Strain coefficient of resistivity in thin metallic films

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Strain coefficient of resistivity in thin metallic films

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Abstract. General Fuchs–Sondheimer expressions for the strain gauge coefficients of thin metallic films having different specular coefficients at the two surfaces have been derived by incorporating all thermal expansion effects. New expressions predict large deviations in the temperature coefficient of resistivity and γ-values specially for low values of the ratio of the film thickness to the mean free path.

Introduction

The strain coefficient of resistivity has been investigated theoretically as well as experimentally by many workers [1–8]. The theoretical expressions have usually been derived on the basis of either Fuchs–Sondheimer model or Mayadas–Shatzkes theory of grain boundary resistivity. Verma and Sharma [3] have derived F–S expressions for the strain coefficient of resistivity, where they have considered the specular reflection of electrons from both the surfaces of the film, but have overlooked the thermal expansion effects. Metallic thin films deposited onto different substrates by various methods, are known to be in a state of stress. A contribution from the expansion of the film thickness and the mismatch between the expansion of the film and the substrate (in the case of supported films) may also add to the strains in the films [9], which in turn may substantially affect the resistivity and hence its strain coefficient. In the present paper an attempt has been made to derive some more general relations for the strain coefficient of resistivity by incorporating the entire expansion effects. These expressions are seen to be more general than those previously derived by Verma and Sharma [2, 3].

Theory

The Fuchs–Sondheimer function has been extended to include different surface properties as [3]:

$$f(k, p_1p_2) = \frac{\rho_0}{\rho_f} = \frac{(1-p_1)(1-p_2)}{(1-\sqrt{p_1p_2})^2} f(k, \sqrt{p_1p_2}) + \left[1 - \frac{(1-p_1)(1-p_2)}{(1-\sqrt{p_1p_2})^2}\right] \cdot f(2k, p_1p_2) \tag{1}$$

where $k(=t/\lambda_0)$ is the ratio of the film thickness to the bulk mean free path; ρ_0 and ρ_f are the bulk and the film resistivity respectively; and p_1 and p_2 are specularity parameters for two surfaces of the film.

Verma and Sharma [3] have suggested that since metallic films are usually in contact with an insulating substrate on one surface, and the other is exposed to a vacuum or an atmosphere or covered with a protective coating, it is worth while to assume different specularity parameters for the two film surfaces. In such a situation the contribution from the thermal expansion of the film thickness and the expansion mismatch can not be neglected. Expressing the expansion coefficient of the film material as follows [9]:

$$\frac{d\ln(d)}{dT} = \alpha_{1f}\left[1+\frac{2\mu_f}{1-\mu_f}\left(1-\frac{\alpha_{1s}}{\alpha_{1f}}\right)\right] \tag{2}$$

and incorporating this relation into equation (1), we obtain following expression for the reduced TCR.

$$\beta_f/\beta_0 = 1-\frac{k}{f(k, p_1p_2)}\cdot\frac{df(k, p_1p_2)}{dk} \times\left[1+\frac{\alpha_{1f}}{\beta_0}\left\{1-\frac{2\mu_f\left(1-\frac{\alpha_{1s}}{\alpha_{1f}}\right)}{1-\mu_f}\right\}\right] \tag{3}$$

where $f(k, p_1p_2)$ is defined by the equation (1).

Tellier [10] has expressed the strain coefficient of resistivity of thin metallic films in terms of TCR as follows:

$$\frac{1}{\rho}\cdot\frac{d\rho}{d\varepsilon} = \frac{1}{\rho_0}\cdot\frac{d\rho_0}{d\varepsilon}+\left(\frac{\beta_f}{\beta_0}-1\right)(\eta-\mu) \tag{4}$$

where the expansion effects have been neglected. Using Verma and Sharma's modified F–S function and expressions (2) and (3) to include entire expansion effects, values of β_f/β_0 have been calculated for supported as well as unsupported films (Table 1). These values of TCR have been used to deduce the values of strain coefficient of resistivity using Tellier's approach.

Table 1.
β_f/β_0 for three different combinations of p-values.

	β_f/β_0								
	Without expansion			With expansion (Unsupported)			With expansion (Supported)		
k	$p_1=0$ $p_2=0$	$p_1=0$ $p_2=\frac{1}{4}$	$p_1=0$ $p_2=\frac{1}{2}$	$p_1=0$ $p_2=0$	$p_1=0$ $p_2=\frac{1}{4}$	$p_1=0$ $p_2=\frac{1}{2}$	$p_1=0$ $p_2=0$	$p_1=0$ $p_2=\frac{1}{4}$	$p_1=0$ $p_2=\frac{1}{2}$
0.1	0.3585	0.4862	0.5853	0.3555	0.4838	0.5853	0.3537	0.4824	0.5823
0.2	0.4595	0.5620	0.6455	0.4570	0.5600	0.6455	0.4555	0.5588	0.6429
0.5	0.5589	0.6432	0.7165	0.5569	0.6416	0.7165	0.5557	0.6406	0.7144
1.0	0.6754	0.7398	0.7985	0.6739	0.7386	0.7985	0.6730	0.7379	0.7969
2.0	0.8022	0.8496	0.8946	0.8013	0.8496	0.8946	0.8007	0.8485	0.8937

Results and discussion

For the sake of comparison with Verma and Sharma's results the values of TCR have been calculated for silver films (suppored as well as unsupported) assuming the known values of the following parameters [11, 12]:

$$\frac{1}{\rho}\cdot\frac{d\rho_0}{d\varepsilon}=2.15;\qquad \mu_f=0.38,\qquad \eta=1.15$$

$$\alpha_{1f}/\beta_0=0.0046,\qquad \alpha_{1s}/\alpha_{1f}=0.5,$$

Since the computed values of the function $f(k, p_1p_2)$ were available only for $0.1\leqslant k\leqslant 2$ and three combinations of p-values, the TCR values could not be calculated for lower values of k, where the expansion effect is expected to become much more prominant [13]. Table 1 lists the calculated values of the TCR ratio for different values of k.

The values of the strain coefficient calculated from equation (4) are found to deviate from those calculated by Verma and Sharma because of an extra contribution due to expansion effects. The percentage deviation in $(1/\rho)\cdot(d\rho/d\varepsilon)$ has been plotted against k for unsupported (Fig. 1) and supported films (Fig. 2). From the

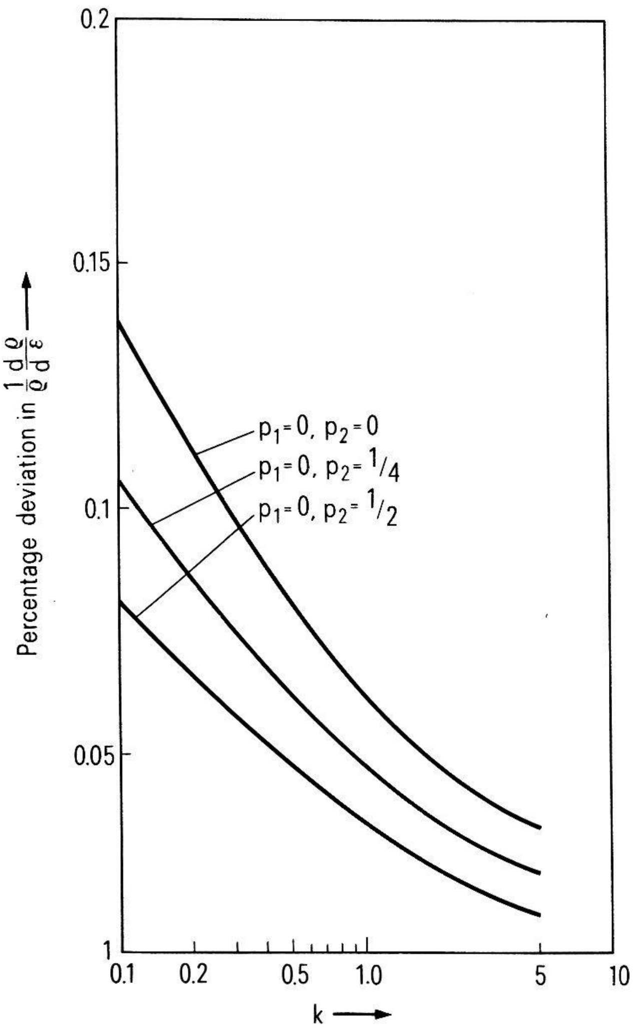

Figure 1
Curves showing percentage deviation in strain coefficient of resistivity of silver films (unsupported) as a function of k for different values of p.

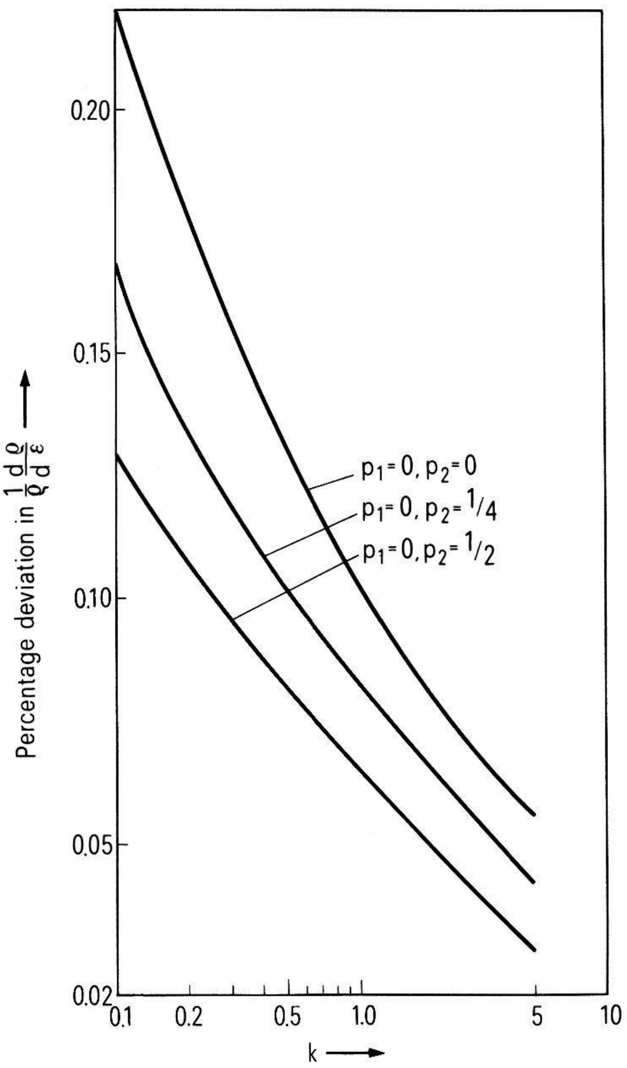

Figure 2
Curves showing percentage deviation in strain coefficient of resistivity of silver films (supported) as a function of k for different values of p.

figures it is clear that the deviation goes on increasing as the k-value is reduced, the rate of increase being more in supported case than in unsupported one. This is expected since in supported films, the expansions mismatch introduces additional resistivity contribution, which becomes more and more sensitive to k as the k-value is reduced. The magnitude of deviation is expected to become remarkably prominant at low temperatures, where very low k-values can be achieved.

Of all the experimental studies carried out on the measurement of strain gauge factor, a quantitative agreement has hardly been obtained. A detailed study in this direction was attempted by Parker and Krinsky [6], who studied the strain dependence of resistivity of a series of films deposited onto different substrates at room temperature. They observed the variation of the γ-values as a function of R_s, the resistance per square. In general, the gauge factor increases with thickness in the low R_s-range, while for higher R_s films the gauge factor is observed to decrease with thickness. As far as this general behaviour is concerned, the present theoretical results do find a qualitative agreement; but a quantitative comparison would remain inevitable unless one can exactly determine the magnitude of specularity at the lower and upper surface of the sample.

Further, Parker et al. have also attempted to see the effect of substrate on gauge factor by depositing Pd films on two different substrates (porcelain and resin). Contrary to our expectation they did not observe any significant substrate effect in the continuous film case. As has already been pointed out by the authors [8], this could be the case if the observations are in higher k-range and secondly if the thermal expansion coefficient and the Poisson's ratio of the two substrates do not differ appreciably in the prevailing conditions.

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