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The missing elements of reality in the description of quantum mechanics of the E.P.R. paradox situation

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Résumé. Nous démonstrons que la mécanique quantique n'est pas complète. Cette incomplétude n'est pas déduite ici par un raisonnement par l'absurde comme l'ont fait Einstein Podolsky and Rosen; nous indiquons explicitement les élements de réalité manquants dans la description par la mécanique quantique des systèmes physiques séparés.

Abstract. We show that quantum mechanics is not a complete theory. We do not as in the case of Einstein Podolsky and Rosen derive this incompleteness by a logical reasoning ex absurdum, but indicate explicitly which are the missing elements of reality in the description by quantum mechanics of separated physical systems.

1. Introduction

Einstein, Podolsky and Rosen show that quantum mechanics is not a complete theory [1]. In a recent study on the description of separated physical systems in a more general theory than quantum mechanics we were able to show that quantum mechanics cannot describe separated physical systems [2] [3] and that this incapacity of quantum mechanics is at the origin of the completeness proof of E.P.R. [4] [5]. Indeed in this incompleteness proof E.P.R. use the physical situation of two separated physical systems and they apply quantum mechanics to describe these two separated physical systems. In doing so they make the hypothesis that quantum mechanics describes correctly two separated systems and from this hypothesis they construct elements of reality of the subsystems that are not contained in the quantum mechanical description of these subsystems. Since these subsystems are arbitrary they can conclude that quantum mechanics is not a complete theory *or* that quantum mechanics does not describe correctly separated systems. Since they say in the beginning of their paper that they suppose quantum mechanics to be correct, and hence also quantum mechanics to give a correct description of separated systems, they can conclude that quantum mechanics is not complete. We think that E.P.R. have touched in their reasoning at a serious deficiency of quantum mechanics. The deficiency of quantum mechanics is however not in the description of the subsystems as is indicated by the reasoning of E.P.R. but in the description of the joint system of two separated systems.

It is the description of this joint system of separated systems by means of the tensorproduct of the Hilbert spaces of the subsystems which is not correct, as we show in [4] and [5]. In this paper we shall show that quantum mechanics is not a complete theory, because it cannot describe separated physical systems. And we will not as in the case of E.P.R. derive this incompleteness by a logical reasoning, but we will explicitly indicate which are the missing elements of reality in the description of separated physical systems.

2. Completeness of a theory

Let us recall the definition of element of reality given by Einstein, Podolsky and Rosen: '*If without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of reality corresponding to this physical quantity*'. The condition of completeness putted forward by E.P.R. is the following: 'A theory is complete if every element of reality has a counterpart in the theory'.

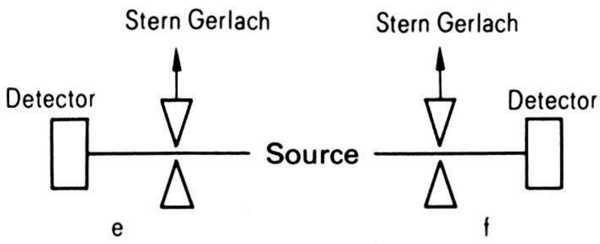
Clearly E.P.R. did not mean that a theory should describe all possible elements of reality of the physical system. Indeed, if this was what they meant, then of course every theory is incomplete, because a theory only gives a model for the physical system and this model describes a well defined set of elements of reality of the physical system. Therefore we would like to put this criterium of completeness in a different way. We would say that: '*A theory is complete if it can describe every possible element of reality of the physical system, without leading to contradictions*'. This completeness criterium should be satisfied by a reasonable physical theory. It means in fact that the theory is flexible enough to provide a model for any well defined set of elements of reality of the physical system. This is not the case for quantum mechanics as we show, because quantum mechanics cannot provide us a model for the description of separated physical systems.

3. Separated physical systems

Two physical systems S_1 and S_2 are separated if an experiment performed on one of the systems does not change the state of the other system. This of course does not mean that there is no interaction between S_1 and S_2. In general there is an interaction between separated systems and by means of this interaction the dynamical change of the state of one system is influenced by the dynamical change of the state of the other system. In classical mechanics for example almost all two body problems are problems of separated bodies (e.g. the Keplerproblem). Two systems are non separated if an experiment on one system changes the state of the other system. For two classical bodies this is for example the case when they are connected by a rigid rod. Let us try to express this idea of separated physical systems in an operational way.

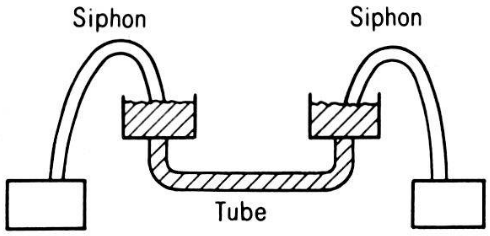
Suppose we consider two experiments e and f on a physical system S with outcome sets E and F. In general it is not possible to perform e and f together. This because often the performance of one of the experiments changes the state of the system in such a way, that it becomes impossible to perform the other experiment. Sometimes however it is possible to perform e and f together. This means that there is a new experiment which we shall denote by $e \times f$ with outcome set $E \times F$. If we perform the experiment $e \times f$ and find the outcome (x, y) then we interpret this as the outcome x for e and the outcome y for f.

Example 1. Consider a system S of two spin $\frac{1}{2}$ particles in the singlet spin state. We perform an experiment e that consists of measuring the spin of one of the particles in a certain direction in one region of space, and a measurement f of the spin of the other particle in the same direction in an opposite region of space. The outcome sets of e and f are $\{0, 1\}$ where 0 means that the electron is absorbed and 1 means that the electron has passed the Stern Gerlach. What we mean is the well known experiment proposed by Bohm [6] and carried out meanwhile several times to test Bell inequalities.



$e \times f$ consists of performing e and f at once. The outcome set of $e \times f$ is $\{(0,0), (0,1), (1,0), (1,1)\}$. As is shown by the experiments [7] and as is also predicted by quantum mechanics, for $e \times f$ we always find one of the outcomes $(0, 1)$ or $(1, 0)$. For e however we can find the outcome 0 and 1 both with probability $\frac{1}{2}$ and also for f we can find the outcome 0 and 1 both with probability $\frac{1}{2}$.

Example 2. Consider a system S consisting of two vessels containing each 10 l. of water connected by a tube as is shown on the figure. The experiment e consists of testing whether the volume of the water contained in the first vessel is more than 10 l. We perform this experiment by emptying the vessel by means of a siphon and collecting the water in a reference vessel. We give the outcome 1 if the water stops flowing after it depasses 10 l. In the reference vessel and we give the outcome 0 if the water stops flowing before it depasses 10 l. The experiment f consists of testing whether the volume contained in the second vessel is more or equal to 10 l.



$e \times f$ consists of performing e and f at once. Again we see that for $e \times f$ we always find the outcomes $(0, 1)$ or $(1, 0)$. For e however we always find the outcome 1 and for f we always find the outcome 1.

In both examples we see that some of the combinations of outcomes of e and f are not possible for the experiment $e \times f$. Indeed in both cases 1 is a possible outcome for e and 1 is a possible outcome for f but $(1, 1)$ is not possible for $e \times f$. This indicates that the experiments e and f are influencing each other in a certain sense.

In both examples this is due to the fact that the system consists of two systems that are not separated. These examples inspire us the following intuitively clear definition of separated experiments

Definition 1. If we have two experiments e and f that can be performed together, hence there exists an experiment $e \times f$, then e and f are separated for $e \times f$ iff

(i) if x is a possible outcome for e and y is a possible outcome for f, then (x, y) is a possible outcome for $e \times f$.
(ii) if (x, y) is a possible outcome for $e \times f$, then x is a possible outcome for e and y is a possible outcome for f.

Clearly in both examples this definition is not satisfied, and thus in both examples e and f are non separated experiments.

As we remarked already, we will say that two physical systems S_1 and S_2 are separated iff an experiment performed on one of the systems does not change the state of the other system. Let us now show that if we have two separated physical systems S_1 and S_2 and if we consider experiments e on S_1 and f on S_2, then e and f are separated. To be able to show this, we have to remark that the state of a physical system determines the possible outcomes of an experiment on this physical system. Indeed, in a classical theory every experiment has only one possible outcome for a state of the system. In quantum mechanics the possible outcomes of an experiment are those outcomes that correspond to eigenstates not orthogonal to the state of the system.

Theorem 1. *Consider two physical systems S_1 and S_2 that are separated (in the sense that an experiment on one of the systems does not change the state of the other system) and experiments e on S_1 and f on S_2, then e and f are separated experiments.*

Proof. Let us define the experiment $e \times f$ as follows. We perform the experiment e which gives us the outcome x and we perform the experiment f which gives us the outcome y. We then give the outcome (x, y) to $e \times f$. We choose freely whether we perform first e and then f or first f and then e or e and f simultaneously. Let us show that e and f are separated. If x is a possible outcome for e and y is a possible outcome for f then (x, y) is a possible outcome for $e \times f$, because the performance of one of the experiments does not change the possible outcomes of the other experiment. For the same reason, if (x, y) is a possible outcome of $e \times f$ then x is a possible outcome of e and y is a possible outcome of f.

In [5] we show that also the inverse of this theorem holds. Namely if all experiments e on S_1 and f on S_2 are separated, then S_1 and S_2 are separated physical systems.

This shows that we can characterize separated physical systems by means of separated experiments, and also that the definition of separated systems given in [2] and [3] is equivalent with the definition adopted in this paper.

4. The missing elements of reality

We shall now proof that an experiment of the type $e \times f$ where e and f are separated experiments cannot be described by quantum mechanics.

Theorem 2. *If e and f are separated experiments on a physical system S described by quantum mechanics in a Hilbert space H, then the experiment $e \times f$ cannot be described by quantum mechanics, in the sense that there does not exist a self adjoint operator corresponding to $e \times f$.*

Proof. Suppose we do try to describe e and f and $e \times f$ by self adjoint operators R, S and O. If E is the outcome set of e and F the outcome set of f then $E \times F$ is the outcome set of $e \times f$. We will first of all show, that if e and f are separated, the R and S commute. To do this, we will use the spectral projections of R, S and O. Consider two arbitrary spectral projections P_A and P_B of R and S. Hence $A \subset E$ and $B \subset F$. With $A \times B$ corresponds the spectral projection $P_{A \times B}$ of O. Consider a state w such that $w \perp P_A(H)$. If the system is in the state w and $x \in A$, then x is not a possible outcome for e. If y is an arbitrary outcome of f, then (x, y) is not a possible outcome of $e \times f$. This shows that $w \perp P_{A \times F}(H)$. So $1 - P_A \subset 1 - P_{A \times F}$. From this follows that $P_{A \times F} \subset P_A$ and $P_{(E/A) \times F} \subset P_{E/A}$. Now clearly $P_{E/A} = 1 - P_A$ and $P_{(E/A) \times F} = P_{E \times F / A \times F} = 1 - P_{A \times F}$. This shows that $P_{A \times F} = P_A$. In an analoguous way we show that $P_{E \times B} = P_B$. From this follows that $[P_A, P_B] = 0$ since $P_{A \times F}$ and $P_{E \times B}$ are spectral projections of the same self adjoint operator O. As a consequence $[R, S] = 0$. We also have $P_{A \times B} = P_{A \times F} \cdot P_{E \times B} = P_A \cdot P_B$.

Consider now the closed subspace $P_A(H) \cap P_B(H)^\perp$. If $P_A(H) \cap P_B(H)^\perp = 0$, then $P_A(1 - P_B) = 0$. Hence $P_A = P_A \cdot P_B$. Since $P_A \neq 0$ we have $P_A \cdot P_B \neq 0$. As a consequence $P_A(H) \cap P_B(H) \neq 0$. In an analoguous way we show that $P_A(H)^\perp \cap P_B(H) \neq 0$ in this case. Hence there are two possibilities.

$$P_A(H) \cap P_B(H)^\perp \neq 0 \quad \text{and} \quad P_A(H)^\perp \cap P_B(H) \neq 0$$

or

$$P_A(H) \cap P_B(H) \neq 0 \quad \text{and} \quad P_A(H)^\perp \cap P_B(H)^\perp \neq 0.$$

We shall show that in both cases the superposition principle allows us to construct a state such that when the system is in this state the experiments e and f are not separated.

Suppose $P_A(H) \cap P_B(H)^\perp \neq 0$ and $P_A(H)^\perp \cap P_B(H) \neq 0$ and take $u \in P_A(H) \cap P_B(H)^\perp$ and $v \in P_A(H)^\perp \cap P_B(H)$. Consider the state $w = u + v$. Then $P_A(w) = u$, $(1 - P_A)(w) = v$, $P_B(w) = v$ and $(1 - P_B)(w) = u$. On the other hand

$$P_{A \times B}(w) = P_A \cdot P_B(w) = P_A(v) = 0$$

$$P_{(E/A) \times (F/B)}(w) = (1 - P_A)(1 - P_B)(w) = (1 - P_A)(u) = 0$$

$$P_{(E/A) \times B}(w) = (1 - P_A)P_B(w) = (1 - P_A)(v) = v$$

$$P_{A \times (F/B)}(w) = P_A(1 - P_B)(w) = P_A(u) = u$$

Suppose now that the physical system is in the state w. Then there is at least one possible outcome $x \in A$ and one possible outcome $z \in E/A$ for e. And there is at least one possible outcome $y \in B$ and one possible outcome $t \in F/B$ for f. But, while (x, t) and (z, y) are possible outcomes for $e \times f$, the outcomes (x, y) and (z, t) are not possible. This shows that e and f are not separated experiments. If $P_A(H) \cap P_B(H) \neq 0$ and $P_A(H)^\perp \cap P_B(H)^\perp \neq 0$ we proof in an analoguous way that e and f are not separated.

It is easy to see that it is the superposition principle that makes it possible to construct states of the system such that e and f are not separated.

Consequence. If e and f are experiments on a physical system described by quantum mechanics, then e and f are never separated experiments.

Suppose now that we have two separated physical systems S_1 and S_2. If e is an experiment on S_1 and f an experiment on S_2, then $e \times f$ is an experiment of the type that cannot be described by quantum mechanics. There are elements of reality of the joint system S consisting of the two separated systems S_1 and S_2 defined by experiments of the type $e \times f$ that cannot be described by quantum mechanics. This incapacity of quantum mechanics leads to the contradiction in the EPR reasoning. Completing quantum mechanics cannot be achieved by changing the description of the subsystems by adding additional variables (hidden variables) to this description because it is the description of the joint system that is wrong and has to be changed. In [2] and [3] we give such a description of separated systems in a more general theory as quantum mechanics and we see that the mathematical structure of the set of states is indeed not a vector space. Such that the superposition principle shall not be valid in this description of the joint system.

5. The E.P.R. reasoning

E.P.R. consider the following two sentences

(1) quantum mechanics is not complete
(2) physical quantities that are not compatible cannot have simultaneous reality.

Obviously these two sentences cannot both be wrong. Indeed, if two non compatible quantities can have simultaneous reality, then quantum mechanics is not complete, because the wave function cannot describe these elements of reality. So we have one of the three cases

A (1) false (2) true

B (1) true (2) false

C (1) true (2) true

Once E.P.R. come to this conclusion they consider the situation of two separated systems S_1 and S_2. By applying quantum mechanics to describe these two separated systems, they can show that it is possible to attach simultaneously elements of reality to non compatible observables. Hence (1) is true. So what

E.P.R. show is the following

Quantum mechanics describes correctly and in a complete way separated systems.
$\Rightarrow$ Quantities that are not compatible can have simultaneous reality.
$\Rightarrow$ Quantum mechanics is not complete.

From this they can conclude that

D the quantum mechanical description of separated systems is not correct or not complete.

or

E quantities that are not compatible can have simultaneous reality and hence quantum mechanics is not complete.

E.P.R. mention in the beginning of their paper that they suppose quantum mechanics to be correct. Hence they then also suppose quantum mechanics to give a correct description of separated systems. Two alternatives remain in this case: the quantum mechanical description of separated systems is incomplete or quantum mechanics is incomplete in the sense that quantities that are not compatible can have simultaneous reality. But in any case quantum mechanics is not complete. Hence E.P.R. can conclude that if quantum mechanics is correct, then it is not complete. What we showed in this paper is that the incompleteness arrives from D and not from E. Quantum mechanics is incomplete because it does not give a complete description of separated systems.

Let us sum up all this and see that there is no paradox left. There are two possible situations for two systems.

First situation. The two systems are separated. Then quantum mechanics does not give a correct description of this situation. Correcting quantum mechanics does not happen by adding states to the subsystems but by taking states away of the joint system (see [2] and [3]).

Second situation. The two systems are not separated. In this case, it is not possible to make the E.P.R. reasoning. Indeed if we try to attribute an element of reality to one of the systems S_1 by making an experiment on the other system S_2, then because the experiment on the system S_2 changes the state of S_1, the element of reality of S_1 is created by the experiment on S_2. Hence we cannot proceed the E.P.R. reasoning and say that the element of reality was already there before we made the experiment on S_2. This step is however necessary if we want to show that S_1 can have two elements of reality corresponding to non compatible observables. So as we can see, the E.P.R. reasoning touches at a major shortcoming of quantum mechanics, namely its incapacity to describe separated systems. Since the E.P.R. reasoning is a reasoning ex absurdum, it however does not indicate the missing elements of reality. We can ask ourselves which of the three cases A, B or C is correct. Then we have to answer the question whether there exists observables that are not elements of reality at the same time. It is possible to find experimental evidence that such kinds of observables exist (see [2] and [8]). Hence C would then be correct choice.

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