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SELF-POLARIZATION OF PROTONS IN STORAGE RINGS

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ABSTRACT

It has been proposed that stored proton or heavy ion beams can be polarized by spatially separating particles with opposite spin directions, using the Stern-Gerlach effect in alternating quadrupole fields. The growth rate of the vertical betatron amplitude is calculated for beam halves with opposite polarizations rotating in the horizontal plane, at intrinsic spin resonance $a\gamma - \nu_y$ = integer. This polarization method would work best with rings having large diameter, low vertical emittance, low vertical betatron tune, and strong superconducting quadrupoles. Provided that suitable strong quadrupoles exist, the method might advantageously replace the present technique for obtaining polarized proton or heavy ion beams, where low energy polarized beams are first generated by a source and then accelerated through numerous depolarizing resonances up to the final energy.

1. Introduction

The acceleration of polarized protons has been studied extensively in theory [1,2] and the techniques have been demonstrated in the ZGS [3], SATURNE [4], and the AGS [5]. In all these machines routine operation with polarized beam has been achieved, but it is clear that in larger machines the number of depolarizing resonances is such that new techniques must be sought for. A global solution has been proposed by Derbenev and Kondratenko [6], who suggest the use of "snake" magnets which invert the spin and eliminate the depolarization in a fashion analogous to spin echo in solids and liquids, familiar to polarized target specialists.

In this paper a different approach for obtaining stored polarized proton beams is presented, in which the unpolarized beam is polarized only at the energy of storage. This self-polarization is based on the Stern-Gerlach effect: in an inhomogeneous magnetic field the particles with spins aligned parallel or antiparallel to the field, are deflected in opposite directions and become spatially separated [7].

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A conventional storage ring has quadrupole magnets with inhomogeneous fields. Considering one single magnet, particles with different spin directions are deflected by different angles in the quadrupole field. Since this difference in the deflection angle is extremely weak, the next quadrupole with opposite polarity would almost exactly cancel the effect of the first. Fortunately, the bending magnets in between the quadrupoles may rotate the spin in such a way that the kicks obtained in the quadrupoles add up, always following the variation of the vertical deflection angle. For a FODO cell, therefore, one might require that the vertical betatron phase advance from the first to the second quadrupole approximately equals the spin phase advance minus π over complete precession cycles in the dipole magnet between the quadrupoles. A more general additional requirement is that the spin and vertical betatron phase advances, modulo 2π, in a complete machine cycle are equal. This can be expressed as

$$a\gamma - n = \nu_y - m, \tag{1}$$

where $a = (g - 2)/2$ (= 1.793 for protons), $\gamma = (1 - \beta^2)^{-1/2}$ is the Lorentz factor, n is the nearest integer below $a\gamma$ (the number of spin cycles in a machine turn), ν_y is the number of vertical betatron oscillations in a machine turn and m is the nearest integer below ν_y. This corresponds to an intrinsic resonance

$$a\gamma = n - m + \nu_y,$$

which may be strong or weak depending on whether n - m is a multiple of the periodicity P of the machine lattice, and on the magnitude of the quadrupole field errors. This resonance quickly destroys any vertical polarization by dephasing the vertical spin components. The horizontal spin components, however, will remain synchronized by the resonant horizontal fields and there is no horizontal depolarization in first order.

This situation can be visualized in analogy with magnetic resonance in solids, where transformation to rotating frame allows to remove the main time dependence of the spin Hamiltonian. It is known experimentally that in solids no depolarization will occur as long as the oscillating transverse field is maintained, in spite of large randomly fluctuating dipolar fields.

Although the above considerations seem straightforward for a monoenergetic beam, it has to be proved that the method is also practical with real beams consisting of particles with different energies. An attempt to show this will be made in the present paper in first order, but many higher order effects remain to be studied. Only a brief outline will be presented on the possible ways of tuning the spin and betatron motions over extended periods.

2. Separation of Particles with Different Spin Directions

The field of a quadrupole magnet is described in first order by

$$B_x = -by,$$

$$B_y = bx,$$

in a coordinate system where x and y are the transverse horizontal and vertical axes, respectively, and z is tangent to the design orbit. The force on the magnetic dipole μ is [8]

$$\delta\bar{F} = \nabla(\bar{\mu}\cdot\bar{B}) = b\nabla(-\mu_x y + \mu_y x) = b(\bar{I}\mu_y - \bar{J}\mu_x) \qquad (2)$$

and the additional deflections of the particle in the x-z and y-z planes are, respectively,

$$\delta x' = -b\mu_y \Delta L/E$$

$$\delta y' = b\mu_x \Delta L/E$$

where μ_x and μ_y are the projections of the magnetic moment along the x and y directions, ΔL is the length of the quadrupole and E is the beam energy. We note that when the spin points in the direction of the beam, there is no deflection; when it points along x, the particle is deflected vertically; when it points up, the deflection is horizontal. The radial field gradient b is positive for quadrupoles focusing in the vertical plane and negative for those focusing in the horizontal plane.

In between the quadrupoles there are bending magnets with homogeneous field. The motion of the spin in a homogeneous field is described by the BMT equation [9]:

$$d\bar{s}/dt = (q/m\gamma)\bar{s}\times\big((1 + a)\bar{B}_\parallel + (1 + a\gamma)\bar{B}_\perp\big).$$

Here $\bar{s}$ is a unit vector describing the spin direction in the laboratory frame, q is the charge of the particle, m is the rest mass, $\gamma = (1 - \beta^2)^{-1/2}$ is the Lorentz-factor and $a = (g - 2)/2$. $\bar{B}_\parallel$ and $\bar{B}_\perp$ are laboratory frame fields parallel and perpendicular to the motion of the particle. The required integrated field strength for a 90° spin rotation for protons is 2.7 Tm, independent of energy in the relativistic limit.

Let us consider a synchrotron with a large diameter, where each FODO cell satisfies the requirement that the horizontal spin components roughly reverse in the interleaved dipole magnet. We shall furthermore assume that the vertical betatron tune ν_y is small so that we may approximate the vertical motion by the homogeneous equation

$$y'' + \omega_y^2 y = 0 \qquad (3)$$

which gives undamped sinusoidal vertical motion

$$y = y_o \sin(\omega_y t + \phi),$$

where $\omega_y = 2\pi\nu_y f$ and f is the cycling frequency of the machine. The inhomogeneous equation including the perturbation due to the Stern-Gerlach force [of Equation (2)] is

$$y'' + \omega_y^2 y = \frac{\delta F(t)}{\gamma m} = - \frac{b(t)\mu_x(t)}{\gamma m} \qquad (4)$$

where the gradient b(t) now depends on the machine lattice. Equation (4) can be solved, for example, by Fourier techniques when the lattice is known. Assuming (for simplicity) an asymmetric arrangement of quadrupoles in a superperiod, we may write

$$b(t) = b\sum_k c_k \sin k\omega_o t, \qquad (5)$$

where,

$$c_k = \frac{2}{T} \int_o^T \frac{b(t)}{b} \sin \frac{k\pi t}{T} \, dt \qquad (6)$$

and $2T = (fP)^{-1} = 2\pi/\omega_o$ is the time of passage through the superperiod. The projection $\mu_x(t)$ of the magnetic moment along x is

$$\mu_x(t) = \mu \cos \left(\int_o^t \omega_s(t') \, dt' \right), \qquad (7)$$

where

$$\omega_s(t) = \frac{qa}{m} B_y(t) \qquad (8)$$

is the spin precession relative to the orbital motion, and $B_y(t)$ is the vertical dipole field. The Fourier spectrum of Equation (7) consists of a large number of lines centered around $\omega = (qa/m)B_o$.

Again for simplicity, let us assume a perfectly round machine with constant dipole field $B_y(t) = B_o$, which gives a constant spin precession frequency ω_s. Equation (4) then becomes

$$y'' + \omega_y^2 y = - \frac{b\mu}{\gamma m} \sum_k \frac{1}{2} c_k \left(\sin(\omega_s + k\omega_o)t - \sin(\omega_s - k\omega_o)t\right). \qquad (9)$$

The complementary solution of this equation has terms such as

$$y = \frac{t \cos \omega_y t}{2\omega_y} \frac{b\mu}{2\gamma m} c_k = y_o(t) \cos \omega_y t \qquad (10)$$

when

$$\omega_s - k\omega_o = \omega_y. \qquad (11)$$

By scaling Equation (11) with the angular frequency $2\pi f$ of the machine, we get

$$a\gamma - kP = \nu_y \tag{12}$$

which is the same as Equation (1) when $n - m = kP$. Because of inevitable small errors in the quadrupole fields, also the periodicity $P = 1$ is possible in Equation (12); there is no discrepency now between our heuristic initial arguments and the somewhat better founded result of Equation (12).

Equation (10) now allows us to write the vertical splitting rate of the beam halves with opposite μ_x:

$$\frac{\Delta y}{\Delta t} = 2\dot{y}_o\ (t) = \frac{\mu b}{4\ \pi f \nu_y\ \gamma m}\ c_k. \tag{13}$$

Because in most synchrotrons b/γ and ν_y are roughly constants, the only energy dependence is in c_k. The maximum value that c_k may obtain is when $b(t)$ and $\sin(k\pi t/T)$ are in phase all over the superperiod, which gives $c_k \leq 4/\pi$. In most of the existing machine lattices c_k may be assumed to be below 1 but above 0.1 at optimum energy. By inserting values of b/γ, ν_y, and f appropriate for existing or planned large colliders such as the CERN SPS, Fermilab Tevatron, and SSC, gives separation speeds below 1 mm/day, which make the method somewhat impractical. It is interesting, however, to extrapolate toward a purpose-built machine, which is optimized for spin separation. Quadrupole magnets can be built with gradient $b = 100\ \mathrm{Tm}^{-1}$ [10]; by assuming alternating gradient magnets with this strength all over the ring with $f = 50$ kHz (such as the CERN SPS); operating at proton energy $\gamma m_p = 100\ \mathrm{GeV}/c^2$ and $\nu_y \cong 10$, we find

$$\frac{\Delta y}{\Delta t} = (4.5\ \frac{\mathrm{mm}}{\mathrm{h}})\ c_k.$$

If the lattice is optimized to give $c_k \approx 1$ at this energy, the separation of the beam into polarized halves may be much faster than the rate of blow-up or loss.

The derivation of a more general expression for the c_k in Equation (13) will be given in a forthcoming publication [11].

We emphasize that the above high rate of beam separation buildup may be unachievable even with very strong quadrupole insertions in any of the existing accelerators.

3. Discussion

At least the following problems must be solved before our suggested scheme may be considered as a serious alternative for polarized beams:

1. Beam energy spread makes a spread both in the spin tune $a\gamma$ and the betatron tune ν_y. Can these be compensated so that all spins will maintain phase coherence with the betatron oscillations?

2. The spin and betatron oscillations must be phase-locked over extended periods of time. Is this possible with existing beam-diagnostic devices?

3. Higher order effects may destroy (dephase) the rotating horizontal polarizaton. One such mechanism may be the coupling of the vertical and horizontal betatron oscillations due to uncompensated axial fields, for example.

Without attempting to find definite answers to these questions, we would like to point out that there are several techniques to make the spin tune energy independent. These are generally based on spin rotation about the longitudinal or the transverse horizontal axis [12]. This can be accomplished with the "Siberian Snakes" [6], which rotate the spin through an angle independent of energy. Other spin manipulation techniques might involve vertical bend spin rotators and closed orbit distortions, which need to be optimized only at the energy of storage, if further acceleration or deceleration of the polarized beam is not required.

The harmonics of the betatron frequency can be measured by specially built pick-ups and low-noise amplifiers. The additional motion due to the spin, Equation (9), might be observable as a sideband to a suitable high harmonic of the betatron frequency [13], and allow the phase locking of the two oscillations. This technique might also allow the monitoring of the separation build-up, and hence polarization.

The energy dependence of the betatron tune is suppressed to a large extent by the averaging due to synchrotron oscillations at low energies. At very high energy, the tune spread of ν_y must be made small, which clearly also requires very good measurement of the betatron oscillations.

The acceleration or deceleration from the exact resonance $a\gamma = n - m + \nu_y$ would turn the rotating horizontal polarization adiabatically in the vertical orientation, conserving the opposite alignments in the beam halves. Returning back to the resonance would restore the original horizontal polarization and, to first order, the phase relationship between the spin and betatron motions. The adiabatic spin manipulation has been proven experimentally during acceleration and deceleration [4].

We conclude that self-polarization of protons in existing colliders and storage rings is impractically slow. However, a purpose-built machine might provide a sufficiently fast polarization speed so that it might be worth examining the outstanding problems related with the method.

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