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Strong field effects in general relativity

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Abstract. We review some aspects of the physics of strong gravitational fields in the context of General Relativity. The discussion concentrates upon the physics of black holes and especially upon the irreversible behaviour of the surface of a black hole which is closely analogous to that of a fluid membrane endowed with finite values of electrical conductivity and shear and bulk viscosities.

§1. Introduction

Non-linear effects, in particular strong-field effects, are of great interest in all branches of physics. Their study has often led to the discovery of new phenomena that could not have been foreseen when working only at the linear (weak-field) level: for instance soliton solutions of non-linear field equations and chaotic behaviour of dynamical systems. General Relativity is no exception to the rule and the thorough investigation of non-linear gravity physics in the past twenty five years has led to a rich harvest of new concepts including geons, black holes, white holes, wormholes, chaotically oscillating singularities, etc. (for a comprehensive review see [1]). In the following discussion I wish first, to emphasize the specificity of strong field effects in General Relativity as compared to strong field effects in other relativistic theories of gravity, and second, to focus on the physics of black holes where the non-linear gravitational effects lead to a remarkable irreversible behaviour which is closely analogous to the irreversible thermodynamics of an electrically conducting viscous fluid membrane. Moreover this analogy seems to go beyond the classical level and to extend to the quantum level, thereby suggesting the existence of a deep connection between gravitation, thermodynamics and quantum theory. As the latter three topics were of particular interest to Professor E. C. G. Stueckelberg, it is a great honour for me to dedicate the following discussion of strong field effects in General Relativity to his memory.

§2. Specificity of strong field effects in General Relativity

One might think that it is quite bold, and somewhat premature, to investigate in great detail the strong field effects in the context of General Relativity because this theory has been tested and confirmed mainly through observations done in the Solar system where the gravitational field is everywhere very weak (and quasi-static). Indeed the parametrized post-Newtonian formalism has shown (see e.g. [2]) that many alternative theories of gravity have or can be adjusted to have the same (quasi-static) weak-field limit as General Relativity, and therefore can also explain all the gravitational experiments done in the Solar system, even though they would behave very differently in the (dynamic and/or) strong-field regime. However the purpose of this section is to stress that it can be convincingly argued, thanks to the very peculiar nature of strong field effects in General Relativity, that the recent observations of the binary pulsar PSR 1913 + 16 [3] provide an indirect confirmation of the full non-linear structure of Einstein's equations.

Indeed heuristic arguments ([4], [5], [2]) have indicated that, in the context of most alternative theories of gravity, the motion of a binary system should exhibit an irreversible decay of the orbital period, P, of the following form:

$$\frac{dP}{dt}=\alpha(s_1-s_2)^2\left(\frac{v}{c}\right)^3+\beta\left(\frac{v}{c}\right)^5 \tag{1}$$

where v is some mean orbital velocity, where s_1 and s_2 are 'strength parameters' measuring how strongly self-gravitating each member of the binary system is ($s_a \simeq -E_{\text{grav.}a}/m_a c^2$, $a=1, 2$, $E_{\text{grav.}}=$ gravitational binding energy), and where α and β are dimensionless numbers of order unity. Now for a binary system containing strongly self-gravitating bodies (as is the case of the binary pulsar PSR 1913 + 16) the difference between the strength parameters will be of order unity or at least a few tenths (barring an improbable finely tuned compensation between s_1 and s_2). As $v/c \sim 10^{-3}$ in the case at hand, this means that the first term on the RHS of equation (1) (which is a *strong field effect*) will dominate by several orders of magnitude the second one, thereby predicting a period decay $dP/dt \sim (v/c)^3 \gg (v/c)^5$, *except* if $\alpha \equiv 0$.

Now most alternative theories of gravity are heuristically expected to have $\alpha \neq 0$, $\alpha \sim 1$ (see [2]), while it has been shown by detailed kinematical calculations that in General Relativity one has $\alpha \equiv 0$ ([6], [7]) and that the second term of equation (1) is given by ([8], [9]):

$$\left[\beta\left(\frac{v}{c}\right)^5\right]^{\text{G.R.}}=-\frac{192\pi}{5c^5}\left(\frac{2\pi G}{P}\right)^{5/3}\frac{m_1 m_2}{(m_1+m_2)^{1/3}}\frac{1+73e^2/24+37e^4/96}{(1-e^2)^{7/2}}. \tag{2}$$

It should be noted that the result $\alpha \equiv 0$, is a direct consequence of a peculiar property of the strong-field regime in General Relativity, namely the 'effacing property' to the effect that the general relativistic motion of two strongly self-gravitating objects depends, up to a very high accuracy, only on two parameters that is the 'Schwarzschild' masses of the objects into which all the strong field effects have been absorbed (similar to the absorption of some of the divergencies of Quantum Field Theory into the 'dressed' masses). On the contrary in most alternative theories of gravity the result $\alpha \neq 0$ signals a distinct influence of strong field effects on the motion of a binary system (as is clear from equation (1)). Now the observations of the binary pulsar PSR 1913 + 16 have led to an estimate of the period decay which agrees with the general relativistic prediction (2) within 4% [3]. This close agreement is therefore a strong indirect confirmation of the strong field behaviour of General Relativity.

§3. Global mechanics of black holes

In the previous section we considered a particular type of strong gravitational field: the one generated by a compact object such as a neutron star. Now one of the main open problems of the physics of strong gravitational fields is to understand what happens when one tries to generate even stronger gravitational fields by further condensing a compact object. The current view, based on a paradigmatic work of Oppenheimer and Snyder [10], is that such a further condensation will lead to gravitational collapse resulting in the formation of a black hole: i.e. a region of space-time which may contain singular ('infinitely strong') gravitational fields but which is 'invisible' (in the sense that no signals can escape from the black hole region and reach infinity). This view has been given a precise formulation by Penrose as the 'cosmic censorship hypothesis' (see e.g. [11]). Some recent work of Christodoulou has shown that the generic collapse of a spherically symmetric dust ball [12] or scalar wave packet [13] ultimately leads to the formation of a black hole although it may also form, at some intermediate stage, a somewhat mild (infinitely redshifted) 'naked singularity'. On the whole this can be considered as a confirmation of the spirit, if not of the letter, of the cosmic censorship hypothesis. In the following we shall *assume* the cosmic censorship hypothesis and concentrate upon the physics of black holes after their formation.

One of the basic results of black hole theory is that the boundary of the black hole region, also called 'black hole surface' or 'horizon' is a null (i.e. everywhere tangent to the local light cone) hypersurface H admitting compact sections and generated by non terminating null geodesics [14], [15]. It has proven both mathematically convenient and physically suggestive to adopt a '2 + 1' view of the horizon, i.e. to split the space-time structure of H (a three-dimensional manifold) into (two-dimensional) space + 'time'. More precisely let us consider an (arbitrary) family of two-dimensional spatial sections S of the horizon and an (arbitrary) parameter, t (playing the role of time) labelling the sections. On each section S ($t = \text{const.}$) we introduce two arbitrary surface coordinates x^A ($A = 2, 3$) (similar to θ and φ on a sphere). The full four-dimensional coordinate system x^a ($a = 0, 1, 2, 3$) will be chosen to be regular near H and such that $x^0 \equiv t$, $x^1 = 0$ on H and, when $a = A = 2, 3$, $x^a = x^A$ on H.

The space-time metric $ds^2 = g_{ab}(x)\, dx^a\, dx^b$ induces on each section S a time-dependent positive–definite metric

$$ds^2\,|_S = \gamma_{AB}(x^C, t)\, dx^A\, dx^B \tag{3}$$

(where numerically $\gamma_{AB} = g_{AB}$ for $A, B = 2, 3$). One can then define the total area of the surface of the black hole at 'time' t as:

$$S(t) = \oint_S \sqrt{\gamma}\, dx^2 \wedge dx^3 \tag{4}$$

(where $\gamma := \det \gamma_{AB}$).

One of the most remarkable results of black hole theory is that, assuming the cosmic censorship hypothesis and a positivity property of the stress-energy tensor, it follows necessarily from Einstein's equations that $S(t)$ must be a monotonically increasing function of time [16], [17], [18]:

$$\frac{dS(t)}{dt} \geqslant 0. \tag{5}$$

The *irreversibility* inherent in this result is quite remarkable when one considers that no statistical hypothesis entered into its derivation and that a black hole is essentially made out of pure curved space-time! (however it must be recalled that the mere definition of a black hole already selects a preferred sense of time because it speaks only of *emitted* signals and not of received signals). The result (5) is often named the 'second law of black hole mechanics' in view of its obvious analogy with the second law of thermodynamics. The analogy has been extended by deriving the 'first law of black hole mechanics' [19], [20], [21], [22] giving the total mass–energy variation between two neighbouring *equilibrium states* of a black hole:

$$\delta M = \Omega \delta J + V \delta e + \frac{g}{8\pi} \delta S, \tag{6}$$

where we have considered for the sake of simplicity an isolated black hole. In equation (6) δM, δJ, δe and δS are respectively the variations in the total mass, total angular momentum, total electric charge and total area of the black hole, Ω is the angular velocity of the hole (physically defined as the limiting angular velocity of any test mass moving very near the hole and being dragged around it by its strong gravitational field), V the (comoving) electric potential, and g the 'surface gravity' of the hole. The surface gravity is defined by:

$$l^a_{;b} l^b = g l^a \tag{7}$$

where the semi-colon denotes the space-time covariant derivative and where l^a is the null vector normal (and tangent) to the horizon. One normalizes l^a by:

$$l^a = \frac{dx^a(t)}{dt}, \tag{8}$$

where $x^a = x^a(t)$ are the integral space-time-curves of the null vector $\vec{l}$ ('generators' of H) expressed in terms of the time parameter t (for stationary black holes one chooses t so that it is linked to the stationary character of the black hole geometry: i.e. $\partial/\partial t$ = time-translation Killing vector).

Comparing equations (5) and (6) with their obvious thermodynamical analogues *suggests* [23] that there exists a constant α such that, in some sense, αS measures the 'entropy' and $g/8\pi\alpha$ the 'temperature' of a black hole, and that $g\delta S/8\pi$ can be thought of as being the 'heat' generated in the black hole during a transition between two neighbouring equilibrium states.

§4. Surface mechanics of black holes

In the previous section we compared the global behaviour of a black hole with the global laws of a classical thermodynamical system. Remarkably enough it is possible to push this analogy further by going from the global to the local point of view, and also by dealing no longer with the 'thermostatic' properties (transition between equilibrium states) but rather with the 'irreversible thermodynamical' properties (phenomena occurring in non-equilibrium states where entropy is continuously being generated). Indeed it has been shown that the local evolution of the surface of a black hole (as deduced from Einstein's equations) admits a very precise analogy with the local irreversible mechanics of an electrically conducting fluid membrane endowed with surface electrical conductivity [24], [25] and surface viscosities [26], [27], [28], [29].

In order to develop this analogy let us first discuss the *kinematical* properties of the horizon. In the $2+1$ view of the horizon the 2-surface S can be considered as a '*membrane*' (or 'bubble') constituted of (fictitious) '*particles*' whose trajectories are the 'generators' (i.e. the integral curves $x^a(t)$ of the null normal vector $\vec{l}$, see equation (8)). We can then introduce the concept of the *surface velocity field* of the membrane:

$$v^A := \frac{dx^A(t)}{dt} \quad (A = 2, 3), \tag{9}$$

so that

$$\vec{l} := l^a \frac{\partial}{\partial x^a} = \frac{\partial}{\partial t} + v^A \frac{\partial}{\partial x^A}. \tag{10}$$

In the case of isolated stationary black holes it can be verified that ($\partial/\partial t$ being chosen to be the time Killing vector) the magnitude of v^A is everywhere $\leqq c$. It is an open conjecture [29] to know whether this inequality holds for general distorted stationary holes.

Returning to a general dynamic black hole evolution one can measure the rate of change of the relative distances between the 'particles' constituting the black hole 'membrane' by means of the deformation tensor:

$$D_{AB} := \frac{1}{2}\frac{D\gamma_{AB}}{dt} = \frac{1}{2}\left(\frac{\partial \gamma_{AB}}{\partial t} + v_{A|B} + v_{B|A}\right), \tag{11}$$

where $\gamma_{AB}(x, t)$ is the 2-metric of S, the vertical bar (in $v_{A|B}$) denotes the covariant derivative with respect of γ_{AB}, and D/dt denotes the 'convective derivative' (or 'material derivative'), i.e. in mathematical terms the 'Lie derivative' with respect to $\vec{l}$ (equation (10)). It is usual to decompose the deformation tensor into its trace:

$$\theta := \gamma^{AB} D_{AB}, \tag{12}$$

called the *expansion,* which measures the local rate of change of area, and its trace-free part, the *shear* tensor:

$$\sigma_{AB} := D_{AB} - \tfrac{1}{2}\theta\gamma_{AB}. \tag{13}$$

Moreover if some electromagnetic field lines (described by the electromagnetic tensor F^{ab}) and/or some electric currents (described by the four-current $J^a = F^{ab}_{;b}/4\pi$) are running through the black hole it is natural to attribute to the black-hole 'membrane' both a (fictitious) *surface charge density* [30]

$$\sigma_H := \frac{1}{4\pi} F^{0a} l_a, \tag{14}$$

a (fictitious) *surface current density* [24], [25]

$$K^A := \frac{1}{4\pi} F^{Aa} l_a, \tag{15}$$

and a (fictitious) *surface conduction current density*

$$C^A := K^A - \sigma_H v^A. \tag{16}$$

These definitions are natural in the sense that the total electric charge of the hole (defined by a Gauss integral around it) is given by

$$e = \oint_S \sigma_H \, dS, \tag{17}$$

and that the current $\vec{K}$ 'flowing on the surface of the black hole' allows one to close any (real) external currents penetrating the hole in the sense that the following law of conservation of electricity holds:

$$\frac{1}{\sqrt{\gamma}} \frac{\partial}{\partial t} (\sqrt{\gamma}\, \sigma_H) + \frac{1}{\sqrt{\gamma}} \frac{\partial(\sqrt{\gamma}\, K^A)}{\partial x^A} = \text{injected current} = -J^a l_a. \tag{18}$$

In order to display more clearly the 'irreversible thermodynamical' properties of the black-hole membrane let us now restrict ourselves, for a moment, to the *quasi-stationary* states of a black hole, i.e. to black hole geometries of the type $g_{ab} = g^{(0)}_{ab} + \lambda g^{(1)}_{ab} + \lambda^2 g^{(2)}_{ab} + \cdots$, and electromagnetic fields of the type $F_{ab} = \lambda F^{(1)}_{ab} + \lambda^2 F^{(2)}_{ab} + \cdots$ (where λ is an expansion parameter measuring the strength of small external perturbations) such that the first order field configuration $g^{(0)}_{ab} + \lambda g^{(1)}_{ab}$, $\lambda F^{(1)}_{ab}$, is stationary, the time dependence being rejected at the second order. We normalize the evolution parameter of the black hole, t, in such a way that $\partial/\partial t$ is the time-translation Killing vector of the first order geometry. Then one finds that the area of the black hole is slowly increasing and that the total 'rate of generation of heat' (or '*dissipation*') in the black hole membrane,

$$D := \frac{g^{(0)}}{8\pi}\frac{dS}{dt}, \tag{19}$$

is given by

$$D = \lambda^2 \oint dS\{2\eta_H \sigma^{(1)}_{AB}\sigma^{(1)AB} + \rho_H C^{(1)}_A C^{(1)A}\} \tag{20}$$

where the indices are moved with the surface metric γ_{AB}. Equation (20) has precisely the form expected for the dissipation in a viscous, electrically conducting membrane endowed with a surface shear viscosity η_H and a surface electrical resistivity ρ_H. Because of their surface nature the quantities η_H and ρ_H are dimensionless (contrarily to their familiar volume analogues). Their values are: $\eta_H = 1/16\pi$ [26], [27], [28] and $\rho_H = 4\pi = 377$ ohms (the impedance of the vacuum) [24], [25].

What is even more remarkable is that the analogy with a physical membrane can be extended even to the level of the local dynamical laws constraining the field variables appearing in the dissipation D. Indeed on the one hand there exists an Ohm's law connecting the conduction current to the surface electromagnetic field of the hole:

$$E_A + \varepsilon_{AB} B_\perp v^B = \rho_H C_A \tag{21}$$

where ε_{AB} is the antisymmetric Levi Civita tensor on S, and where the tangential electric field E_A and the normal magnetic induction $B_\perp$ are defined by restricting the electromagnetic two-form to the horizon $((\frac{1}{2}F_{ab}\,dx^a \wedge dx^b)_H = (E_A\,dx^A) \wedge dt + B_\perp\,dS)$. This surface electromagnetic field satisfies the Faraday law:

$$\operatorname{curl} \vec{E} = -\frac{1}{\sqrt{\gamma}}\frac{\partial}{\partial t}(\sqrt{\gamma}\,B_\perp). \tag{22}$$

On the other hand the shear tensor σ_{AB} satisfies an equation which is analogous to the Navier–Stokes equation [28], [29]. This equation is simplest in the case of a slowly rotating ($v^2/c^2 \ll 1$), quasi-stationary, uncharged black hole when it reads:

$$2\eta_H \sigma^B_{A|B} - p_{|A} = 0(v^2/c^2). \tag{23}$$

The 'surface pressure' p of the black hole 'membrane' appearing in the Navier–Stokes equation (23) is proportional to the surface gravity.

$$p = +\frac{g}{8\pi}. \tag{24}$$

It can also be checked that the very general connection ('minimum entropy production principle') found by Prigogine [31] between the dissipative phenomena and the dynamical equations of a classical stationary thermodynamical system is still valid in the case of a slowly rotating black hole weakly perturbed by stationary external 'thermodynamical forces'. This means that the dissipation D considered as a functional of the unconstrained degrees of freedom of the velocity field and electric field is *minimum* when the dynamical equations (Ohm, Navier-Stokes) are satisfied [28], [29].

It should be noted that the full set of 'membrane analogies' can be generalized to an arbitrary dynamic evolution (even very violent) of a black hole [28], [29]. The only new features that arise are:

1) the appearance of a (negative) bulk viscosity for the membrane: $\zeta_H = -1/16\pi$;

2) a full Navier–Stokes equation:

$$\frac{D\pi_A}{dt} = -p_{|A} + 2\eta_H \sigma^B_{A|B} + \zeta_H \theta_{|A} + f_A, \tag{25}$$

linking the surface stresses with the external tangential 'force' density (flux of impulsion: $f_A = -l^a T_{aA}$) and with the convective derivative of the surface density of momentum π_A defined by:

$$l^a_{;A} = -(8\pi)\pi_A l^a + D^B_A \partial_B x^a; \tag{26}$$

3) the appearance of a negative response time $-\tau = -1/g$ in the equation connecting the local area increase to the viscous and Joule dissipations:

$$\frac{D}{dt}(dS) - \tau \frac{D^2}{dt^2}(dS) = \frac{8\pi}{g}[2\eta_H \sigma_{AB}\sigma^{AB} + \zeta_H \theta^2 + \rho_H C_A C^A]\, dS. \tag{27}$$

The preceding 'membrane' approach to black hole physics has recently been extended, and connected to the physics happening in the external space-time by means of the introduction of the concept of a 'stretched horizon' [32], [33], [34].

§5. Quantum theory and black hole physics

Stueckelberg [35], [36] has introduced a new way of looking at the phenomenon of pair creation by strong electromagnetic fields, namely the idea that particles (and antiparticles) are 'moving' in the arena of space-time according to the flow of some 'proper time' parameter λ running from $-\infty$ to $+\infty$, and that the only difference between particles and antiparticles is that particles are going forward in time ($dx^a/d\lambda$ future directed) while antiparticles are going backwards ($dx^a/d\lambda$ past directed) (see the Fig. 1 of [35]). Then, at the level of first quantization, he wrote a Schroedinger-type equation for the 'motion' in space-time of wave-packets $\psi(x^a, \lambda)$:

$$i\hbar \frac{\partial}{\partial\lambda}\psi(x, \lambda) = \tfrac{1}{2}g^{ab}(x)\left(\frac{\hbar}{i}\frac{\partial}{\partial x^a} - \varepsilon A_a(x)\right)\left(\frac{\hbar}{i}\frac{\partial}{\partial x^b} - \varepsilon A_b(x)\right)\psi(x, \lambda). \tag{28}$$

In this approach the phenomenon of pair creation means that some wave-packet incident *from the future* on some strong field region of space-time will be partially *reflected* towards the future (see the Fig. 2 of [35]). Then the *reflection coefficient*, say R (square of the relative reflected amplitude) gives directly the relative probability for creating one pair (in the state considered). The absolute probability for creating n pairs is

$$p_n = p_0 R^n \tag{29}$$

where p_0 is obtained by requiring that the total probability is one. This yields

$$p_0 = (1 \mp R)^{\pm 1} \tag{30}$$

(upper sign for bosons and lower sign for fermions). The mean number of pairs created in the state considered ($\langle n \rangle = \sum np_n$) is then easily found to be:

$$\langle n \rangle = \frac{R}{1 \mp R}. \tag{31}$$

These results can be obtained equivalently by using as a basic mental picture the Dirac sea of negative states. The role of the 'reflection' coefficient R is then played by the 'transmission' (or better 'transmutation') coefficient $T = R/(1 \mp R)$ between an ingoing negative-energy state and an outgoing positive-energy state ('Klein paradox', for a review of this approach and its link with Stueckelberg's see e.g. [37]).

The preceding approaches can be applied to the phenomenon of particle creation by the strong gravitational field of a black hole. This was done [38], [37] in an attempt to better grasp the physical and mathematical assumptions underlying Hawking's prediction [39] of black hole quantum evaporation. The outcome of such calculations is that if one *assumes* that the space-time geometry, and the wave function, are *analytic* near the horizon then the 'reflection' coefficient R can be easily computed (see the à-la-Stueckelberg Fig. 1 of [38]) and is found to be:

$$R = \exp\left\{-\frac{2\pi}{g}\left(\omega - m\Omega - \frac{\varepsilon}{\hbar}V\right)\right\}, \tag{32}$$

where ω is the frequency at infinity of the wave function, m its azimuthal quantum number, and ε the electric charge of the particle. The exponential nature of $R(\omega)$ together with the algebraic form (31) for $\langle n \rangle$ makes is evident that (disregarding the 'greying' transmission factor of the combined potential and centrifugal barrier outside the hole) the corresponding spectrum of created particles is Planckian, with a temperature:

$$T = \frac{\hbar}{k}\frac{g}{2\pi}. \tag{33}$$

This is Hawking's result [39] which is beautifully consistent with the previously discussed classical investigations of the thermodynamic-like properties of black holes. This result suggests that one should take seriously the view that αS (with α now determined to be $k/4\hbar$) is really a measure of the physical entropy of a black hole. This would mean that the physics of black holes is realizing a profound synthesis between gravity physics, thermodynamics and quantum theory. Such a conclusion is certainly very appealing, however it must be stressed that it is somewhat premature for the following reasons:

1) All existing derivations of the Hawking quantum evaporation (for a review and references see e.g. [40]) deal, at some intermediate stage, with physically infinitesimally small distances near the horizon (e.g. the analyticity property of the space-time geometry in the derivation quoted above); Any cut-off at small distances (or high frequencies) (even for distances much smaller than the Planck length $l_P = (\hbar G/c^3)^{1/2} = 1.616 \times 10^{-33}$ cm) would probably destroy or at least considerably alter the phenomenon; On the other hand many different arguments show that one should not trust the existence of a continuum below the length scale l_P. This casts a doubt on the mere existence of Hawking's thermal evaporation process.

2) Even if some 'thermal' radiation is coming out of the hole this does not prove that $kS/4\hbar$ measures the 'entropy' of the black hole. One must still a) prove the validity of the 'Generalized Second Law of Thermodynamics' [41] for the total system: black hole + external universe, and b) interpret statistically $kS/4\hbar$ (for recent proposals see [42], [43], [44] and references therein).

However, whatever becomes of the 'thermal evaporation' of black holes and of its associated synthesis between gravitation and quantum theory, it is clear that, among all non-linear phenomena, strong field effects in General Relativity deserve a special place by their ability to give rise to a *qualitatively* new physics (irreversible behaviour of a membrane) when using as basic ingredient only the curved geometry of space-time.

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