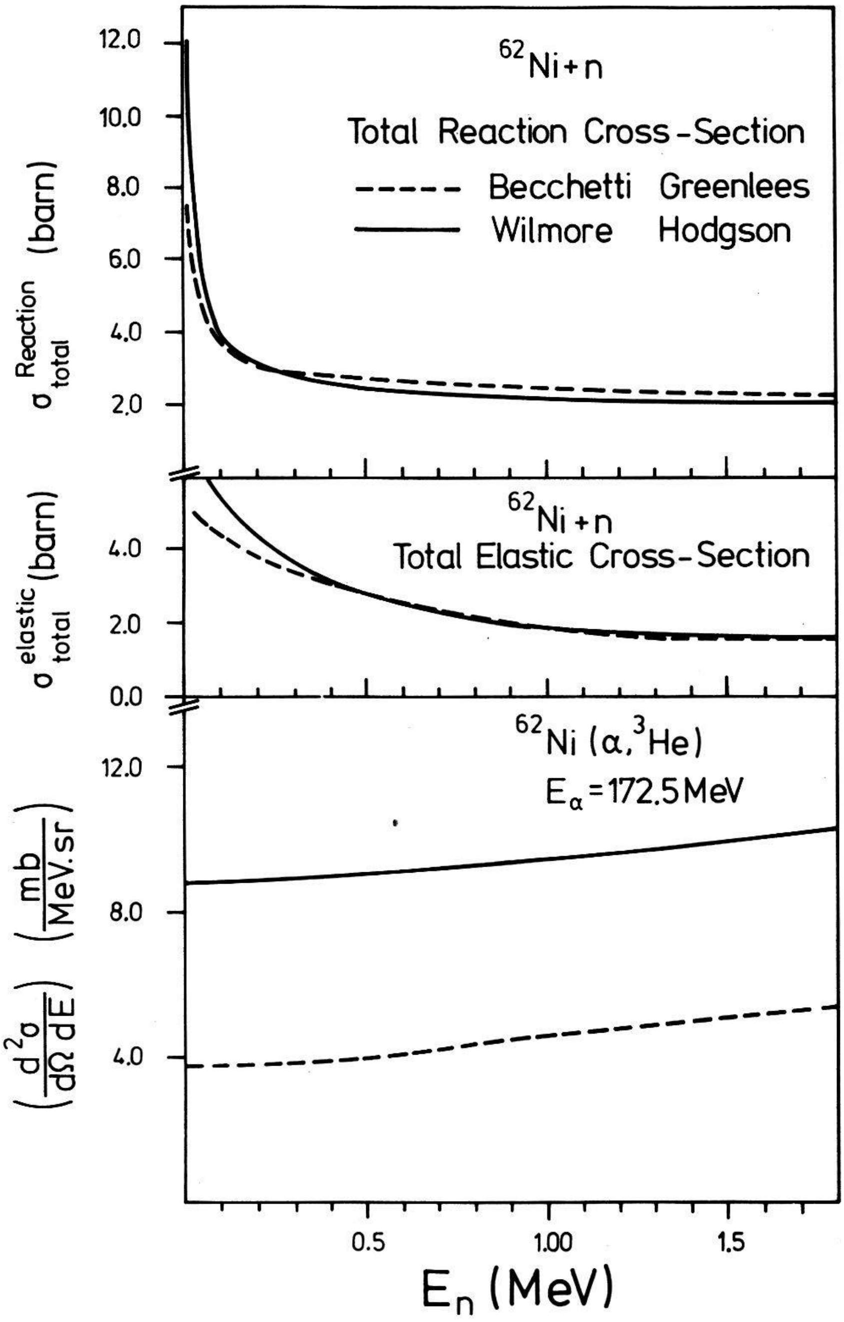

Figure 1
Calculation of ^{62}Ni$+n$ elastic and reaction cross section (top part) as compared to the ^{62}Ni (α, ^{3}He) stripping into the continuum cross section (bottom part) at $\theta_{^3\text{He}} = 5°$ as a function of E_n.

(b) *Numerical results for the impact parameter dependence of the break-up probability. Scaling laws and factorization properties*

Whereas coincidence cross sections calculated from our break-up theory can be rather sensitive to special details of optical model parameters [10] we have found rather simple, geometry dependent universal properties of the integrated break-up cross sections.

We define the probability of break-up $T_{l_a}^{\text{b-up}(a,b)}$ by

$$\sigma_{\text{total}}^{\text{b-up}}(a, b) = \int d\Omega_b\, dE_b \frac{d^2\sigma(a, b)}{d\Omega_b\, dE_b} = \frac{\pi}{q_a^2} \sum_{l_a} (2l_a + 1) T_{l_a}^{\text{b-up}(a,b)} \tag{20}$$

where $\frac{d^2\sigma(a, b)}{d\Omega_b\, dE_b}$ is the double differential inclusive cross section, calculated according to equations (7) and (13). We relate the angular momentum l_a of the incoming projectile to the impact parameter b by $b=(l_a+\frac{1}{2})/q_a$. The break-up probability defined in equation (20) is only a partial probability for break-up into a specific fragment b. For the total break-up probability all possible decay modes would have to be added up. Since we use a partial wave expansion of the break-up T-matrix, we can perform the integration over Ω_b analytically by means of the orthogonality of the spherical harmonics. The integration over the energy E_b of the emitted particle b is performed numerically.

Now we shall present our numerical results for deuteron, ^{3}He and α-induced break-up. We find that a rather simple parametrization can reproduce the numerical results. This is discussed along with some properties of this parametrization at the end of this chapter.

(i) *Deuteron break-up probabilities*

In this paragraph we calculate deuteron break-up probabilities for $E_d=$ 25 MeV and $E_d=80$ MeV. The optical model parameter sets used are described

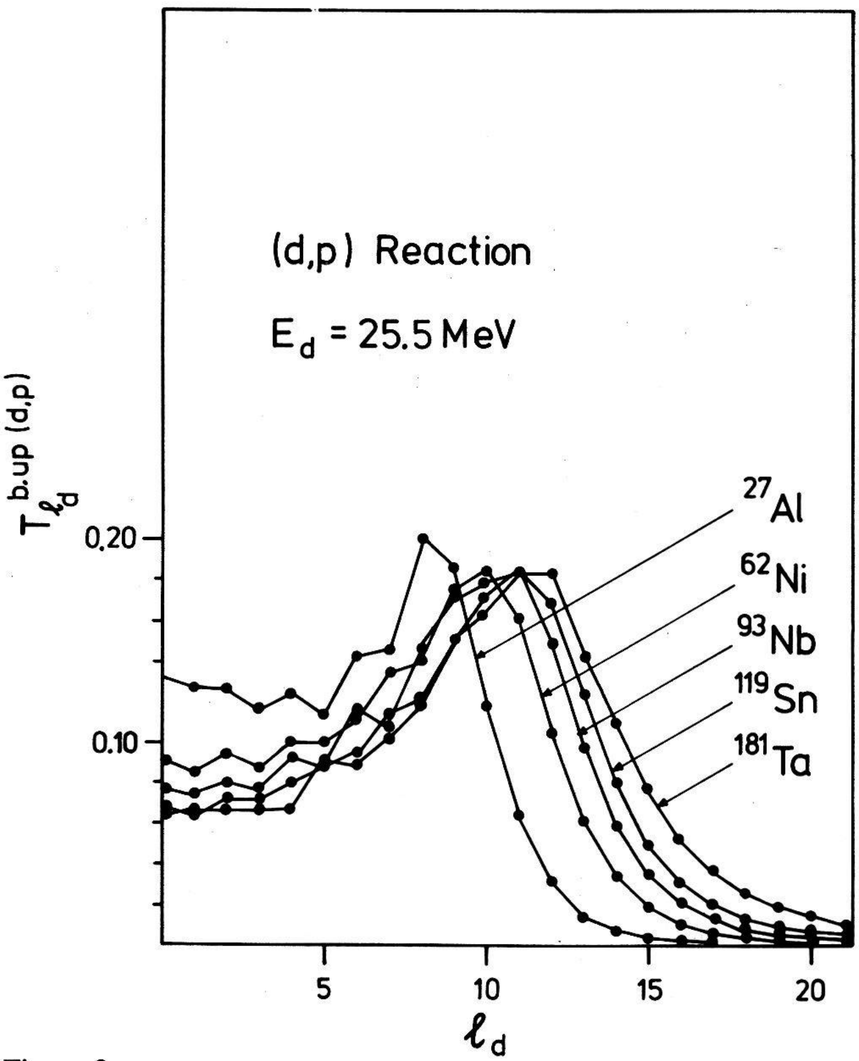

Figure 2
Deuteron break-up probabilities for a range of target nuclei as a function of the deuteron angular momentum.

in Refs. 6 and 26, respectively. The deuteron serves as a good testing ground for the other particles, where very much the same phenomena are found. In Fig. 2 we show the (d, p) break-up probabilities for $E_d = 25.5$ MeV deuterons incident on a range of target nuclei from $A = 27$ to 181. Despite the still rather large wave length of the incident particle, there is a definite localization of the break-up probability in the surface region.

With increasing projectile energies, the wave length becomes smaller and the surface localization becomes even more pronounced. This is shown in Fig. 3 for $E_d = 80$ MeV incident deuterons on ^{93}Nb and ^{197}Au, respectively. The transmission coefficient as calculated from the deuteron-nucleus optical model potential is also shown, it shows the expected smooth cut-off behaviour for small l-values, the break-up probability becomes rather small, for grazing partial waves it is an appreciable part of the total absorption.

In order to stress the geometrical nature of the break-up process we introduce the impact parameter $b = (l_a + \frac{1}{2})/q_a$. In Fig. 4 we show the total break-up cross section $2\pi b T^{\text{b-up}(d,p)}$ as a function of the impact parameter. We can see that in both cases, $E_d = 25$ and 80 MeV, the break-up probability is localized around the surface. With increasing deuteron energy, we also note an increase in the break-up probability.

The surface nature of the break-up process, which we find in our numerical investigation, implies a rather simple dependence of the total break-up cross section on the mass number A of the target nucleus. It is proportional to the length of a ring with radius $R = r_0(A^{1/3} + a^{1/3})$. The total (d, p) break-up cross section as a function of the mass number is shown in Fig. 5. We can see that the

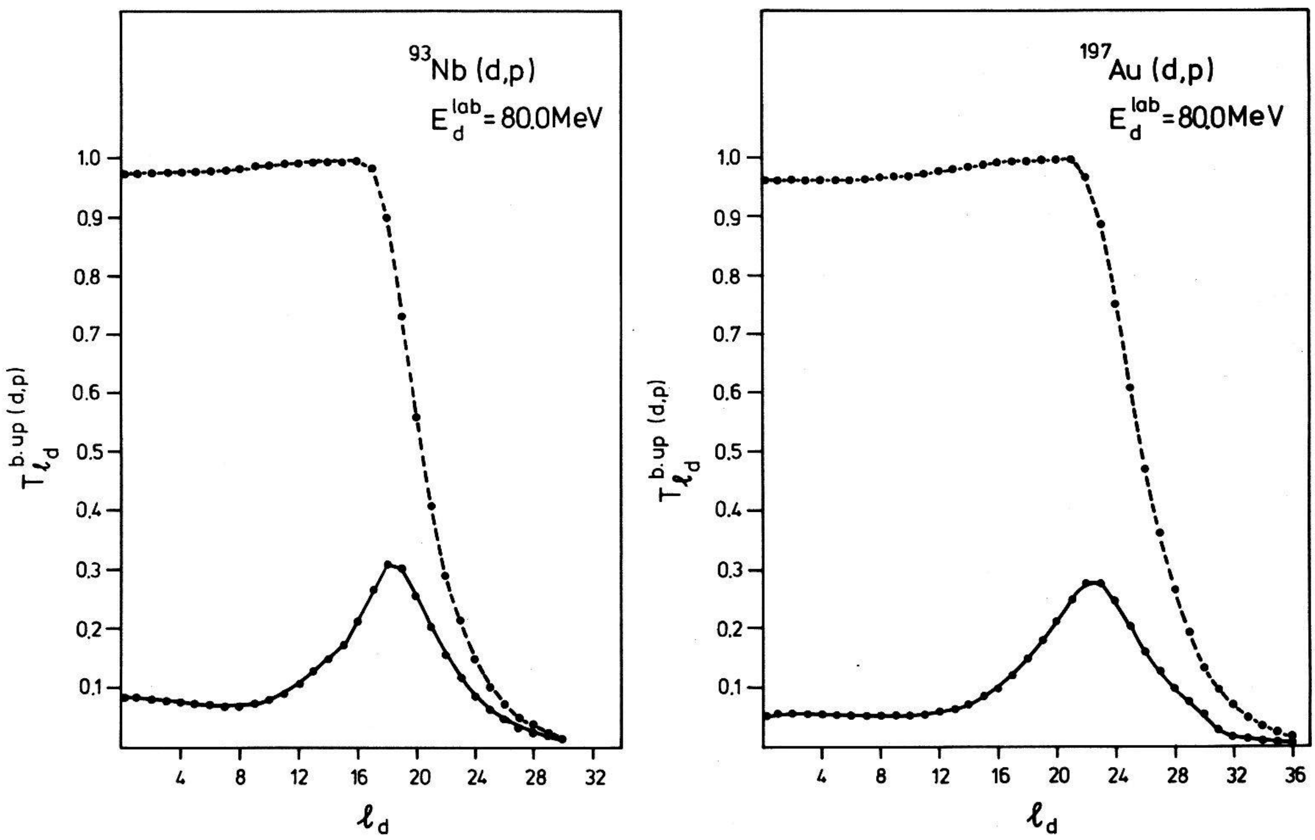

Figure 3
Deuteron break-up probabilities for $E_d = 80$ MeV induced deuteron break-up (continuous line). The dashed line denotes the transmission coefficient calculated with the usual optical potential.

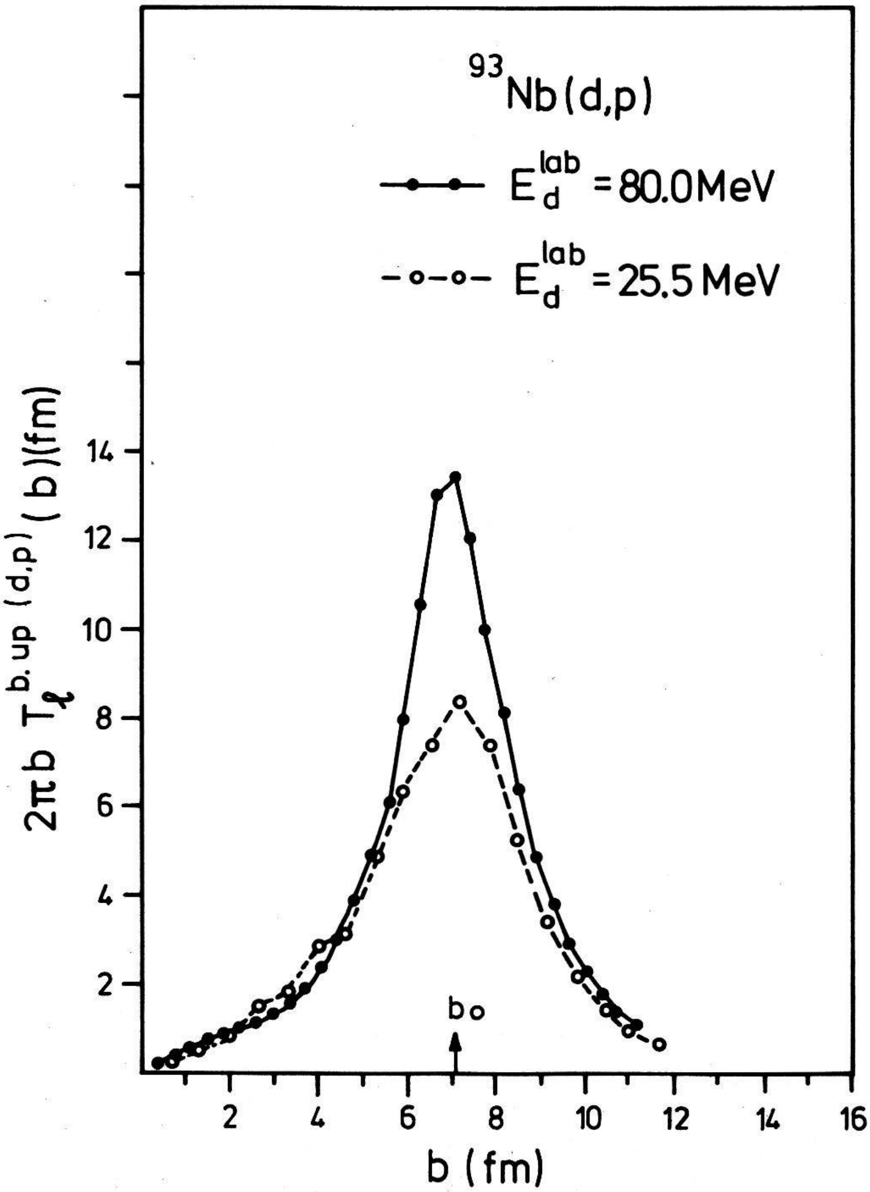

Figure 4
Partial break-up cross section $2\pi b T^{\text{b-up}(d,p)}$ as a function of the impact parameter for $E_d = 25$ and 80 MeV deuteron incident on ^{93}Nb. The area under the curves gives the total (d, p) break-up cross section. The impact parameter b_0 denotes the grazing impact parameter.

expected proportionality to $(A^{1/3} + a^{1/3})$ shown by the continuous line, is rather well fulfilled. Even for the simplest kind of projectile with which the break-up process can be studied, the deuteron, there are more modes of fragmentation than we have studied up to now: it is also possible that the neutron interacts inelastically with the target nucleus. This cross section can also be calculated quite analogously to the (d, p) case. If we would now add up both the (d, p) and (d, n) inclusive cross sections in order to obtain the total deuteron break-up probability, we would count the elastic part twice. Therefore, we have to define the total break-up probability as follows:

$$T_l^{\text{b-up},d} = T_l^{\text{b-up}(d,pn)}(\text{el}) + T_l^{\text{b-up}(d,p)}(\text{inel}) + T_l^{\text{b-up}(d,n)}(\text{inel}). \quad (21)$$

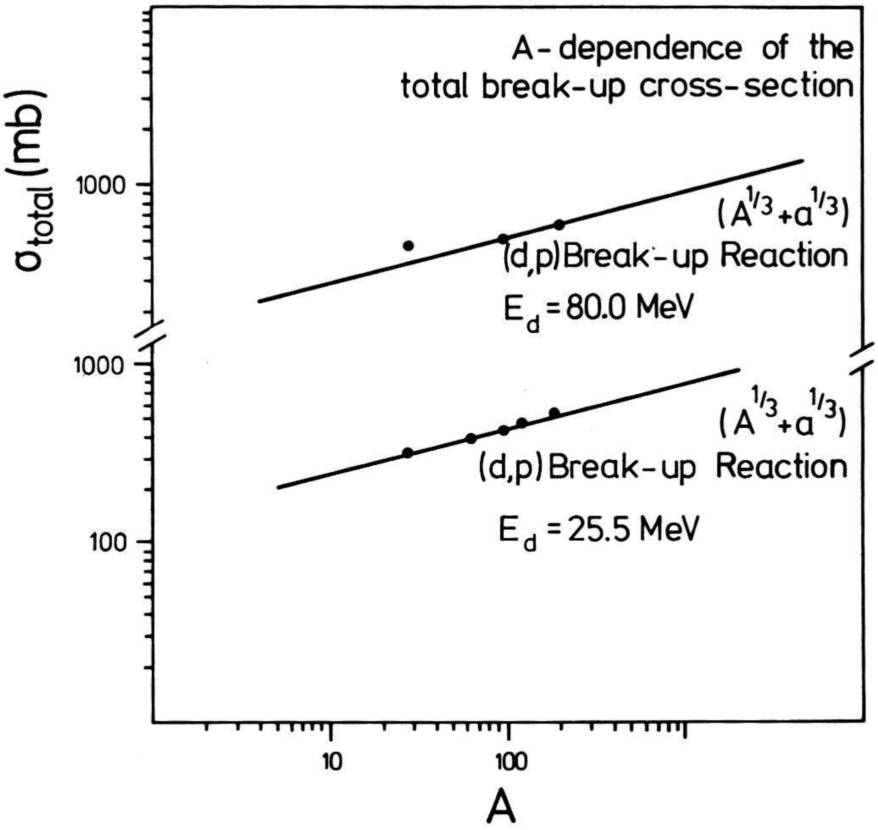

Figure 5
Total (d, p) break-up probabilities as a function of target mass A. The continuous lines are proportional to $(A^{1/3}+a^{1/3})$.

As an example we show in Fig. 6 the deuteron break-up of $E_d = 25.5$ MeV on ^{181}Ta. On the left hand side we show the break-up probabilities and on the right hand side the (partial) break-up cross section. We can see that the (d, p) and (d, n) break-up probabilities are similar in shape and absolute magnitude. The elastic (d, pn) break-up probability is much smaller and shows also a different behaviour as a function of the impact parameter. It is responsible for the long range part of the absorption. This is due to the long range Coulomb force. In this mode, there is no excitation of the target nucleus involved. It is interesting to compare the situation to the corresponding one for high energy fragmentation: for peripheral collisions, inelastic modes dominate strongly, for even more distant collisions Coulomb dissociation is the reaction mechanism, the target is not excited (monopole-multipole interaction is dominant), i.e. those fragmentations are of the elastic type.

To prove that it is really the Coulomb force which is responsible for the long range part of the break-up probability, we have performed model calculations, where we switched off the Coulomb force between the deuteron and the proton with the target nucleus. This is shown in Fig. 7. The dashed line shows the calculation where the nuclear interactions of the d, p and n with the target nucleus is switched off. The dashed–dotted line shows calculations where the Coulomb interactions are switched off, the full curve shows the case where all interactions are switched on. We can see now explicitly that the Coulomb force determines the break-up probability for the high partial waves. The analogous situation is shown for ^{27}Al as a target in Fig. 8. In this case, of course, the

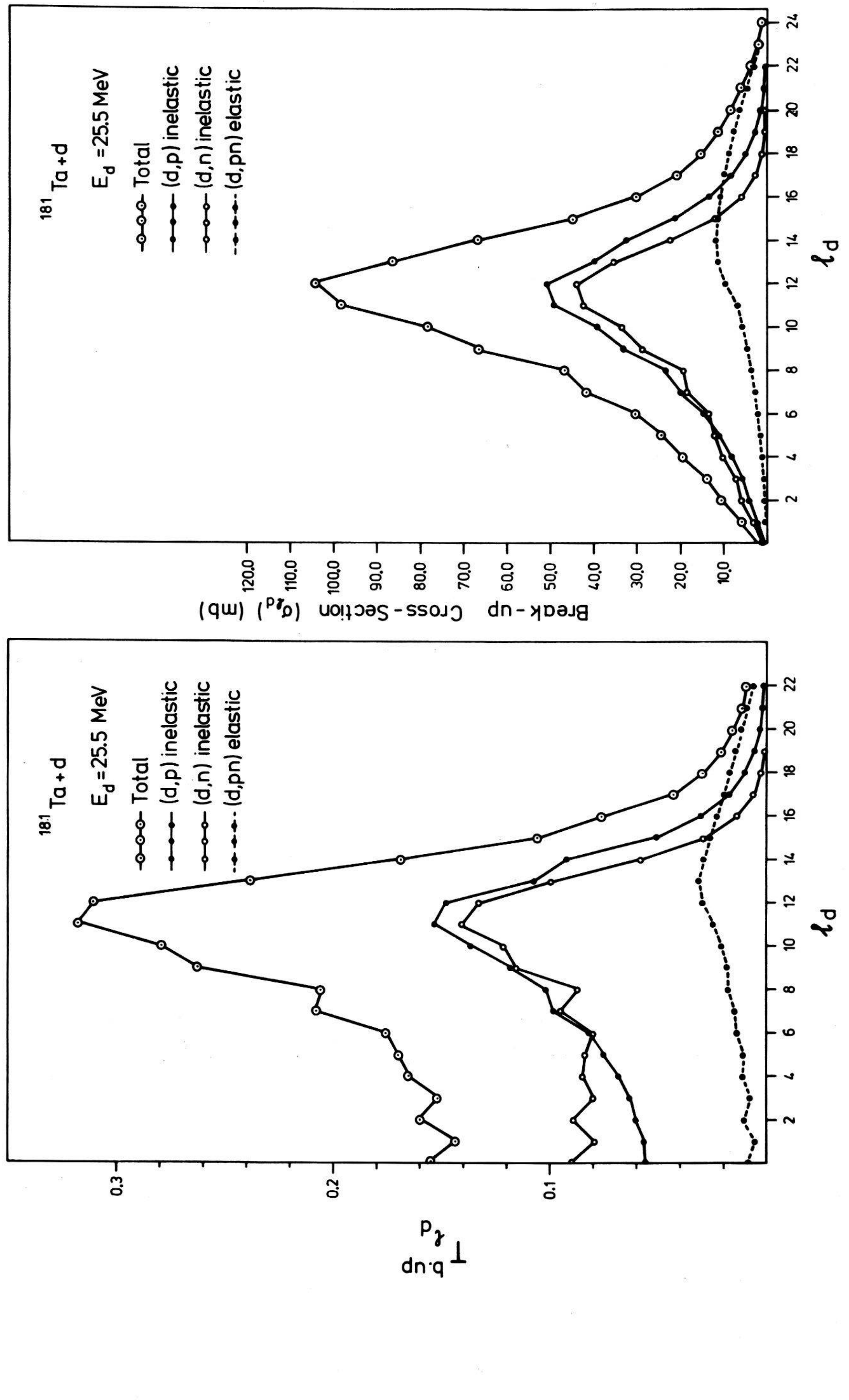

Figure 6
Break-up probabilities (left hand side) and partial break-up cross sections (right hand side) for 25.5 MeV deuteron incident on ^{181}Ta.

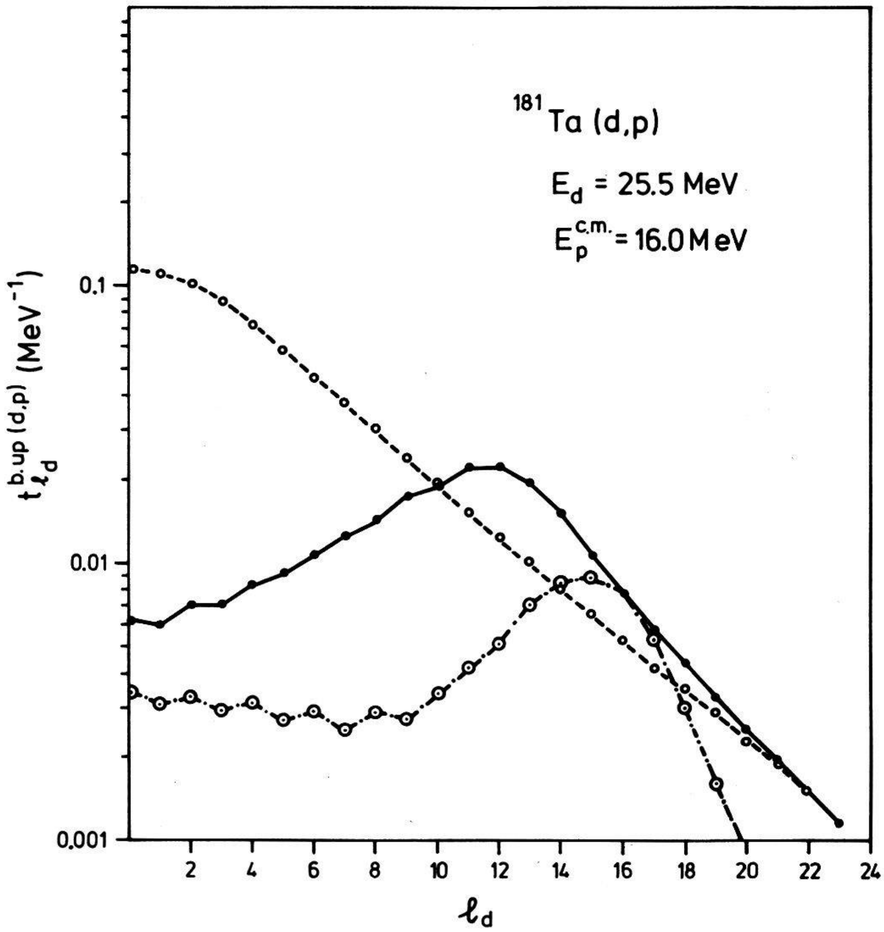

Figure 7
(d, p) break-up probability for 25.5 MeV deuterons on ^{181}Ta. For this model calculation we do not perform the integration over the proton energy spectrum, but take a representative proton energy, $E_p = 16$ MeV in the peak region. For the continuous line, all interactions are included, for the dashed line, the nuclear interactions of d, p and n with the target are switched off, for the dashed–dotted line, the corresponding Coulomb forces are switched off.

Coulomb effect is very small. Even there we can see the long-range character of the Coulomb force, as expected.

In Fig. 9 we show a model calculation where only the neutron target interaction is switched off. We can see that it is responsible for a large part of the break-up probability in the peak region, as it is expected. It becomes unimportant for the very high partial waves, where the Coulomb forces dominate.

(ii) *Break-up of the* ^{3}He *particle*

Although the deuteron break-up is the classical testing ground, we can extend our theory to the break-up of any projectile. Let us now study the break-up of the next, more complicated nuclear particle, the ^{3}He. In this case one can study (^{3}He, d), (^{3}He, p), and (^{3}He, n) inclusive spectra. The simplest one is the (^{3}He, d) spectrum, in this case the only possible mechanism is the transfer of a proton. For the (^{3}He, p) spectrum, more possibilities can occur: again we have a ^{3}He $\rightarrow p + d$ break-up, where the deuteron can interact elastically or inelastically. Our calculations are based on this mechanism. There are also other modes, which

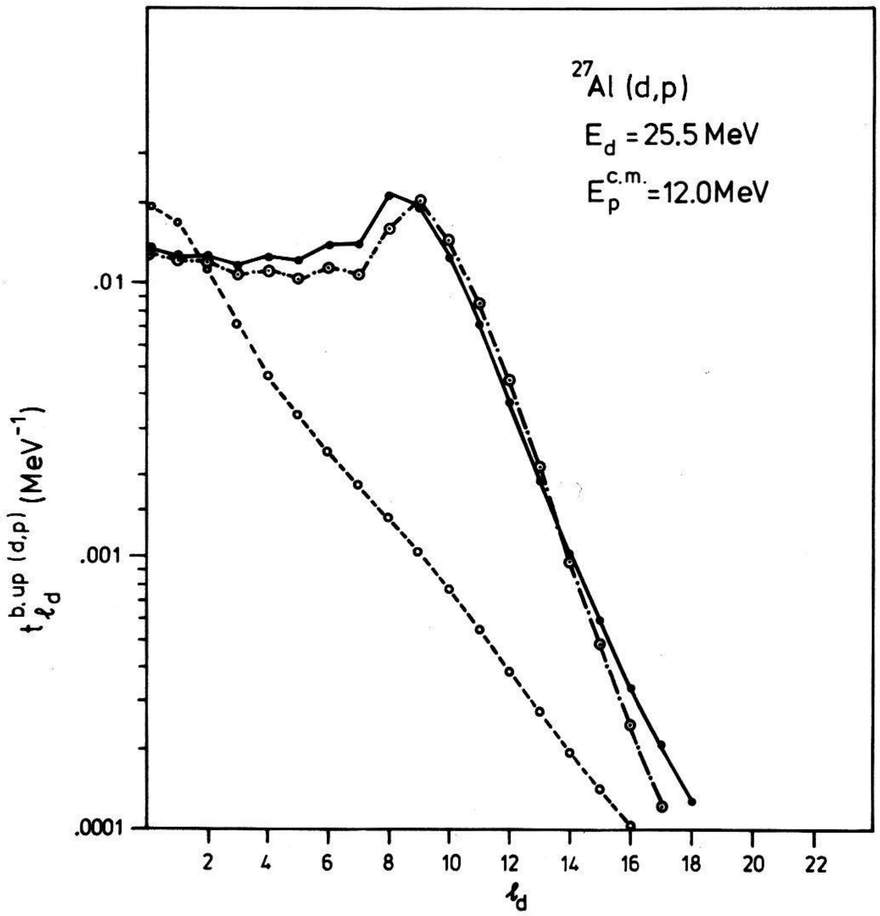

Figure 8
The same as in Fig. 7 but with ^{27}Al as a target.

we have not taken into account here: neutron transfer with subsequent decay of the diproton system into 2 protons, or a proton transfer with the formation of unbound $n-p$ pairs also.

In Fig. 10 we show the break-up probabilities for (^{3}He, p) and (^{3}He, d) on Ni. The optical potentials for ^{3}He–Ni interaction are taken from Ref. 27 and those for deuteron and the proton channels are taken from the compilation of Perey and Perey [28].

(iii) *Break-up of the α-particle*

Let us treat now the α-particle break-up, where experimental results have recently become available [7, 29]. Of course, with more complex particles more complicated decay modes may occur. For instance, a given projectile may disintegrate into two parts, where one (or both) are unbound and will decay subsequently. We cannot solve the problem in its general form here.

Let us now deal with the (α, ^{3}He) process, where such complications are not present. In these calculations the optical model potential parameters for the α-Ni interaction have been taken from Refs. 7 and 10 for the α-energy of 172.5 MeV, and those for 140 and 100 MeV have been taken from Refs. 30 and 31, respectively. The ^{3}He-potential has been taken from Chant et al. (e.g. see Ref.

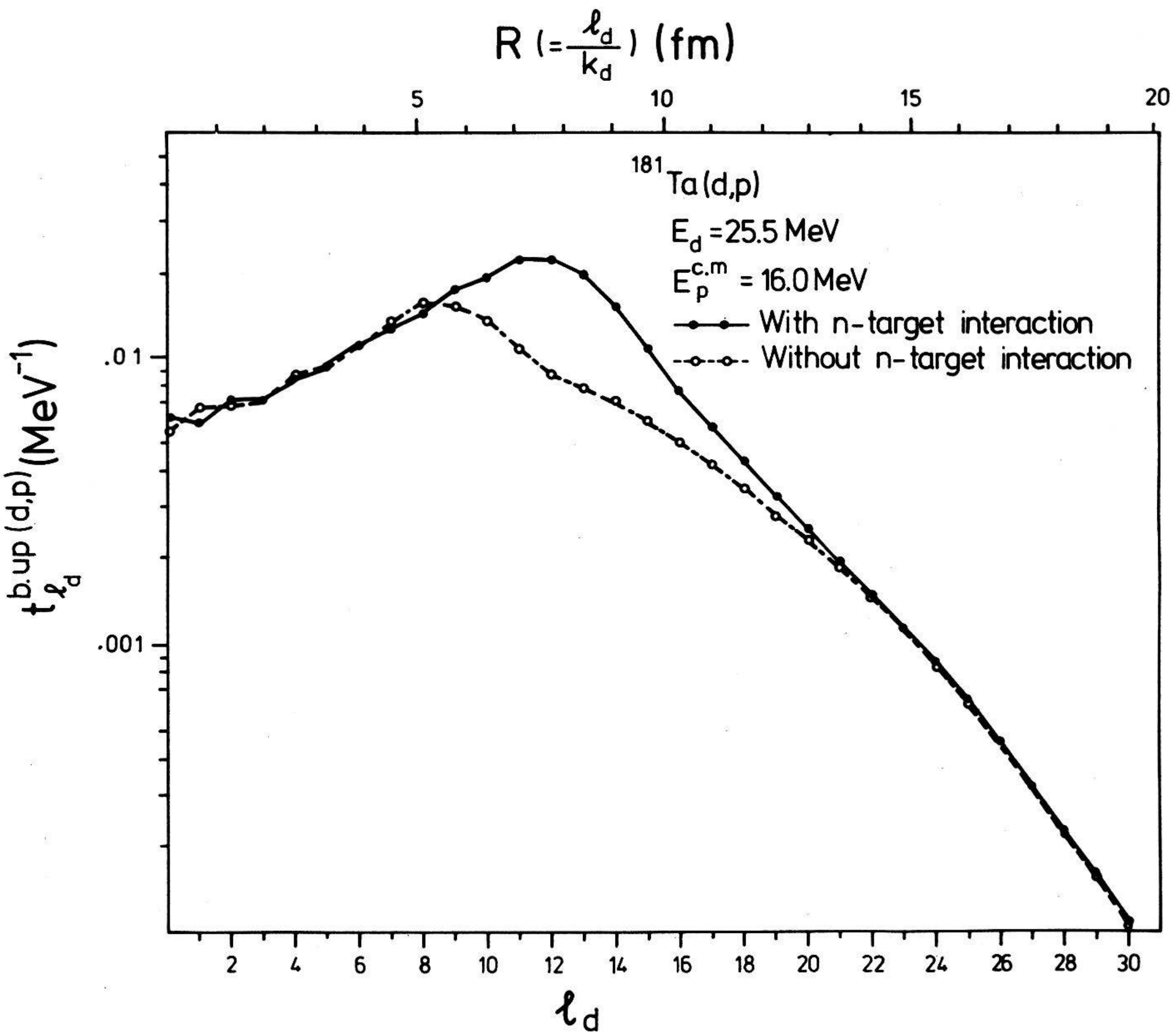

Figure 9
The same as in Fig. 7, for the dashed line, however, only the nucleon-target interaction in the final state is switched off.

10) and from Goldberg and Smith [32]. We use standard Becchetti–Greenlees potentials for the neutron target interaction.

In Fig. 11 we show our results for the (α, ^{3}He) break-up on Ni for two different α-energies. The optical model transmission coefficients show the expected smooth cut-off behaviour; the break-up probability is peaked around grazing l-values, which will increase with increasing energy.

In Fig. 12 we show the A-dependence of the (α, ^{3}He) break-up reaction at $E_\alpha = 140$ MeV. Quite similar to the situation for the deuteron break-up, the peak moves to higher l-values just as the grazing angular momentum increases with the nuclear radius.

In Fig. 13 we show the impact parameter dependence of the elastic and inelastic (α, ^{3}He) break-up modes. Contrary to our previous example of the deuteron break-up, Coulomb effects play a minor role here. Thus, we do not see in our calculation a long range elastic component. It can also be seen that the elastic mode has a much smaller absolute magnitude than the inelastic one, as we already know.

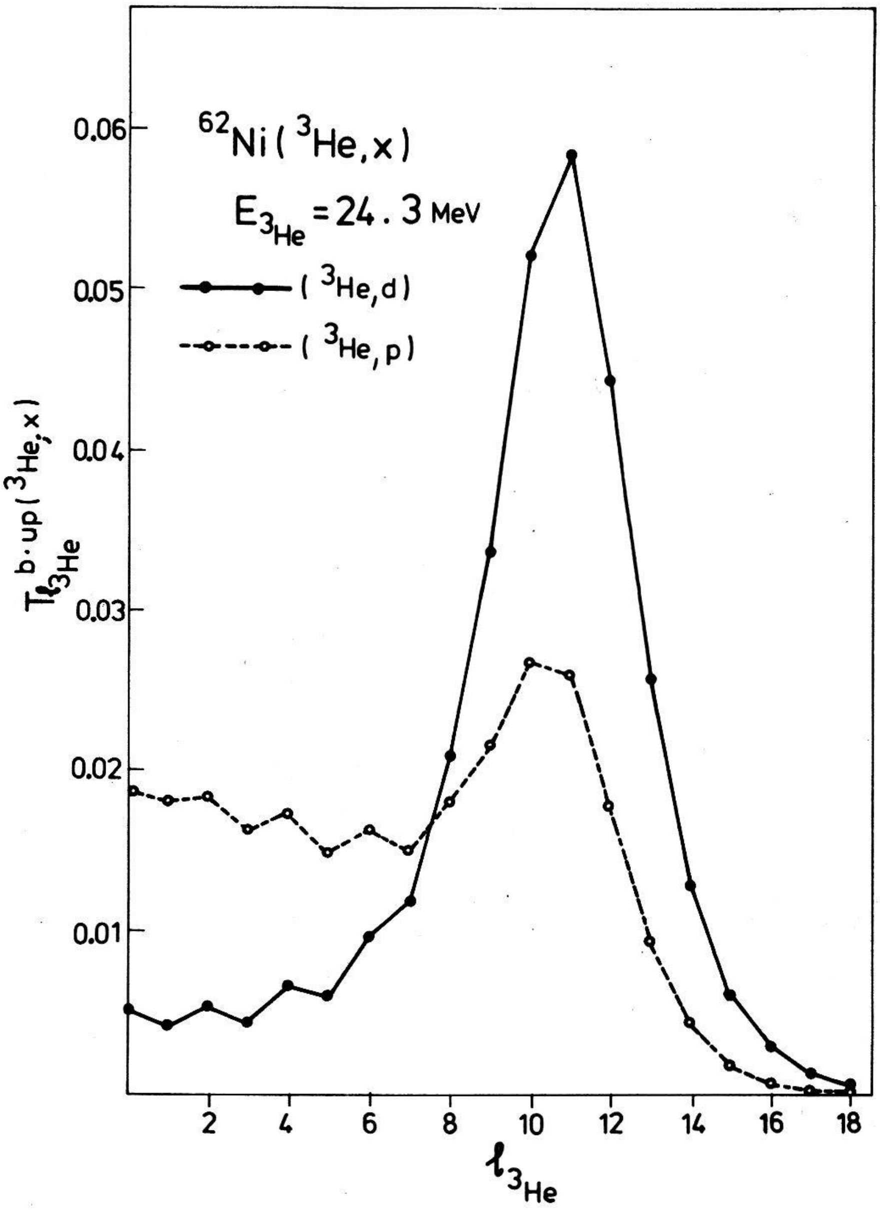

Figure 10
(^{3}He, d) and (^{3}He, p) break-up probabilities.

(iv) *Gross properties*

From the numerical calculations presented above, we can now try to extract some 'gross properties' of the break-up reaction. These calculations suggest a simple parametrization of the break-up process. We introduce the parametrization (see also Ref. 33)

$$T^{\text{b-up}(a,b)} = \beta(E_a)\exp-\frac{(l-l_0)^2}{(\Delta l)^2} = \beta(E_a)\exp-\frac{(b-b_0)^2}{(\Delta R)^2} \tag{22}$$

where $b = l/q_a$, $b_0 = l_0/q_a$ and $\Delta R = \Delta l/q_a$. The factor β describes the strength of the break-up process, which is expected to show a saturation for sufficiently high incident energies (limiting fragmentation [8]) and should vanish for incident energies comparable to the binding energy of projectile a. We relate b_0 to the size

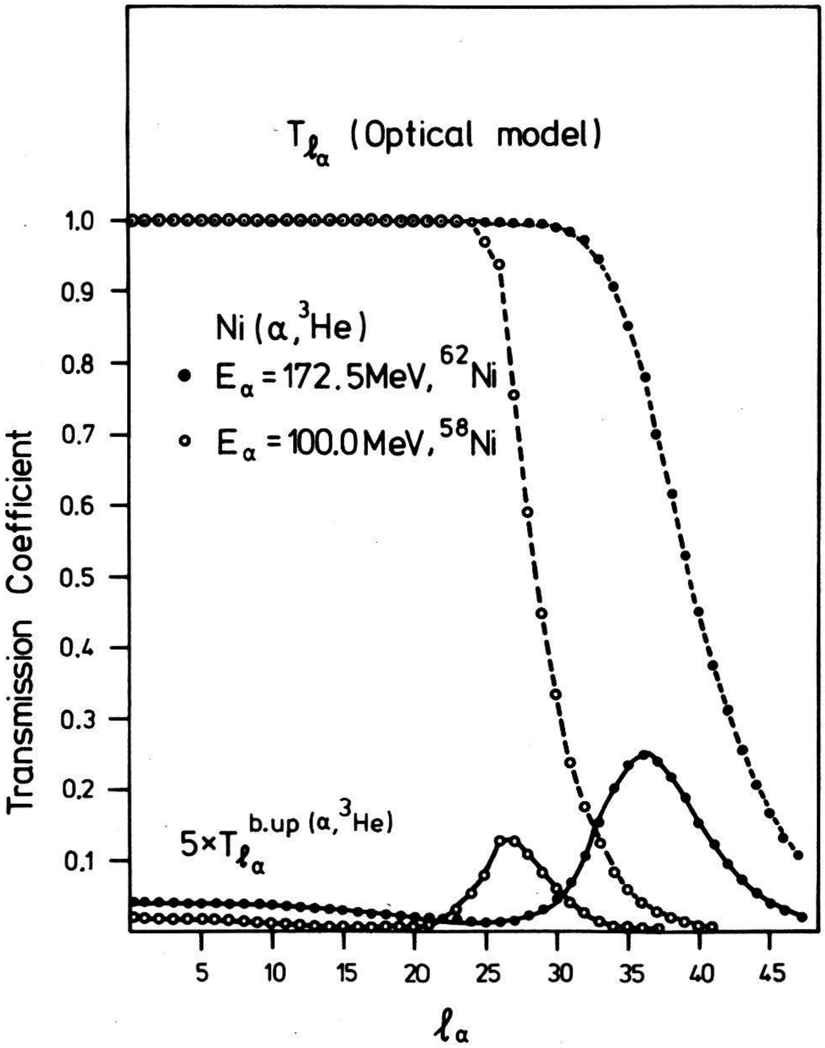

Figure 11
Break-up probabilities and transmission coefficients for α particle incident on Ni isotopes at $E_\alpha = 100$ and 172.5 MeV.

of the target and projectile by $b_0 = r_0(A^{1/3} + a^{1/3})$. The value for r_0 is of the order of 1.2 fm and it is remarkably independent of A and E_a (see Ref. 33). The quantity ΔR depends only on the projectile a and the fragment b.

These break-up probabilities, calculated in the framework of modern direct reaction theories, correspond very closely to those which one expects from the old classic and intuitively very appealing model of Serber [34]. It can roughly be explained as follows: consider the nucleus as a completely black (absorbing) sphere with radius R. If a projectile with the dimension of the order of ΔR hits the nucleus too closely ($b < R - \Delta R$) it gets completely absorbed (no break-up). If the impact parameter or the collision is too large ($b > R + \Delta R$) there is no break-up either (disregarding Coulomb-effects [35, 36]). In peripheral collisions, however, there is a chance that part of the projectile hits the nucleus and gets absorbed, whereas the remaining part continues to fly by practically undisturbed with the velocity it had before. This velocity consists of the beam velocity of the projectile and the Fermi motion. This old Serber [34] model is also very much in

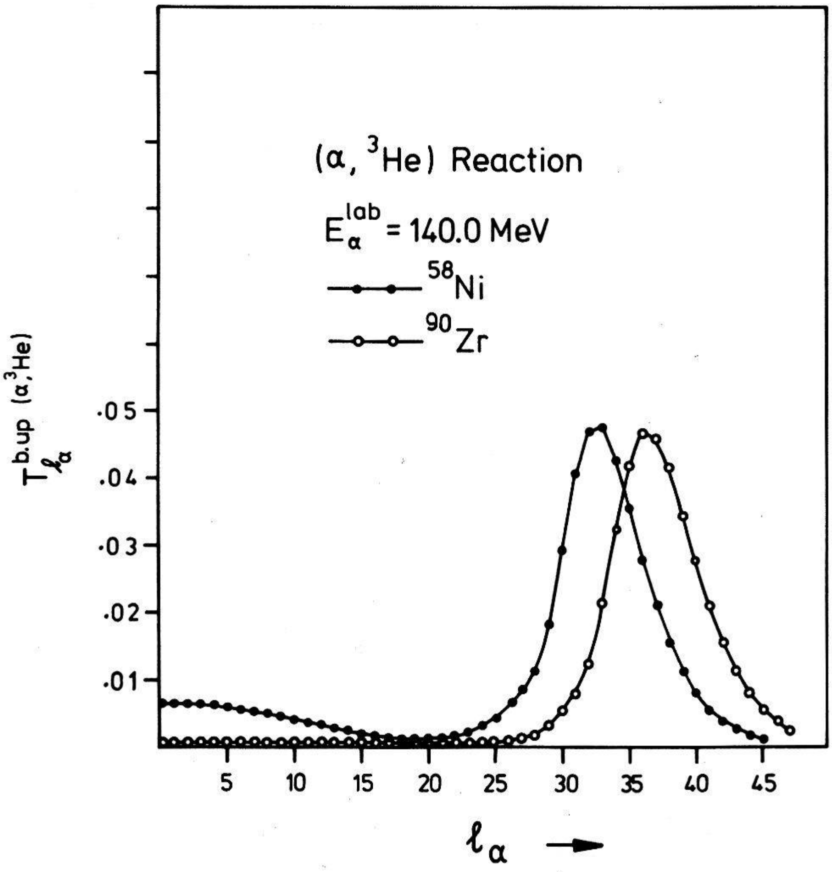

Figure 12
(α, ^{3}He) break-up probability for $E = 140$ MeV and two different target nuclei, ^{58}N and ^{90}Zr.

agreement with the recently discovered fragmentation processes in relativistic heavy ion collisions [8].

With the simple parametrized form of $T_l^{\text{b-up}(a,b)}$ equation (22) we can directly calculate the total (a, b) break-up probability

$$\sigma_{\text{total}}^{\text{b-up}}(a, b) = 2\pi\beta \int_0^\infty dbb \exp - \frac{(b-b_0)^2}{(\Delta R)^2} \cong 2\pi^{3/2}\beta b_0 \, \Delta R. \tag{23}$$

This formula has a close analogy to the formula given by Serber [34]. It also shows the factorization property found in heavy ion fragmentations [8]. The total cross section factorizes into a part ΔR, which depends only on the projectile and fragment, and 'target factor' $b_0 = r_0(A^{1/3} + a^{1/3})$ with $r_0 = 1.2$ fm, which is directly related to the size of the colliding systems.

4. Conclusions

In this paper we have studied fragmentation, one of the most important reaction mechanisms for peripheral collisions of nuclei on nuclei. Especially important is the inelastic fragmentation mode. It accounts for the continuous transitions from bound to unbound state stripping.

In our numerical calculations we have found a simple parametrization of break-up cross sections, which seems to be of universal validity. However, it

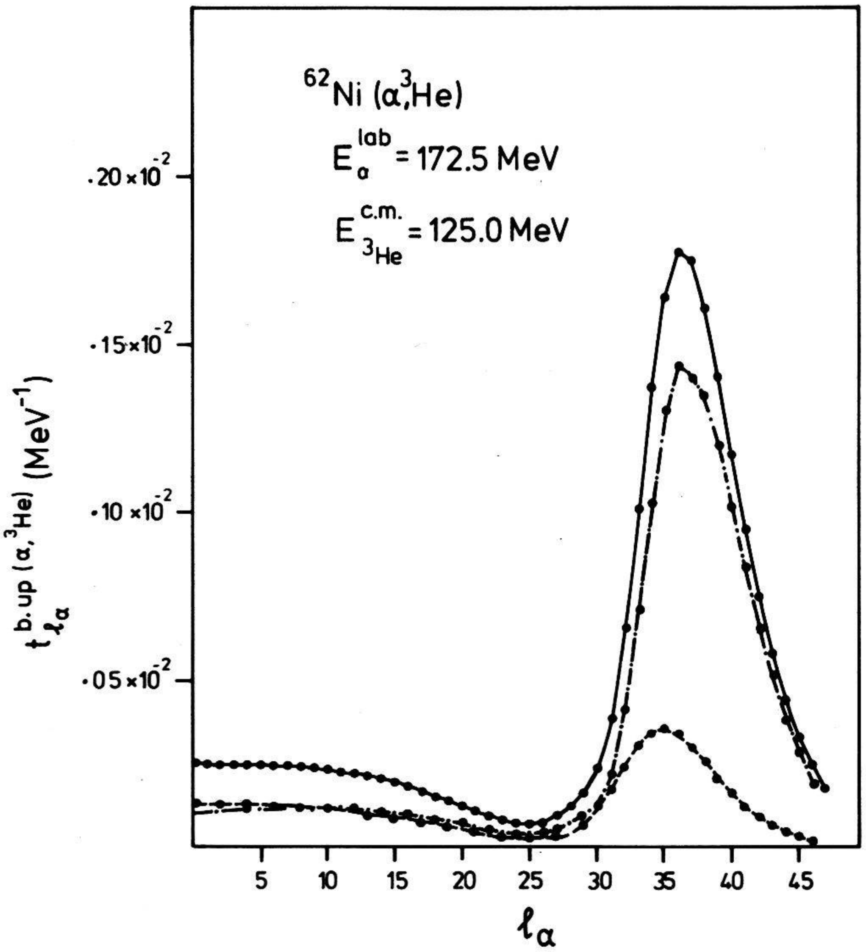

Figure 13
Elastic and inelastic (α, ^{3}He) break-up probability for $E_\alpha = 172.5$ MeV on ^{62}Ni. For the ^{3}He particle we choose a representative energy in the peak region $E_{^3\mathrm{He}} = 125.0$ MeV. The dashed line represents the elastic break-up, a dashed–dotted line the inelastic break-up and the full line represents the total inclusive cross section.

remains to be seen how the empirically found properties can be derived directly from our theory.

Our results show that the break-up is a dominant source of absorption in the surface region. Thus it is especially important to consider break-up effects in the theoretical calculation of optical model potentials.

Appendix A. Threshold behaviour of the break-up cross section for the charged particle transfer

Here, we shall specialize our discussion for the case of the proton transfer. However, the formulation can be carried over to any other charged particle.

Starting from equation (x. 42) of Ref. (37) it is easy to show that the phase shift for the scattering of the proton from a target has the following form in the limit $E_p \to 0$,

$$\delta_{l_p} \xrightarrow[E_p \to 0]{} (2l_p+1)^2 C_{l_p}^2 \frac{1-l_p-\hat{q}}{\hat{q}+l_p} (k_p R_0)^{2l_p+1} \tag{A.1}$$

where

$$C_{l_p} = 2^{l_p} e^{-1/2\pi\eta} \frac{\Gamma(l_p+1+i\eta)}{(2l_p+1)!} \tag{A.2}$$

with η being the Coulomb parameter. R_0 in equation (A.1) represents the range of the nuclear potential and $\hat{q}$ is defined as

$$\hat{q} = \left(\frac{\kappa\, du_{l_p}(\kappa)}{d\kappa}\right)_{\kappa=k_pR_0} \tag{A.3}$$

where $u_{l_p}(\kappa)$ is the wave function for the proton.

In equation (7) for the elastic break-up cross sections the quantities depending on the proton energy are the form factor occurring in the T-matrix and the wave vector for the proton occurring in the phase space factor. The asymptotic form of the form factor is

$$f_{l_p}(E_p, r) \sim \frac{1}{k_pr} |F_{l_p}(k_pr) + \tfrac{1}{2}(e^{2i\delta_{l_p}-1}) H^+_{l_p}(k_pr)| \tag{A.4}$$

where $H^+_{l_p} = G_{l_p} + iF_{l_p}$ with F_{l_p} and G_{l_p} being the regular and irregular Coulomb wave functions, respectively.

With the help of the threshold property ($E_p \to 0$) of the Coulomb wave functions and equation (A.1) one can show that the modulus square of the form factor (equation (A.4)) is

$$|f_{l_p}(E_p, r)|^2_{E_p\to 0} \simeq B^2_{l_p} k_p^{2l_p} \left(\frac{1}{r}\right)^{2l_p-2} \tag{A.5}$$

where

$$B_{l_p} = (2l_p+1) C_{l_p} \left(\frac{1-l_p-\hat{q}}{l_p+\hat{q}}\right) R_0^{2l_p+1} \tag{A.6}$$

It is easy to show with equations (A.6) and (A.1) that the elastic break-up cross section near the threshold behaves as

$$\frac{d^2\sigma(\mathrm{el})}{d\Omega\, dE} \propto k_p^{2l_p+1}$$

which is zero at $E_p = 0$, even for S-waves.

To investigate the threshold behaviour of the inelastic break-up cross section, we rewrite the equation (13) for the inelastic break-up cross sections as

$$\frac{d^2\sigma(\mathrm{inel})}{d\Omega\, dE} = \rho(\text{phase-space}) \sum_{l_pm_p} (1-|S_{l_pl_p}|^2)\, |T^+_{l_pm_p}|^2 \tag{A.7}$$

where the T-matrix $T^+_{l_pm_p}$ contains $H^+_{l_p}$ as the form factor. Using equation (A.1) it can be shown that

$$1-|S_{l_pl_p}|^2 = 4(2l_p+1)^2 C^2_{l_p} R_0^{2l_p+1}\, \mathrm{Im}\left(\frac{1-l_p-\hat{q}}{\hat{q}+l_p}\right) k_p^{2l_p+1} \tag{A.8}$$

With the help of the threshold property of the Coulomb functions H^+_{lp}

and the additional k_p-term present in the phase factor one gets

$$\frac{d^2\sigma(\text{inel})}{d\Omega\, dE} \propto \text{Im}\left(\frac{1-l_p-\hat{q}}{\hat{q}+l_p}\right)R_0^{2l_p+1} \tag{A.9}$$

The right hand side of equation (A.9) is in general a finite quantity at the threshold, hence the inelastic cross section, unlike the elastic one, does not vanish at the threshold.

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