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# Infrared regularization of supersymmetric quantum electrodynamics

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*Abstract.* We have treated the off-shell IR divergences of this supersymmetric gauge theory by introducing a mass parameter $\mu$, which preserves a modified gauge invariance but breaks softly the supersymmetry. We have shown by explicit one-loop's computation that certain Green's functions of gauge-invariant operators are independent of the parameter $\mu$.

## 1. Introduction

It is known that infrared divergences (IR divergences) appear in supersymmetric (SUSY) gauge theories because of a non-integrable singularity of the gauge superpropagator at the origin of the momentum space. In SQED – an extension of ordinary QED – the problem was first solved by giving a mass to the vector superfield, which takes the place of the photon field in QED. (Ref. 4).

The non-abelian case is being treated presently (Ref. 8) such divergences – at least in a pure Yang–Mills model, Ref. 8 – have been removed by using an IR regularization which breaks explicitly and softly the supersymmetry but preserves the gauge invariance. The validity of that regularization has been demonstrated for all orders of the perturbation theory through modified Slavnov identities. Furthermore, it was shown that the final quantities – i.e. Green's functions of gauge-invariant operators did not depend on the SUSY breaking parameter. Instead, such a general argument has not been found yet for the abelian case.

Our purpose is to test the IR regularization mentioned above in a SQED model by computing up-to-second-order contributions to the Green's functions of the gauge superfields' strength (The SUSY extension of the $F_{\mu\nu}$ tensor of QED), and to show that the same regularization works equally well, at least up to second order diagrams.

For conventions and notations, see appendix.

## 2. A supersymmetric extension of QED (SQED)

This model, presented by Wess and Zumino (see Ref. 2), is built up from three superfields belonging to several representations of a graded (super) group.

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It contains a vector supermultiplet $V$, which may be expanded in powers of the Grassmann variables $\theta$ and $\bar{\theta}$ as:

$$V(x,\theta,\bar{\theta}) = C(x) + \theta\chi(x) + \tfrac{1}{2}\theta\theta M(x) + \tfrac{1}{2}\bar{\theta}\bar{\theta}\bar{M}(x) + \tfrac{1}{2}\theta\sigma^{\mu}\bar{\theta}v_{\mu}(x) + \tfrac{1}{2}\bar{\lambda}(x)\bar{\theta}\theta\theta + \tfrac{1}{2}\lambda(x)\theta\bar{\theta}\bar{\theta} + \tfrac{1}{4}\theta\theta\bar{\theta}\bar{\theta}D(x)$$

and two chiral conjugated supermultiplets $S_+$ and $S_-$, which may be expanded, in the chiral representation, as:

$$S_+(x,\theta,\bar{\theta}) = A_+(x) + \theta\psi_+(x) + \theta\theta F_+(x)$$

and

$$S_-(x,\theta,\bar{\theta}) = A_-(x) + \theta\psi_-(x) + \theta\theta F_-(x)$$

Under a local (infinitesimal) gauge transformation, we have $\delta V = i(\Lambda - \bar{\Lambda})$ and $\delta S_{\pm} = \mp ig\Lambda S_{\pm}$, $\Lambda$ being a chiral superfield. The invariant action is given by:

$$\Gamma_{\mathrm{INV}} = 2\int dSW^{\alpha}W_{\alpha} + \int dV(e^{gV}S_+\bar{S}_+ + e^{-gV}S_-\bar{S}_-) - 4m\left\{\int dSS_+S_- + \int d\bar{S}\bar{S}_+\bar{S}_-\right\}$$

where the Lagrangian is integrated over the Grassmann space using, respectively, a chiral $(dS)$, an antichiral $(d\bar{S})$, and a vectorial $(dV)$ measure.

$W^{\alpha}$ is a gauge-invariant quantity defined as:

$$W^{\alpha} = \bar{D}\bar{D}D^{\alpha}V$$

$D^{\alpha}$ and $\bar{D}^{\alpha}$ may be regarded as covariant derivatives.

In order to define the SUSY propagators, we must add a gauge-breaking term, given by:

$$\Gamma_B = \int dVDDV\bar{D}\bar{D}V$$

It may be shown, using either an argument about the residuum's sign of the Green's functions, or by analogy with QED, that the antichiral superfield $DDV$ contains a set of ghost fields, i.e. fields defining negative norm states.

The action is completed with source terms $J$ and $J_+$ which are respectively vector and chiral superfields.

It is relatively easy to derive a local Ward identity of the $U(1)$ group:

$$\frac{8}{\alpha}\Box\,\bar{D}\bar{D}V = \tfrac{1}{4}\bar{D}\bar{D}J + \frac{g}{8}(S_-J_+ - J_-S_+)$$

which implies that the ghost field $\overline{DDV}$ (as well as $DDV$, taking the corresponding conjugate equation) behave as free fields, i.e. can be decoupled from the physical sector of Fock space. This fact is every important to define, at least formally, a unitary $S$ matrix acting on that subspace.

The vector superfield propagator in momentum space is given by (Ref. 4):

$$\langle T(V(k,\theta_1)V(-k,\theta_2))\rangle = \frac{i}{4^5}\frac{e^{\bar{\theta}_1\gamma\theta_2 k}}{k^4}[4(1-\alpha) + (1+\alpha)\theta_{12}^4k^2]$$

The $1/k^4$ factor yields, when calculating the Green's functions, to IR divergences, i.e. to integrals of the type

$$\int d^4k \frac{f(p)+g(k,p)}{k^4}$$

$p$ being the external momentum, which manifestly present a non-integrable singularity at the origin.

## 3. SQED ($\mu^2$)

From part 2, it is clear that an IR regularization, independently of a UV renormalization, is required for defining Green's functions.

We will build up a massive*) SQED model in which the supermultiplet is no longer a superfield, i.e. an explicit and soft SUSY's breaking, but we will redefine the gauge transformations in order to preserve the gauge invariance.

The new field $\hat{V}$ is defined from the vector superfield as follows:

$$\hat{V} = V(1+u), \qquad u = u(\theta, \bar{\theta}) = \tfrac{1}{2}\theta^4\mu^2 \tag{3.1}$$

$u$ being a real parameter. In fact, we have generalized the wave function renormalization.

The new local (infinitesimal) gauge transformation $\hat{\delta}$ is defined by:

$$\hat{\delta}\hat{V} = \delta V = i(\Lambda - \bar{\Lambda}) \quad \text{and} \quad \hat{\delta}S_\pm = \mp ig\Lambda S_\pm \tag{3.2}$$

From (3.1) and (3.2) it is clear that:

$$\hat{\delta}V = \frac{1}{1+u}\hat{\delta}\hat{V} \tag{3.2'}$$

because $u$ is a gauge-invariant parameter. Recalling the fact that $(\theta^4)^n = 0$ for $n > 1$, we obtain that

$$\hat{\delta}V = (1-u)\,\hat{\delta}\hat{V} = (1 - \tfrac{1}{2}\mu^2\theta^4)\,\hat{\delta}\hat{V} \tag{3.3}$$

The new gauge-invariant action reads:

$$\hat{\Gamma}_{\text{INV}} = 2\int dS\hat{W}^\alpha \hat{W}_\alpha + \int dV(e^{g\hat{V}}S_+\bar{S}_+ + e^{-g\hat{V}}S_-\bar{S}_-)$$
$$-4m\left\{\int dSS_+S_- + \int d\bar{S}\bar{S}_+\bar{S}_-\right\}; \quad \hat{W}^\alpha = \bar{D}\bar{D}D^\alpha\hat{V} \tag{3.4}$$

It is clear that, as long as $\hat{\delta}\hat{V} = \delta V$ and $\hat{\delta}S_\pm = \delta S_\pm$,

$$\hat{\delta}\hat{\Gamma}_{\text{INV}} = \delta\Gamma_{\text{INV}} = 0$$

As before, we must add a gauge-breaking term:

$$\Gamma_B = \frac{2}{\alpha}\int dVDDV\bar{D}\bar{D}V \tag{3.5}$$

*) As long as the matter's fields $S_+$ and $S_-$ are concerned.

We have not written it in terms of the new field $\hat{V}$, because it would have cancelled the effect of the $\mu^2$ regularization, as it will be shown later.

The main problem that we have found throughout the calculation is that the field $\hat{V}$ is 'partially' supersymmetric, not allowing us to use the powerful superfield's technic, so we had to consider the single components (the $x$-functions in the development on powers of $\theta$ and $\bar{\theta}$). If we develop $W^\alpha$, we obtain:

$$W_\alpha = \bar{D}\bar{D}D_\alpha V = -2\lambda'_\alpha + 2\theta^\beta(\varepsilon_{\alpha\beta}D' - \sigma^{\mu\nu}_{\alpha\beta}v_{\mu\nu}) - 2i\theta^2\sigma^\nu_{\alpha\dot{\beta}}\,\partial_\nu\bar{\lambda}^{\dot{\beta}} \tag{3.6}$$

with $\lambda' = \lambda + i\sigma^\nu\,\partial_\nu\bar{\chi}$, $D' = D + \partial^2 C$ and $v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu$. As long as $\hat{D} = D + 2\mu^2 C$ is the only new component which differs from the old ones, the only component to be taken into account for the calculation of Green's function will be $\hat{D}' = D + \partial^2 C + 2\mu^2 C$. In fact, it is known that all the other quantities which are present in $W_\alpha$ – such as $\lambda'$ and $v_{\mu\nu}$ – do not lead to IR divergent Green's functions, at least up to second order diagrams.

A very important question to be taken into account is whether a 'good' broken Ward identity may be found. We had demonstrated that the identity found in our case leads to the existence of decoupled ghost fields.

In fact, the (infinitesimal) variation of the action (3.4) and (3.5) may be written as:

$$\hat{\delta}\hat{\Gamma} = \hat{\delta}(\hat{\Gamma}_{\rm INV} + \Gamma_B) = \hat{\delta}\Gamma_B$$

which is, by definition, equal to

$$i\int dS\Lambda w_g\hat{\Gamma} - i\int d\bar{S}\bar{\Lambda}\bar{w}_g\hat{\Gamma} \tag{3.7}$$

On the other hand, we have that

$$\hat{\delta}\hat{\Gamma} = \int dV\,\hat{\delta}V\frac{\delta\hat{\Gamma}}{\delta V} + \int dS\left(\hat{\delta}S_+\frac{\delta\hat{\Gamma}}{\delta S_+} + \hat{\delta}S_-\frac{\delta\hat{\Gamma}}{\delta S_-}\right) + \int d\bar{S}\left(\hat{\delta}\bar{S}_+\frac{\delta\hat{\Gamma}}{\delta\bar{S}_+} + \hat{\delta}\bar{S}_-\frac{\delta\hat{\Gamma}}{\delta\bar{S}_-}\right) \tag{3.8}$$

(the other terms cancel between them).

Recalling the fact the $dV = d^4x d^4\theta$ and $dS = d^4x d^2\theta$ and the equivalence between $d^2\theta$ and $DD$, (3.8) becomes:

$$\int \delta S\bar{D}\bar{D}\left(\hat{\delta}V\frac{\delta\hat{\Gamma}}{\delta V}\right)$$

and using (3.2) and (3.2'),

$$\hat{\delta}\hat{\Gamma} = \int dS\bar{D}\bar{D}\left\{(1 - \tfrac{1}{2}\mu^2\theta^4)i\Lambda\frac{\delta\hat{\Gamma}}{\delta V}\right\} - \int d\bar{S}DD\left\{(1 - \tfrac{1}{2}\mu^2\theta^4)i\bar{\Lambda}\frac{\delta\hat{\Gamma}}{\delta V}\right\}$$

Using the fact that $\Lambda$ commutes with $\theta^4$ and is cancelled by the $\overline{DD}$ operator due to his chiral character, we can establish that:

$$\begin{aligned} w_g\hat{\Gamma} &= \bar{D}\bar{D}\left\{(1 - \tfrac{1}{2}\mu^2\theta^4)\frac{\delta\hat{\Gamma}}{\delta V}\right\} \\ &= \bar{D}\bar{D}\left[(1 - \tfrac{1}{2}\mu^2\theta^4)\frac{2}{\alpha}\{DD, \bar{D}\bar{D}\}V\right]^{*)}. \end{aligned} \tag{3.9}$$

*) $$w_g = \bar{D}\bar{D}\frac{\delta}{\delta V} - igS_+\frac{\delta}{\delta S_+} + igS_-\frac{\delta}{\delta S_-}$$

Using the definition of $\{DD, \bar{D}\bar{D}\}$,*) we find that

$$\bar{D}\bar{D}(2/\alpha\{DD, \bar{D}\bar{D}\}V) = -\frac{32}{\alpha}\Box\,\bar{D}\bar{D}V$$

Furthermore, we have that:

$$\theta^4\{DD, \bar{D}\bar{D}\}V = \theta^4\{DD, \bar{D}\bar{D}\}_{\theta^0}V = 8D''\theta^4$$

with $D'' = D - \Box C$ and $\bar{D}\bar{D}(\theta^4 D''(x)) = -4\theta^2 D''$ (in the chiral representation). Then (3.9) becomes:

$$w_g\hat{\Gamma} = -\frac{32}{\alpha}\Box\,\bar{D}\bar{D}V + \frac{32}{\alpha}\,\theta^2\mu^2 D''$$

Taking into account that $\bar{D}\bar{D}V = -2\bar{M} - 2\theta\lambda - \theta^2(D'' - 2i\,\partial^\mu v_\mu)$, with $\lambda'' = \lambda - i\sigma^\mu\,\partial_\mu\bar{\chi}$, we find the following equations:

$$w_g\hat{\Gamma}|_{\theta^0} = \frac{64}{\alpha}\Box\,M \qquad (3.12)$$

$$w_g\hat{\Gamma}|_{\theta} = \frac{64}{\alpha}\Box\,\lambda'' \qquad (3.13)$$

$$w_g\hat{\Gamma}|_{\theta^2} = \frac{32}{\alpha}(\Box + \mu^2)D'' - i\frac{64}{\alpha}\Box\partial^\mu n_\mu \qquad (3.14)$$

Using the same arguments pointed out in part (2), it can be shown that the components of the chiral superfield $\bar{D}\bar{D}V$ are ghost fields. The equations (3.12) and (3.13) as well as the imaginary part of (3.14) imply that the ghost fields $\bar{M}$, $\lambda''$ and $\partial_\mu v^\mu$ behave as free massless fields. The real part of (3.14) implies that the ghost field $D''$ behaves as a free field with mass $\mu^2$, i.e. all the ghost fields can be decoupled from the physical sector of Fock space.

With the $\mu^2$ regularization, the Green's function of the gauge-invariant operator $\hat{D}'$ is no longer IR divergent. We will show that it would not be dependent on that SUSY breaking parameter, at least up to second order diagrams.

## 4. Green's function of $\hat{D}'$

The action given by (3.4) and (3.5) is completed with source (external) fields: the chiral superfield $J_{S_+}$ coupled to $S_+$ and the antichiral superfield $\bar{J}_{S_-}$ coupled to $\bar{S}_-$. From the completed action, we can deduce the free field's equation of the matter's fields:

$$\bar{D}\bar{D}\bar{S}_+ + 4mS_- = -J_{S_+} \qquad (4.1)$$

and

$$DDS_- + 4m\bar{S}_+ = -\bar{J}_{S_-} \qquad (4.2)$$

We make the operator $DD$ act on the two sides of equation (4.1) and multiply

*) $\{DD, \bar{D}\bar{D}\} = -8i\bar{D}\bar{\sigma}^\mu D\,\partial_\mu - 16\Box$

(4.2) by $-4m$. Using the fact that

$$[DD, \bar{D}\bar{D}]\bar{S}_+ = -(8i\bar{D}\bar{\sigma}^\mu D\,\partial_\mu + 16\partial^2)\bar{S}_+ = -16\partial^2\bar{S}_+$$

(because $\bar{S}_+$ is antichiral, which implies also that $\bar{D}\bar{D}DD\bar{S}_+ = 0$), we obtain the following equation for $\bar{S}_+$:

$$-16(\partial^2 + m^2)\bar{S}_+ = -DDJ_S + 4m\bar{J}_S \tag{4.3}$$

and the corresponding complex-conjugate equation for $S_+$:

$$-16(\partial^2 + m^2)S_+ = -\bar{D}\bar{D}\bar{J}_S + 4mJ_S \tag{4.4}$$

From (4.4) we may derive the propagators by taking functional derivatives:

$$\langle T(S_+(1)\bar{S}_-(2))\rangle = i\frac{\delta S_+(1)}{\delta \bar{J}_S(2)} = \frac{i}{16(\partial^2 + m^2)}\bar{D}\bar{D}_1\,\bar{\delta}_S(1,2) \tag{4.5}$$

and

$$\langle T(S_+(1)S_-(2))\rangle = i\frac{\delta S_+(1)}{\delta J_S(2)} = \frac{-i4m}{16(\partial^2 + m^2)}\delta_S(1,2) \tag{4.6}$$

where (1) and (2) are two points of Grassmann space. From (4.3) we obtain:

$$\langle T(\bar{S}_+(1)S_+(2))\rangle = \Delta_{\bar{S}_+S_+} = \frac{i}{16(\partial^2 + m^2)}DD_1\,\delta_S(1,2) \tag{4.7}$$

and

$$\Delta_{S_+S_-} = \frac{-i4m}{16(\partial^2 + m^2)}\delta_S(1,2) \tag{4.8}$$

Furthermore, we can find the equivalent equations for the $S_-$ and $\bar{S}_-$ superfields. We may deduce as well that

$$\Delta_{S_+S_+} = \Delta_{S_-S_-} = \Delta_{S_+\bar{S}_-} = \Delta_{S_-\bar{S}_+} = 0 \tag{4.9}$$

Taking into account the expression of $\delta$-functions, we obtain:

$$\Delta_{S_+\bar{S}_+} = \Delta_{S_-\bar{S}_-} = \frac{i}{16(k^2 - m^2)}e^{-(\bar{\theta}_1\gamma\theta_2 - \theta_{12}\sigma\bar{\theta}_{12})k}$$

$$\Delta_{S_+S_-} = \frac{-im}{16(k^2 - m^2)}\theta_{12}^2; \qquad \Delta_{\bar{S}_+\bar{S}_-} = \frac{-im}{16(k^2 - m^2)}\bar{\theta}_{12}^2 \tag{4.10}$$

We must compute also the propagators of the component fields. In the chiral representation, we have:

$$S_+ = A_+ + \theta\psi_+ + \theta^2F_+; \qquad S_- = A_- + \theta\psi_- + \theta^2F_-$$

We obtain the following results:

$$\Delta_{A_+\bar{A}_+} = \Delta_{A_-\bar{A}_-} = \Delta_{S_+\bar{S}_+}|_{\theta_1^0\theta_2^0} = \frac{i}{16(k^2 - m^2)}$$

$$\Delta_{F_+\bar{F}_+} = \Delta_{F_-\bar{F}_-} = \Delta_{S_+\bar{S}_+}|_{\theta_1^2\theta_2^2} = \frac{ik^2}{16(k^2 - m^2)}$$

$$\Delta_{A_+F_-} = \Delta_{S_+S_-}|_{\theta_1^0\theta_2^0} = \frac{im}{16(k^2 - m^2)}\ {}^{*)} \tag{4.11}$$

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*) $\Delta_{A_+F_+} = \Delta_{A_-F_-} = \Delta_{A_+A_-} = \Delta_{F_+F_-} = \Delta_{A_+\bar{A}_-} = \Delta_{A_+\bar{F}_-} = 0$

In order to compute the $C-D$ propagators, we take apart the part of the Lagrangian involving such fields. Taking into account the properties of chiral and vectorial measures, we find:

$$\Gamma_{(C,D)} = 32\int d^4x\left[-\hat{D}'^2 + \frac{1}{\alpha} D''^2\right] + \int d^4x\{e^{gC}[S\bar{S}]_{\theta^4} + 4g\hat{D}e^{gC}A\bar{A}\}$$

$$= 32\int d^4x\left[-\hat{D}'^2 + \frac{1}{\alpha} D''^2\right] + \int d^4x\{e^{gC}(16F\bar{F} + 4i\psi\overleftrightarrow{\partial}^{\mu}\sigma_{\mu}\bar{\psi}$$

$$+ 8\partial^{\mu}A\,\partial_{\mu}\bar{A} - 4A\,\partial^2\bar{A} - 4\partial^2 A\bar{A}) + 4g\hat{D}e^{gC}A\bar{A}\} \tag{4.12}$$

We build up the $\Gamma_{(C,D)}$ matrix by taking functional derivatives; for instance $\Gamma_{CD} = \delta\hat{\Gamma}_{\text{BIL}}(C, D)/\delta C\,\delta D$ and we obtain:

$$\Gamma_{(C,D)} = 32\begin{bmatrix} 2(\beta-1) & 2k^2(\beta+1)-4\mu^2 \\ 2k^2(\beta+1)-4\mu^2 & 2(\beta-1)k^4+8k^2\mu^2-8\mu^4 \end{bmatrix}\beta = \frac{1}{\alpha} \tag{4.13}$$

It is known that in $k$-space $\Gamma_{(C,D)}\Delta_{(C,D)} = i$ (we have taken $\hbar = 1$), where $\Delta_{CD}$ are the free Green's functions of such fields. Inverting $\Gamma_{(C,D)}$, we find:

$$\Delta_{(C,D)} = \frac{i}{32}\frac{1}{\text{DET}}\begin{bmatrix} 2(\beta-1)k^4+8k^2\mu^2-8\mu^4 & -2k^2(\beta+1)+4\mu^2 \\ -2k^2(\beta+1)+4\mu^2 & 2(\beta-1) \end{bmatrix} \tag{4.14}$$

with $\text{DET} = -16\beta(k^2-\mu^2)^2$.

If we had taken the gauge-breaking term $\int dVDD\hat{V}\bar{D}\bar{D}\hat{V}$ instead of $\int dVDDV\bar{D}\bar{D}V$, we would have got a determinant proportional to $k^4$, i.e. we would not have removed the IR divergences.

From (4.14), we obtain the following propagators in $k$-space:

$$\Delta_{CD} = i\frac{k^2(\alpha+1)-2\alpha\mu^2}{32.8(k^2-\mu^2)+i\varepsilon}$$

$$\Delta_{CC} = i\frac{\alpha-1}{32.8(k^2-\mu^2)+i\varepsilon}$$

$$\Delta_{DD} = i\frac{(\alpha-1)k^4+4\mu^2\alpha-4k^2\mu^2\alpha}{32.8(k^2-\mu^2)^2+i\varepsilon} \tag{4.15}$$

In order to compute the free Green's function of the $\hat{D}'$ operator, which in $k$-space is equal to $D+(2\mu^2-k^2)C$, we calculate first the following quantities:

$$\langle T(\hat{D}'(p)D(-p))\rangle = \Delta_{DD} + (2\mu^2-p^2)\,\Delta_{CD} = i\frac{p^2}{4.32(\mu^2-p^2)+i\varepsilon}$$

$p$ being an external momentum, and

$$\langle T(\hat{D}'(p)C(-p))\rangle = \Delta_{CD} + (2\mu^2-p^2)\,\Delta_{CC} = \frac{ip^2}{4.32(p^2-\mu^2)+i\varepsilon} \tag{4.16}$$

It is interesting to point out that the last quantities do not depend any longer on the gauge-breaking parameter $\alpha$.

Finally, we find that:

$$\langle T(\hat{D}'(p)\hat{D}'(-p))\rangle = \frac{2i(\mu^2-p^2)}{4.32(p^2-\mu^2)} = -\frac{i}{64} \tag{4.17}$$

As expected, we have removed the IR divergence. Furthermore, the free (non perturbated) Green's function of such gauge-invariant operator does not depend on the SUSY's breaking parameter $\mu$.

For computing diagrams of the $\hat{D}'$ operator in a perturbation theory, we put:

$$\langle T(\hat{D}'(1)\hat{D}'(2))\rangle = \left\langle T\left(\hat{D}'_{(0)}(1)\hat{D}'_{(0)}(2)\exp i\int dV\mathscr{L}_{\mathrm{INT}}\right)\right\rangle \tag{4.18}$$

in which $\hat{D}'_{(0)}$ is a free operator. $\mathscr{L}_{\mathrm{INT}}$, which will be indicated at each order, is a functional of the free fields. In order to simplify the writing, all the (0) indices will be omitted. Usually, (4.18) holds a factor which compensates the vacuum diagrams, but is unnecessary to keep it as long as those diagrams will not be considered.

At the first order of the development, we will meet the so-called tad-poles diagrams. Once the integration over $\theta$ and $\bar{\theta}$ has been performed, the interaction term reads:

$$i\int d^4x\left\{\frac{g^2}{2}C^2[S_+(x)\bar{S}_+(x)]_{\theta^4} + 4\hat{D}g^2[S_+(x)\bar{S}_+(x)]_{\theta^0}C + \frac{g^2}{2}C^2[S_-(x)\cdots\right\} \tag{4.19}$$

The $[\overline{S_\pm \bar{S}_\pm}]_{\theta^4}$ tad-poles (see Fig. 1) are given by:

$$16^2\int d^4k[\Delta_{S_\pm \bar{S}_\pm}(k, \theta, \bar{\theta})]_{\theta^4} = 0$$

The contribution of the tad-poles $[\overline{S_\pm \bar{S}_\pm}]_{\theta^0} = [\overline{A_\pm \bar{A}_\pm}]$ is shown in Fig. 2 (the factor 2 coming from the two possible contractions with external fields). In the momentum space, we will obtain:

$$ig^2 2.4\langle T(\hat{D}'(p)\hat{D}(-p))\rangle\int d^4k\langle T(A_\pm(k)\bar{A}_\pm(-k))\rangle\langle T(C(-p)\hat{D}'(p))\rangle$$

From (4.11) and (4.16) the last expression becomes:

$$ig^2 2.4\left(\frac{i}{4.32(p^2-\mu^2)}\right)^2(2\mu^2-p^2)\frac{i}{16}\int d^4k\frac{1}{k^2-m^2}$$

Then, the tad-poles' contribution (in fact the $A_+\bar{A}_+$ contribution times two) will be:

$$\frac{g^2(2\mu^2-p^2)}{(4.32(p^2-\mu^2))^2}\int d^4k\frac{(p+k)^2-m^2}{D_m} \tag{4.20}$$

where $D_m$ stands for $((p+k)^2-m^2)(k^2-m^2)$. Indeed, the last integral is UV divergent, so the present result must be taken formally, until the UV renormalization will be performed.

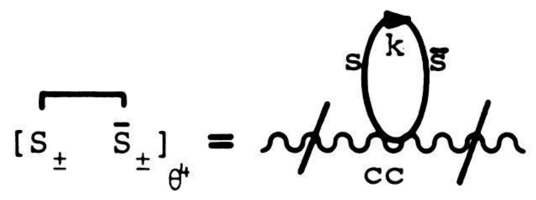


Figure 1

$$[\overbrace{A_\pm \; \bar{A}_\pm}] = ig^2 2.4$$

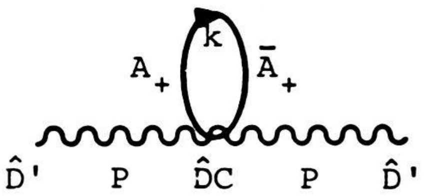


Figure 2

For the second order (two-legs) diagrams, the interaction term reads:

$$\frac{i^2}{2}\int d^4x_3 \mathscr{L}_{\rm INT}(3)\int d^4x_4 \mathscr{L}_{\rm INT}(4) \tag{4.21}$$

where

$$\mathscr{L}_{\rm INT} = gC[S_+\bar{S}_+]_{\theta^4} - gC[S_-\bar{S}_-]_{\theta^4} + 4g\hat{D}(A_+\bar{A}_+ - A_-\bar{A}_-) \tag{4.22}$$

All the possible contractions are given by the diagrams shown in Fig. 3. We find that the amputated SUSY diagrams (see Fig. 4) become:

$$16\int d^4k \langle T(S_\pm(p+k)\bar{S}_\pm(-p-k))\rangle\langle T(\bar{S}_\pm(k)S_\pm(-k))\rangle|_{\theta_1^4\theta_2^4}$$

$$= -\tfrac{1}{16}\int d^4k [e^{-\bar{\theta}_1\gamma\theta_2 p+\theta_{12}\sigma\bar{\theta}_{12}p+2\theta_{12}\sigma\bar{\theta}_{12}k}]_{\theta_1^4\theta_2^4}\cdot\frac{1}{D_m} \tag{4.23}$$

It is relatively easy to calculate the $\theta_1^4\theta_2^4$ term of the exponential. It gives $p^4$. So (4.23) becomes:

$$-\tfrac{1}{16}\int d^4k \frac{p^4}{D_m} \tag{4.24}$$

The one-loop diagrams $A_+$ and $A_-$ (see Fig. 5) are equivalent; they yield to

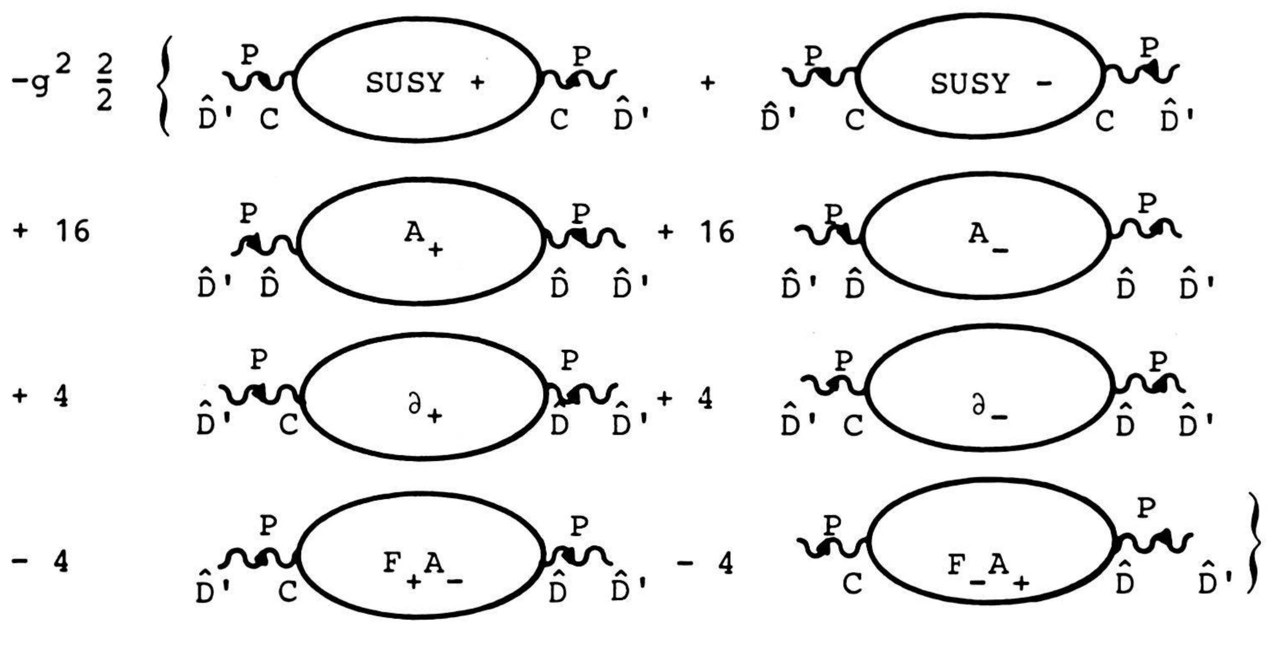


Figure 3

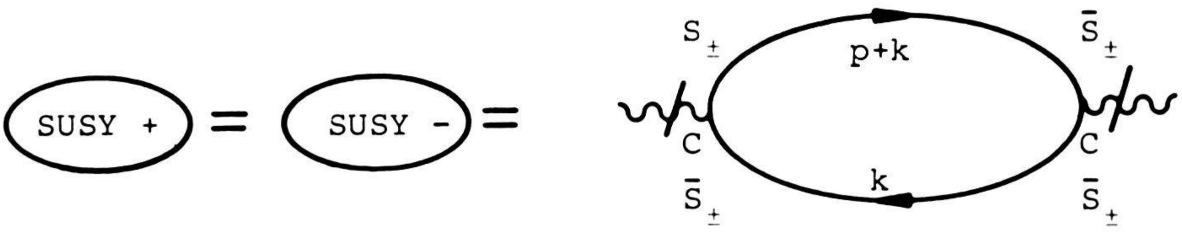


Figure 4

the same convolution product:

$$\int d^4k\, \Delta_{A,\bar{A}}(p+k)\, \Delta_{\bar{A},A}(k) = \left(\frac{i}{16}\right)^2 \int d^4k \frac{1}{D_m} \tag{4.25}$$

The one-loop diagrams $\partial_+$ and $\partial_-$ (see Fig. 6) are also equivalent; for each one of them the result is:

$$-\tfrac{1}{32} \int d^4k \frac{(p+2k)^2}{D_m} \tag{4.26}$$

Finally, the diagrams $F_+A_-$ and $F_-A_+$ (see Fig. 7), which are equivalent, yield to the following convolution product:

$$\int d^4k\, \Delta_{F,A}(p+k)\, \Delta_{\bar{A},\bar{F}}(k) = -\frac{m^2}{16^2} \int d^4k \frac{1}{D_m} \tag{4.27}$$

Using (4.24) through (4.27) and the external propagators (4.16), the two-legs diagrams contribution will read:

$$-\frac{g^2}{8(4.32(p^2-\mu^2))^2} \int d^4k \frac{1}{D_m} \{2p^4 + 4\mu^4 - 4\mu^2p^2 + 2(2\mu^2 - p^2) \times [(p+2k)^2 - 4m^2]\} \tag{4.28}$$

With the contribution of the tad-poles (4.20), the integrand of (4.28) becomes:

$$\frac{1}{D_m}\{8p^4 + 4\mu^4 - 16\mu^2p^2 - 8(2\mu^2 - p^2)pk\}$$

It is convenient to shift the internal (integration) impulsion: $k = l - p/2$. The shifted integrand is given by INV + ANTISYM, where

$$\mathrm{INV} = \frac{4(p^2-\mu^2)^2}{D_{symm}}, \qquad \mathrm{ANTISYM} = \frac{8(p^2-2\mu^2)pl}{D_{symm}}$$

$D_{symm}$ stands for $\{(l+p/2)^2 - m^2\}\{(l-p/2)^2 - m^2\}$ (4.29)

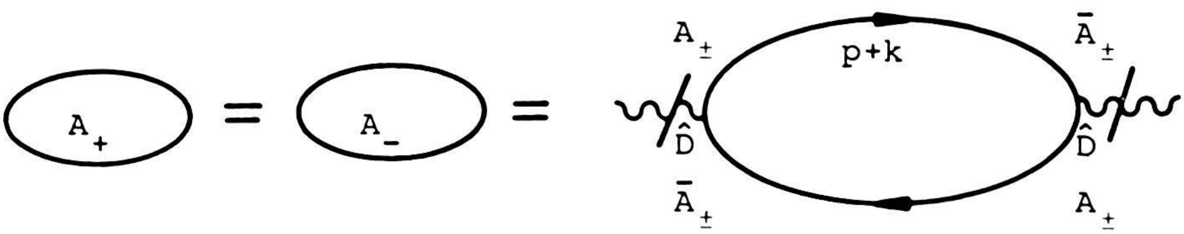


Figure 5

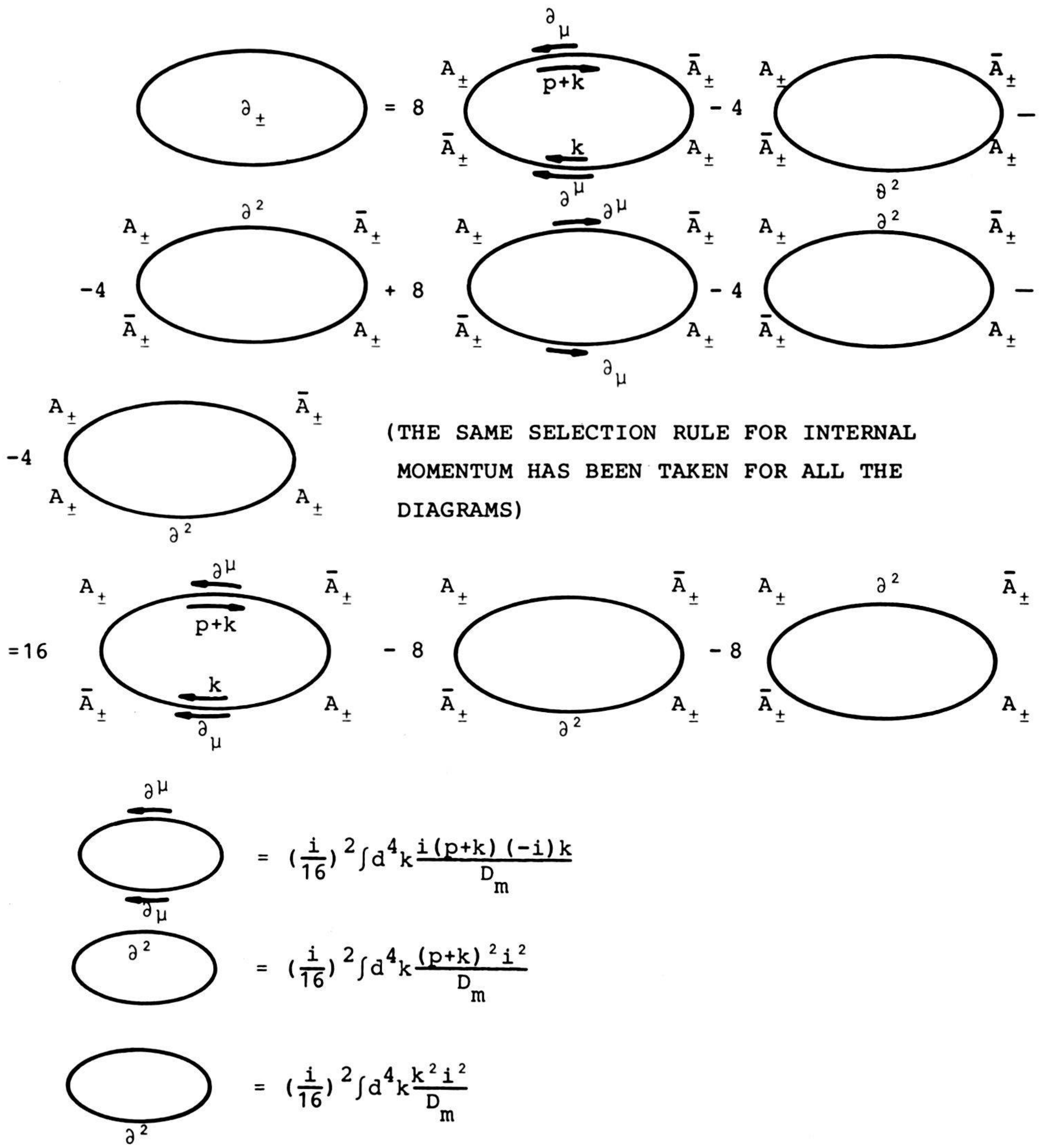


Figure 6

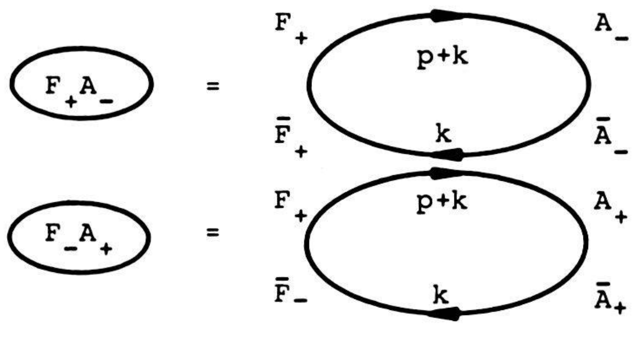


Figure 7

We will perform an UV renormalization following the Zimmerman's method, i.e. subtracting to the integrand a Taylor's development of the integrand in powers of $p$, the external momentum, up to the respective degree of divergence $UV(d_{UV})$.

For the INV term ($d_{UV}=0$), we have that

$$\left.\frac{1}{D_{symm}}\right)_{REN}=\frac{1}{D_{symm}}-\left.\frac{1}{D_{symm}}\right|_{p=0}$$

$$=\frac{1}{D_{symm}(l^2-m^2)^2}\left\{(pl)^2-\frac{p^4}{16}-\frac{p^2}{2}(l^2-m^2)\right\} \quad (4.30)$$

For the ANTISYM term ($d_{UV}=1$), we have that:

$$\left.\frac{l^\mu}{D_{symm}}\right)_{REN}=\frac{l^\mu}{D_{symm}}-\left.\frac{l^\mu}{D_{symm}}\right|_{p=0}-p^\nu\left[\frac{\partial}{\partial p^\nu}\left(\frac{l^\mu}{D_{symm}}\right)\right]_{p=0}$$

$$=\frac{l^\mu}{D_{symm}}\left\{(pl)^2-\frac{p^4}{16}-\frac{p^2}{2}(l^2-m^2)\right\}-p^\nu\left[\frac{\partial}{\partial p^\nu}\left(\frac{l^\mu}{D_{symm}}\right)\right]_{p=0} \quad (4.31)$$

It can be shown that the second term of the r.h.s. of (4.31) vanishes:

$$\left[\frac{\partial}{\partial p^\nu}\left(\frac{l^\mu}{D_{symm}}\right)\right]_{p=0}=-l^\mu\cdot\frac{l_\nu(l^2-m^2)-l_\nu(l^2-m^2)}{(l^2-m^2)^4}$$

The first term keeps his antisymmetric character, then it will vanish if integrated over the momentum space, because $d^4k$ is a symmetric measure.

The renormalized (UV) Green's function reads:

$$-\frac{2g^2}{(4.32)^2}\int d^4l\frac{1}{D_{symm}(l^2-m^2)^2}\left\{(pl)^2-\frac{p^4}{16}-\frac{p^2}{2}(l^2-m^2)\right\}$$

The IR divergence has been removed and the result does not depend any longer on the SUSY's breaking parameter.

It would be interesting to briefly comment the results obtained in a model which was not gauge invariant.

Taking into account (3.2) for $S_+=S$, $S_-=0$, the action given by

$$2\int dS\hat{W}^\alpha\hat{W}_\alpha-4m\left[\int dSS^2+\int d\bar{S}\bar{S}^2\right]+\int dVe^{8\hat{V}}S\bar{S}+\frac{2}{\alpha}\int dVDDV\bar{D}\bar{D}V$$

will not be gauge-invariant, even if we do not consider the gauge-breaking term, because of the massive term. The contribution of first and second order diagrams is given (at least formally, the UV renormalization has not been performed yet) by:

$$-\frac{g^2}{16(4.32(p^2-\mu^2))^2}\int d^4k(\text{INDEP}+\text{ANTISYM}+\text{DEP}(m^2))$$

where

$$\text{INDEP}=\frac{8(p^2-\mu^2)}{D_{symm}}.\qquad \text{ANTISYM}=\frac{8(p^2-2\mu^2)pk}{D_{symm}}\quad\text{and}$$

$$\text{DEP}(m^2)=\frac{12m^2(2\mu^2-p^2)}{D_{symm}}$$

The ANTISYM term will be vanished at UV renormalization as well as the dependence on $\mu$ of the INDEP term will cancel with the denominator, but it cannot be removed – even after UV renormalization – from the DEP($m^2$) term.

The fact that the $\mu$-dependence is removed in a gauge-invariant model suggests a possible link between gauge-invariance and the independence on the regularization parameter.

## Acknowledgements

I am grateful to O. Piguet for helpful discussions and comments.

## Appendix

Throughout this work we have used the metric (1, −1, −1 −1) and the $\partial^\mu$ operator is equivalent – in momentum space – to $ik^\mu$. Sometimes, the tensor indices have been omitted. For instance:

$$p^2 = p_\mu p^\mu, \qquad pk = p^\mu k_\mu \qquad \sigma k = \sigma^\mu k_\mu$$

The matrices $\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ may form a basis for the two-times-two complex matrices $\sigma^\mu_{\alpha\dot\alpha}$. $\sigma^{\mu\nu}_{\alpha\beta}$ stands for

$$\frac{i}{2}[\sigma^\mu, \sigma^\nu]_{\alpha\beta}$$

The product between spinors is defined through the antisymmetric tensors $\varepsilon^{\alpha\beta}$ and $\varepsilon^{\dot\alpha\dot\beta}$ ($\varepsilon_{21} = \varepsilon^{12} = 1$, $\varepsilon_{12} = \varepsilon^{21} = -1$, $\varepsilon = \varepsilon_{22} = 0$):

$$\psi\chi = \psi^\alpha \chi_\alpha = \varepsilon^{\alpha\beta}\psi_\beta\psi_\alpha$$

The tensor $\varepsilon^{\alpha\beta}$ may also be used to raise the indices of the $\sigma$-matrices:

$$\bar\sigma^{\mu\dot\alpha\alpha} = \varepsilon^{\dot\alpha\dot\beta}\varepsilon^{\alpha\beta}\sigma^\mu_{\beta\dot\beta}$$

The product $\bar\psi\bar\chi = \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha}$ is defined in such a way that

$$(\psi\chi)^\dagger = \bar\psi\bar\chi$$

An important result about $\sigma$-matrices:

$$(\sigma^\mu\bar\sigma^\nu + \sigma^\nu\bar\sigma^\mu)^\beta_\alpha = 2g^{\mu\nu}\,\delta^\beta_\alpha$$

We have also the following completeness relation:

$$\mathrm{Tr}\,(\sigma^\mu\bar\sigma^\nu) = 2g^{\mu\nu}$$

The Grassmann variables $\theta$ and $\bar\theta$ are Weyl spinors. $\theta_{12}$ stands for $\theta_1 - \theta_2$.

We have used some Fierz rearrangement formula:

$$\theta^\alpha\theta^\beta = -\tfrac{1}{2}\varepsilon^{\alpha\beta}\theta\theta \qquad \bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac{1}{2}\varepsilon^{\alpha\beta}\bar\theta\bar\theta$$

$$\theta\sigma^\mu\bar\theta\theta\sigma^\nu\bar\theta = \theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\theta^\beta\sigma^\nu_{\beta\dot\beta}\bar\theta^{\dot\beta} = \tfrac{1}{2}g^{\mu\nu}\theta\theta\bar\theta\bar\theta = \tfrac{1}{2}g^{\mu\nu}\theta^4$$

$\theta^2$ stands for $\theta\theta$, $\bar{\theta}^2$ for $\bar{\theta}\bar{\theta}$ and $\theta^4$ for $\theta\theta\bar{\theta}\bar{\theta}=\theta^2\bar{\theta}^2$. The expression $\bar{\theta}_1\gamma\theta_2$ (appearing in the chiral superfield propagator) stands for $\theta_1\sigma\bar{\theta}_2-\theta_2\sigma\bar{\theta}_1$. The covariant derivatives $\bar{D}_{\dot{\alpha}}$ and $D_\alpha$ are defined in the chiral representation by:

$$\bar{D}_{\dot{\alpha}}=-\frac{\partial}{\partial\bar{\theta}^{\alpha}}\qquad D_\alpha=\frac{\partial}{\partial\theta^\alpha}-2i\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\,\partial_\mu$$

About integration in Grassmann space: an indefinite integral over a Grassmann variable is defined as follows:

$$\int d\eta=0;\qquad \int\eta\,d\eta=1$$

so

$$\delta(\eta)=\eta.$$

The volume elements in superspace are given by:

$$d^2\theta=-\tfrac{1}{4}d\theta^\alpha\,d\theta^\beta\varepsilon_{\alpha\beta};\qquad d^2\bar{\theta}=-\tfrac{1}{4}d\bar{\theta}_{\dot{\alpha}}\,d\bar{\theta}_{\dot{\beta}}\varepsilon^{\dot{\alpha}\dot{\beta}}$$

dV stands for $d^4x\,d^4\theta$, $dS$ and $d\bar{S}$ for $d^4x\,d^2\theta$ and $d^4x\,d^2\bar{\theta}$ respectively. The operators $DD=D^\alpha D_\alpha$ and $\bar{D}\bar{D}=\bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}$ and the $d^2\theta$ and $d^2\bar{\theta}$ measures are respectively equivalent. We have that:

$$\int dSf(x,\theta,\bar{\theta})=-4\int d^4xf(x,\theta,\bar{\theta})|_{\theta^2}$$

$$\int d\bar{S}f(x,\theta,\bar{\theta})=-4\int d^4xf(x,\theta,\bar{\theta})|_{\bar{\theta}^2}$$

$$\int dVf(x,\theta,\bar{\theta})=16\int d^4xf(x,\theta,\bar{\theta})|_{\theta^4}$$

About the $\delta$ distributions:

$$\delta_S(1,2)=-\tfrac{1}{4}\theta_{12}^2\,\delta(x_1-x_2)\to\tilde{\delta}_S(1,2)=-\tfrac{1}{4}\theta_{12}^2\quad\text{(Fourier transform)}$$
$$\bar{\delta}_S(1,2)=-\tfrac{1}{4}\bar{\theta}_{12}^2\,\delta(x_1-x_2)\to\tilde{\bar{\delta}}_S(1,2)=-\tfrac{1}{4}\bar{\theta}_{12}^2$$

(in the chiral-antichiral representations)

$$\delta_V(1,2)=\tfrac{1}{16}\theta_{12}^4\to\tilde{\delta}_V(1,2)=\tfrac{1}{16}\theta_{12}^4$$

In the real representation we have that:

$$\bar{D}_1\bar{D}_1\,\bar{\delta}_S(1,2)=\delta(x_1-x_2-i\theta_1\sigma\bar{\theta}_1-i\theta_2\sigma\bar{\theta}_2+2i\theta_1\sigma\bar{\theta}_2)$$
$$\xrightarrow{\mathcal{F}}\exp\left[(-\bar{\theta}_1\gamma\theta_2+\theta_{12}\sigma\bar{\theta}_{12})k\right]$$

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