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On the ideal Boson ground state of excitons in two dimensions in a strong magnetic field

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In honor of Emanuel Mooser's 60th birthday

Abstract. Lerner and Lozovik have shown that excitons confined to two dimensions in a strong magnetic field have a ground state which has an ideal Bose character. The origin of this result is discussed as a cancellation of the matrix elements of the interaction connecting the ground state with all other excited states in the manifold of the lowest Landau level.

1. Introduction

For some years Prof. Mooser and his coworkers have made many interesting experiments examining the nature of excitons in layered semi-conductors where the masses are anisotropic [1]. In this communication we would like to discuss the limit of such anisotropic systems, namely the limit of electrons and holes moving in two dimensions with their transverse motion quantized in the lowest level. Then we consider the additional presence of a strong magnetic field perpendicular to the plane in which the electrons and holes move. In this limit the electrons and holes are in their lowest Landau levels and they have no kinetic energy but occupy a set of macroscopically degenerate levels. The discovery of the fractional quantum Hall effect [2] has shown that the single component system of electrons moving in a rigid background in two dimensions in a magnetic field has some exciting and unexpected properties as the concentrations of electrons in the lowest Landau level is varied. The question naturally arises whether a two-component system of electrons and holes is also of comparable interest. In this paper we will review and explore the physical basis of the remarkable properties of the electron-hole system under the same conditions. Lerner and Lozovik [3] have shown that this is a remarkable system too in that it is a manybody system whose ground state is exactly soluble for any concentration of electrons and holes. Further the ground state is a simple Bose condensate of excitons!

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2. Properties of a single exciton

The problem of a single exciton in a very strong magnetic field was considered first by Gorkov and Dzyaloshinskii [4] some years ago. In this paper we will limit ourselves to the strong magnetic field limit and consider the electron and hole to be confined to a two dimensional plane perpendicular to the magnetic field. The strong magnetic field limit means that we can confine ourselves to the lowest Landau level for the electrons and for the holes. In this Landau level the form of the wavefunctions depends on the choice of the electromagnetic gauge. Choosing a Landau gauge, i.e. a vector potential $\vec{A}=(0, Hx, 0)$, the electron and hole wavefunctions take the form

$$\begin{aligned} &\text{electrons} \quad \phi_k(\vec{r})=\frac{1}{(\sqrt{\pi} lL)^{1/2}} \exp\left[iky-(kl^2+x)^2/2l^2\right] \\ &\text{holes} \quad \chi_k(\vec{r})=\frac{1}{(\sqrt{\pi} lL)^{1/2}} \exp\left[iky-(kl^2-x)^2/2l^2\right] \end{aligned} \tag{1}$$

The electron and holes are labelled by a one dimensional vector k which runs over $L^2:2\pi l^2$ values where L is the sample dimension and l is the Larmor radius, $l^2=c/(|e|H)$. Note that the value of l is the same for both electrons and holes independent of the ratio of electron and hole masses. The latter influences the relative energy distance to the next Landau level but since we are considering only the lowest Landau level the mass ratio does not enter. The width of the Gaussian term and the total number of states in the Landau level are the same for the electrons and holes independent of the mass ratio.

The single electron and hole energies are independent of the label, k. Therefore the Hamiltonian can be written simply as the Coulomb interaction term. For a single electron and hole we can write it as

$$H_{eh}=\sum_q v(q)\rho_e(\vec{q})\rho_h(\vec{q}) \tag{2}$$

with

$$\rho_{e,h}(\vec{q})=\int d\vec{r}e^{-i\vec{q}\vec{r}}\rho_{e,h}(\vec{r}) \tag{3}$$

$$=\int d\vec{r}e^{-i\vec{q}\cdot\vec{r}}\psi^+_{e,h}(\vec{r})\psi_{e,h}(\vec{r}) \tag{4}$$

and the creation operation for an electron or hole is

$$\psi^+_e(\vec{r})=\sum_k \phi^*_k(\vec{r})a^+_k \tag{5}$$

$$\psi^+_h(\vec{r})=\sum_k \chi^*_k(\vec{r})b^+_k \tag{6}$$

The Hamiltonian (2) is then expressed as

$$H_{eh}=2\sum_{q_y k_1 k_2} F(q_y, k_1+k_2)a^+_{k_1}b^+_{k_2}b_{k_2-q_y}a_{k_1+q_y} \tag{7}$$

with the Coulomb matrix element written as

$$F(q_y, k_1+k_2) = \frac{1}{2L^2} \sum_{q_x} e^{-q^2l^2/2} v(q) e^{iq_x(k_1+k_2)l^2} \tag{8}$$

The exact eigenstates of this Hamiltonian can be written down as exciton wave functions with finite total wave vector, $\vec{K}$. They take the form

$$\Psi_{ex}(\vec{K}) = \sum_q e^{iK_x q l^2} a^+_{K_y+q} b^+_{-q} \mid vac\rangle \tag{9}$$

Operating with the Hamiltonian H_{eh} leads to

$$H_{eh}\Psi_{ex}(\vec{K}) = 2 \sum_{\substack{q\, q_y \\ k_1 k_2}} F(q_y, k_1+k_2)\, \delta_{k_1+q_y,\, K_y+q_y}\, \delta_{k_2-q_y,\, -q} a^+_{k_1} b^+_{k_2} |vac\rangle \tag{10}$$

which with the substitution $\tilde{q} = q - q_y$ gives

$$H_{eh}\Psi_{ex}(\vec{K}) = 2 \sum_{q_y} F(q_y, K_y) e^{-iK_x q_y l^2} \sum_{\tilde{q}} e^{iK_x\tilde{q}} a^+_{K_y+\tilde{q}} b^+_{-\tilde{q}} \mid vac\rangle \tag{11}$$

$$= \left(2 \sum_{q_y} F(q_y, K_y) e^{-iK_x q_y l^2}\right) \Psi_{ex}(\vec{K}) \tag{12}$$

The exciton energy $E_{ex}(\vec{K})$ then has the form

$$E_{ex}(\vec{K}) = 2 \sum_{q_y} F(q_y, K_y) e^{-iK_x q_y l^2} \tag{13}$$

$$= 2 \sum_{q_x q_y} e^{-q^2l^2/2} v(q) e^{-i(K_x q_y + K_y q_x)l^2} \tag{14}$$

The integral can be written as

$$E_{ex}(\vec{K}) = -\frac{e^2}{\epsilon l}\left(\frac{\pi}{2}\right)^{1/2} e^{-K^2l^2/4} I_0\left(\frac{1}{4}K^2l^2\right) \tag{15}$$

$$\underset{K\to 0}{=} E_{ex}(0)[1 - K^2l^2/4 + \cdots]$$

Note that the dispersion is independent of the masses as expected since the coupling to the higher Landau levels is neglected. In the limit $K \to \infty$ the dispersion approaches zero – the value of the lowest Landau levels asymptotically as

$$E_{ex}(K) = -e^2/\epsilon K l^2 + \cdots \tag{16}$$

There are $(L^2/2\pi l^2)^2$ values of K_x and K_y in total so that the whole Hilbert space of electrons and holes is spanned by the exciton states.

An interesting property of the excitons is the fact that they possess a dipole moment. The case $K_x = 0$, $K_y \neq 0$ is particularly easy in the Landau gauge and it is clear from equations (1) and (9) that the centers of the Gaussian are separated in the x-direction by a distance $K_y l^2$. This relative displacement leads to a dipole moment oriented perpendicular to the direction of motion with a value which is proportional to the velocity. The origin of the dipole is clear in a semi classical description. The electron and hole experience equal and opposite electric fields from their mutual attraction. These electric fields cancel the Lorentz force enabling the electrons and holes to proceed in straight lines. In the higher energy states of the exciton the electron and hole are far separated and the overlap of their wavefunctions is negligible. The group velocity of the excitons vanishes in both $K \to 0$ and $K \to \infty$ limits and has a maximum value. As $K \to 0$ the electric field vanishes as the charge densities overlap while as $K \to \infty$ it vanishes as the charges become infinitely separated.

3. Many excitons

We turn now to consider the case of many excitons. In usual systems [5] we can only discuss this system in two limits, namely low and high densities. In the former limit the system is described in terms of dilute gas of excitons and biexcitons etc. The free energy can be calculated as an expansion in powers of the density. In the high density limit the system is a metallic electron-hole liquid composed of separate electron and hole Fermi liquids and the free energy can be obtained in inverse powers of the density. The transition between the two limits however is not an easy one to describe and only approximate descriptions exist. In the present case there is no kinetic energy for the electrons and holes and the dimensionless parameter that characterizes the density is ν the ratio of the number of electrons (or holes) to the number of states in the Landau level or equivalent the electron (or hole) density in units of $(2\pi l^2)^{-1}$.

In the presence of many electrons and holes the Hamiltonian contains the Coulomb interaction between the electrons and also between the holes in addition to the electron-hole interaction. The explicit form is

$$H = H_{ee} + H_{hh} + H_{eh} \tag{17}$$

with

$$H_{ee} = -\sum_{qk_1k_2} F(q, k_1 - k_2) a^+_{k_1-q} a^+_{k_2} a_{k_2-q} a_{k_1} \tag{18}$$

$$H_{hh} = -\sum_{qk_1k_2} F^*(q, k_1 - k_2) b^+_{k_1-q} b^+_{k_2} b_{k_2-q} b_{k_1} \tag{19}$$

The electron-hole term was defined previously in equation (7) and the Coulomb matrix element F in equation (8).

The high density limit in this system corresponds to the limit $\nu \to 1$ and in this case the lowest Landau levels are completely filled. The wave-function is then simply

$$\Psi(\nu = 1) = \prod_{k_1} a^+_{k_1} \prod_{k_2} b^+_{k_2} |vac\rangle \tag{20}$$

The energy can be evaluated at once as

$$E_G(\nu = 1) = 2 \sum_{k_1 q} F(q, 0) \tag{21}$$

The energy per electron-hole pair then is exactly the energy of a single exciton in the $\vec{K} = 0$ state (see equation (13)). At first this seems curious but coincidental since in the filled Landau level the energy gain arises from the exchange hole around the electrons and holes rather than from the electron hole interaction. We can also easily proceed further by examining the energy of the system when a single electron and hole are missing from the filled Landau Level. As with the single exciton we can write down an exact wavefunction as

$$\Psi'(\nu=1)=\sum_q e^{iK_x q l^2} a_{K_y+q} b_{-q} \Psi(\nu=1) \tag{22}$$

Again it is straightforward to show that this is an eigenstate of the Hamiltonian with an energy exactly of $E_{ex}(\vec{K})$. This is the result of a general property. Namely we can exchange the operators $a_k^+ \to A_k$ and $b_k^+ \to B_k$, which corresponds to a change from electron to 'holes' in the electron Landau level and a corresponding change in the hole Landau level. After this substitution we find that a general relation can be written down for the energy per electron hole pair $E_{eh}(\nu)$ namely $\nu E_{eh}(\nu)=(1-\nu)E_{eh}(1-\nu)+(2\nu-1)E_{ex}$, from which the result (21) follows.

For a general value of ν, Lerner and Lozovik [3] have shown that the Hartree-Fock approximation in which the electrons and holes are paired firstly leads to an energy per electron-hole pair that is independent of ν and secondly that the corrections to the Hartree-Fock approximation cancel completely. The first result is easy to see. In the Hartree-Fock approximation the ground state wavefunction is written as

$$\Psi(\nu)=\prod_k (u+va_k^+b_{-k}^+)\mid vac\rangle \tag{23}$$

Note that in contrast to usual systems the mixing variables, u, v are constants independent of k. The normalization of the wavefunction gives $u^2+v^2=1$. The density of electrons (or holes) is given by

$$\langle\Psi(\nu)\,|\sum_k a_k^+a_k|\Psi(\nu)\rangle=\sum_k v^2 \tag{24}$$

leading to the result

$$v^2=\nu \tag{25}$$

The evaluation of the energy is straightforward.

$$\langle\Psi(\nu)|\,H\,|\Psi(\nu)\rangle=\langle\Psi(\nu)|\,H_{ee}+H_{hh}\,|\Psi(\nu)\rangle+\langle\Psi(\nu)|\,H_{eh}\,|\Psi(\nu)\rangle \tag{26}$$

$$=-2\sum_{qk} v^4(F(0,k)-F(q,0))$$

$$+2\sum_{q,k}((v^2-v^4)F(q,0)+v^4F(0,k)) \tag{27}$$

$$=2v^2\sum_{q,k}F(q,0) \tag{28}$$

Comparing with equation (13) and (21) we see at once that the energy per electron-hole is simply $E_{ex}(0)$ independent of ν – the density of electron-hole pairs as found by Lerner and Lozovik [3].

Lerner and Lozovik [3] have also shown by a diagrammatic argument that there is a complete cancellation among the corrections to the Hartree-Fock formula. Such behavior is unique among manybody systems so we would like to understand how it arises. We have seen in Section 2 that the set of exciton states spans completely the states of a single electron and hole. Therefore there are very many excited states of the system corresponding to excitons in other states. If we examine the effect of the Hamiltonian (17) on the state $\Psi(\nu)$ we see that the individual terms couple to excited states in which there are excitons in excited states. For example

$$H_{eh}\,|\Psi(\nu)\rangle = 2\sum_{qk_1k_2} F(q, k_1+k_2)a^+_{k_1}b^+_{k_2}b_{k_2-q}a_{k_1+q}\,|\Psi(\nu)\rangle \tag{29}$$

$$\begin{aligned} &= 2v\sum_{qk_1} F(q,0)a^+_{k_1}b^+_{-k_1}\prod_{k\neq k_1}(u+va^+_kb^+_{-k})\,|vac\rangle \\ &\quad + 2u^2v^2\sum_{qk_1k_2} F(q, k_1-k_2)a^+_{k_1}b^+_{-k_1-q}a^+_{k_2+q}b^+_{-k_2} \\ &\quad \times \prod_{\substack{k\neq k_1,k_1+q\\ k_2,k_2+q}}(u+va^+_kb^+_{-k})\,|vac\rangle \end{aligned} \tag{30}$$

The second term corresponds to the creation of two exciton states with momentum $\pm q$ out of the ground state. It is through such terms that possible corrections to the ground state arise. However when we examine the other terms in the Hamiltonian, they give rise to terms

$$\begin{aligned} (H_{ee}+H_{hh})\,|\Psi(\nu)\rangle &= -2u^2v^2\sum_{qk_1k_2} F(q, k_1-k_2)a^+_{k_1}b^+_{-k_1-q}a^+_{k_2+q}b^+_{-k_2} \\ &\quad \times \prod_{\substack{k=k_1,k_1+q\\ k_2,k_2+q}}(u+va^+_kb^+_{-k})\,|vac\rangle \end{aligned} \tag{31}$$

So these terms exactly cancel the other terms. Therefore there is no matrix element for the total Hamiltonian which connects the ground state to any of the other states in the manifold of the lowest Landau level. The ground is completely isolated from all the other states in this manifold. This is a remarkable result obtained first by Lerner and Lozovik [3]. Note the result applies only to the ground state. Excited states are composed of excitons with finite dipole moments so that interactions are clearly nonzero.

4. Conclusions

In conclusion we have reviewed the remarkable properties of a two dimensional system of electrons and holes in a strong magnetic field and shown how the existence of a non-interacting Boson condensate as an exact ground state of this system can be understood in terms of the isolation of this state from all the excited states in the lowest Landau level. Of course, a comparison to experiments on semiconductors requires consideration of effects such as coupling to higher Landau levels and the lack of electron-hole symmetry due to the coupled nature of the hole bands. Nonetheless experiments to probe this limit of the electron-hole system would be very interesting.

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