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DISORDER AND INTERACTION

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In trying to describe from a theoretical point of view the combined effect of the interaction and the disorder, it is soon recognised that the main difficulty of the problem lies in the absence of a simple description in terms of an effective Hamiltonian, as a preliminary step to build up a standard renormalization procedure.

When the interaction is not present[(1)], this difficulty has been overcome by different approaches. One can, for example, map the problem into a known field theoretical problem, as it has been done[(2)] with the use of the non linear σ-model, or alternatively one can look for additional constraints on the theory stemming from suitable invariance requirements.

In a previous paper[(3)], by imposing general gauge invariance requirements, we have shown that the localization transition of a free fermion gas in the presence of impurities can be adequately described in 2+ϵ dimensions within the framework of the usual many-body theory. In this way the linear response theory to an external electromagnetic field was directly renormalised. The conductivity σ, which shows logarithmic singularities in two dimensions[(4)], has been connected to the diffusive part of the density-density response function. This last quantity is defined as

$$(1)\quad (K_{00})_{+-} = -i\Omega\chi_d(k,\Omega) = -i\frac{\Omega}{2\pi}\,\mathrm{F.T.}\,\overline{G_+(x,x';\omega+\Omega)\,G_-(x',x;\omega)}$$

where k and Ω are the wave vector and the frequency respectively and G_+

and G- are the retarded and advanced Green functions. The average is taken over the impurity distribution. At small k and Ω , χ_d is shown[3] to have the form

$$\chi_d = \frac{N_o}{-i\Omega + D_d k^2} \tag{2}$$

with the full diffusion constant D_d related to the conductivity σ by the relation $\sigma = N_o D_d$, N_o being the single particle density of states for the free system. $\chi_d(k,\Omega)$ plays then the role of the propagator of an effective field theory having the inverse conductance $g^{-1} = \frac{1}{(2\pi)^2\sigma}$ as the only effective coupling. Its renormalisation leads to a one parameter scaling theory and a 2+ϵ expansion.

The situation appears to be considerably more difficult when the interaction among the Fermi particles is turned on. In this case indeed no mapping into a field theoretical model exists and the task of generalizing the "free" effective Hamiltonian to include the interaction looks rather hard. One should mention that a first step in this direction has been recently[5] made for the particular case when a magnetic field is present. The magnetic field introduces a considerable simplification of the diagrammatic structure.

Recently[6] we have extended to the interacting case the general approach pursued for the non interacting system. The gauge invariance requirements even though less stringent, turn out to be a good guiding criterion for the selection of diagrams at any order in perturbation theory.

It is known[7] that in the presence of a static short range interaction V(q) the single particle density of states N becomes also singular and the conductivity σ acquires additional logarithmic singularities . The renormalization procedure becomes therefore much more involved since additional relevant effective couplings are present.

On the assumption that the interaction introduces only one additional effective coupling proportional to N, a two parameter semi-phenomenological scaling theory has been proposed for a three dimensional system[8]. It has been also implicitely assumed that the density of states appearing in the screening length and in the Einstein relation renormalizes as the single particle density of states N.

By means of gauge invariance and the related Ward Identity (connecting the full electromagnetic vertex of the linear response Kernel to the self-energy), both σ and N are expressed in terms of the density-density response function K_{oo}.

$$\sigma(\Omega) = -\lim_{k\to 0} \frac{\Omega}{k^2} \operatorname{Im} K_{oo}(k,\Omega)\,;\quad N(0) = \lim_{\Omega\to 0} \Big(K_{oo}(0,\Omega)\Big)_{+-} \tag{3}$$

Moreover the thermodynamic density of states $\frac{\partial n}{\partial \mu}$ (where n is the particle density and μ is the chemical potential), being the response to a static and homogeneous external field, is given by:

$$\frac{\partial n}{\partial \mu} = -\lim_{k\to 0} K_{oo}(k,0) \tag{4}$$

The screening bubble also is associated to the irriducible (for cutting an interaction line) part of K_{oo}.

Since all the relevant physical quantities are associated to K_{oo}, we can in our model check the assumptions of the semi-phenomenological theory by a direct evaluation of K_{oo} for both k and Ω different from zero.

A gauge invariant calculation at first order in the interaction and $\frac{1}{g}$ shows that $\frac{\partial n}{\partial \mu}$ as given by eq.(4) is finite and, for $D_0 k^2 < \Omega$, K_{oo} assumes the form

$$K_{oo}(k,\Omega) = \frac{-N_0 D_R k^2}{-i\Omega + D_R k^2} \tag{5}$$

All the logarithmic singularities are reabsorbed in the renormalisation of the diffusion constant D_0 according to the following known expression for $d = 2$:

$$D_R = D_0\left[1 + \frac{1}{g}\ln\frac{\Omega}{\Lambda^2} + \frac{1}{g}(V_1 - s V_2)\ln\frac{\Omega}{\Lambda^2}\right] \tag{6}$$

Λ^2 is a suitable cut-off, s is the spin multiplicity. V_1 and V_2 refer to the small and large k limit of the interaction and are here associated to the exchange and the Hartree contribution in the particle-hole channel[9] respectively. Due to the first of equations (3) and to eq.(5) the density of states appearing in the Einstein relation remains finite and coincides at this order with the one given by eq.(4). The single particle density of states N as given by the second of the equations (3) is instead singular in terms of V.

A two parameter scaling theory (even though with different scaling behaviour from that of the semi-phenomenological theory) could still follow. In order to investigate this question we have to go to higher order in perturbation theory.

We concentrate on the perturbative analysis of N. It can be shown that its Fock terms at first order in V and any order in $\frac{1}{g}$ can be obtained by the knowledge of χ_d for the system in the absence of interaction

$$N(\omega)_{Fock} = N_0 + \frac{1}{\pi}\,\mathrm{Im}\, i\int_{\omega}^{\Lambda^2} d\Omega \int \frac{d^d k}{(2\pi)^d}\, V(k)\, \frac{\partial}{\partial\Omega}\chi_d(k,\Omega) \tag{7}$$

The analog parallel part $\chi_{\parallel d}$, defined as

$$\chi_{\parallel d}(k,\Omega) = \mathrm{F.T.}\ \frac{1}{2\pi}\ \overline{G_+(x,x;\omega+\Omega)\, G_-(x',x';\omega)}$$

generates the Hartree terms of N.

From the diagrams of χ in the absence of interaction, we generate in this way all the diagrams of N at first order in V and second order in $\frac{1}{g}$, to obtain the leading logarithmic contributions in two dimensions:

$$N = N_0\left\{1 + \frac{1}{g}\left[(V_1 - s V_2)\ln\frac{\Omega}{\Lambda^2} - \frac{(V_1 - s V_2)}{g}\ln^2\frac{\Omega}{\Lambda^2} + \frac{(s V_1 - V_2)}{g}\ln^2\frac{\Omega}{\Lambda^2}\right]\right\} \tag{8}$$

The last term of eq.(8) is somehow unexpected since in it the role of V_1 and V_2 is interchanged for the Fock and Hartree diagrams. V_1 and V_2 do not appear as the linear combination $V_1 - sV_2$ as in the first order terms. Having a three parameter theory the results achieved so far are no more sufficient to perform the renormalization of the theory[10].

In the special case where no crossing of impurity lines in any form is allowed, the coupling $\frac{1}{g}$ is no more present by itself even in σ and all the $\ln^2$ terms proportional to $\frac{V}{g^2}$ disappear in N. The coefficient of the $\frac{V^2}{g^2}\ln^2\frac{\Omega}{\Lambda^2}$ term is also shown to vanish. The vanishing of all the $\ln^2$ contributions in N is then enough in this particular case to renormalise the theory according to a one parameter scaling. Both the single particle density of states and the conductivity vanish at the transition as a function of the frequency with a critical index equal to $\frac{d-2}{2}$.

In the general case at any given order in $\frac{1}{g}$, infinite resummations in the potential can be carried out. These resummations have to be considered if we want our model to be realistic. This analysis together with a search of the minimal requirements necessary to renormalise the theory is now under progress and will be presented elsewhere.

References and remarks

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(9) We do not consider here the effect of both the exchange and the Hartree contributions in the particle-particle channel(7) since they can be resummed up to zero. See B.L.Altshuler, A.G.Aronov, A.I.Larkin and D.E. Khmelnitzkii, Sov.Phys. JETP 54, 411 (1981) and Ref.10.

(10) Recently G.S.Grest and P.Lee (preprint) approached this problem by matching the perturbative expressions of three physical quantities to a scaling behaviour. In addition to the single particle density of states they consider the conductivity and the spin susceptibility. Their perturbative expression for N however does not contain the last term of eq.(8). We thank the authors for communicating us their results before publication.