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SUPERSTRINGS

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Abstract : Superstrings are the latter-day superstars for physicists in search of the Theory Of Everything (TOE) unifying all known interactions and particles. They were developed in a very different context, and it is only slowly that the discovery of their remarkable properties, briefly presented here, has given rise to such ambitions.

Superstrings are at present a very active field in particle theory. A relativistic string is a generalization of the concept of a point particle. It is the simplest extended object compatible with the axioms of special relativity. Superstrings are the first and so far the only candidates for a consistent quantum theory unifying gravitation together with strong, weak, and electromagnetic interactions, and furthermore having some chance of describing the real world.

Historically, strings were not introduced for such an ambitious enterprise, but appeared in a rather roundabout way at the end of the sixties, to describe some properties of hadronic interactions.

In the charge-exchange scattering of π^- mesons on protons for example, $\pi^- + p \rightarrow \pi^0 + n$, the low-energy cross-section exhibits peaks (Fig. 1) corresponding to the formation of a highly unstable baryonic resonance (Δ, N*, etc.), according to the Feynman diagram of Fig. 2a.

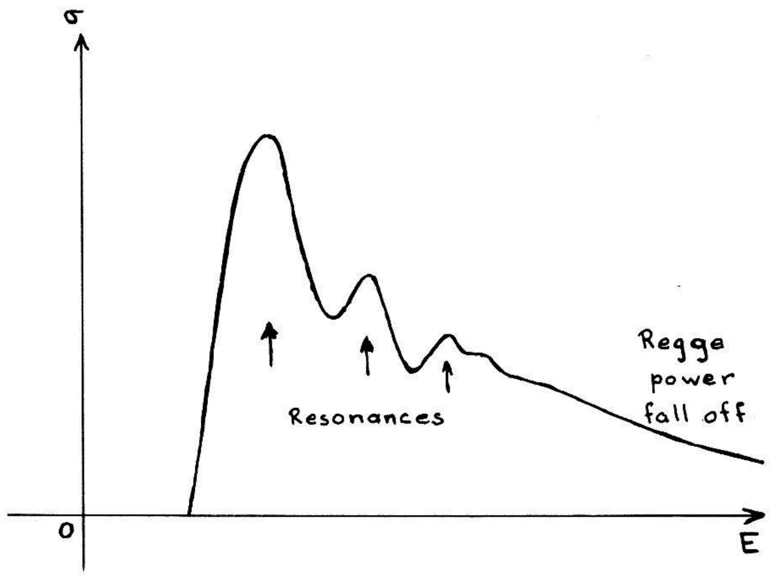

Fig. 1 Behaviour of the cross-section σ for the scattering $\pi^- p \rightarrow \pi^0 n$, as a function of the total centre-of-mass energy E

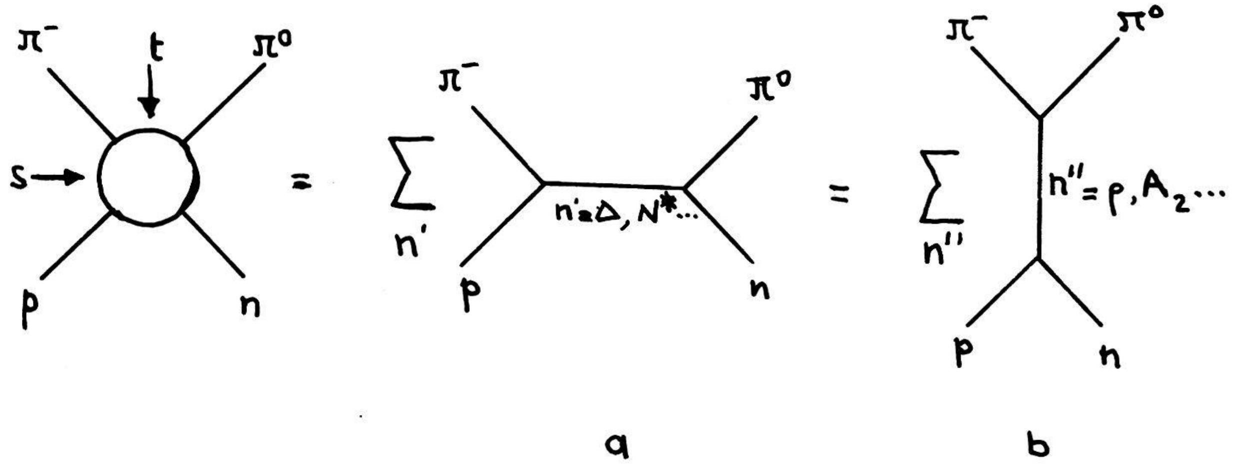

Fig. 2 The scattering $\pi^- p \rightarrow \pi^0 n$ is described either by a) the formation of baryonic resonances $n' = \Delta, N^*, \ldots$, in a unstable intermediate state, or b) the exchange of mesons $n'' = \varrho, A_2, \ldots$.

At high energy, the cross-section becomes smooth, and scattering is well described as being dominated by the exchange of the lightest mesons (ϱ, A_2, etc.) which are the origin of the forces between the pion and the proton (diagram of Fig. 2b). This is not surprising; in quantum electrodynamics, in e^+–e^- scattering, one can also observe at very low energies the effect of positronium bound states, whilst at high energies, the electron and the positron fly past each other so quickly that they do not have time to exchange more than one photon.

What is remarkable for strong interactions is the following experimental fact: as one goes higher in energy, denser and denser baryonic resonances conspire to reproduce precisely the exchange of mesons; similarly, coming down from high energies, and taking into account the exchange of heavier mesons, one reconstructs in the cross-section the peaks due to the formation of baryon resonances. This empirical observation, called duality, is expressed by the equation of Fig. 2: the scattering can be described *either* by a sum of baryonic resonance *or* by the exchange of mesonic resonances.

Actually, to obtain the experimentally observed duality equation of Fig. 2, one must have an infinite set of resonances, and an infinite set of exchanged particles. The first example of duality was found in a simplified theory containing only mesons. This is the Veneziano formula [1], which was the historic starting point of string theories. It describes the scattering amplitude of two identical mesons of mass m and four-momenta p_1^μ, p_2^μ, with two other mesons, identical to the first ones, with four-momenta $-p_3^\mu$ and $-p_4^\mu$. By Lorentz invariance, this amplitude depends only on the kinematic invariants $s = -(p_1 + p_2)^2$ and $t = -(p_2 + p_3)^2$ (Fig. 2); s and t are related to the total energy E and the scattering angle θ in the centre-of-mass frame by ($c = 1$)

$$s = E^2, \quad t = -1/2\,(E^2 - 4m^2)(1 - \cos\theta) \quad .$$

The Veneziano amplitude is ($\hbar = 1$)

$$F(s, t) = g^2\,\{[\Gamma(\alpha' m^2 - \alpha' s)\Gamma(\alpha' m^2 - \alpha' t)]/\Gamma(2\alpha' m^2 - \alpha' s - \alpha' t)\} \quad ,$$

where g is a coupling constant, α' is the slope of Regge trajectories ($\alpha' \sim 1\ \mathrm{GeV}^{-2}$ for hadrons, and Γ is the Euler Γ function).

One trivially has F(s, t) = F(t, s). The Γ function has poles for negative values of its argument, and one can expand, for example, on the s-channel poles:

$$F(s, t) =$$

$$(g^2/\alpha') \sum_{n=0}^{\infty} [(-1)^n/n!][1/m^2 + n - (s/\alpha')](\alpha' m^2 - \alpha' t - 1)(\alpha' m^2 - \alpha' t - 2)\ldots(\alpha' m^2 - \alpha' t - n) \quad .$$

According to Fig. 2a, these poles represent resonances with mass $M_n^2 = m^2 + (n/\alpha')$, n = 0, 1, 2, 3, ... The residue of the n^{th} pole is a polynomial in t, hence $\cos\theta$, of degree n. Decomposing this polynomial on the Legendre polynomials in $\cos\theta$, one thus obtains at the n^{th} pole a set of resonances of maximal orbital momentum (spin) n. Hence, the Veneziano formula uses in a crucial way the experimentally observed fact (Fig. 3) that light quark mesons lie on linear Regge trajectories: their spin increases like the square of their mass.

The Veneziano formula and the duality condition of Fig. 2 were quickly generalized to many-body processes, in order to describe hadron production. It was then found that the set of particles of the model was nothing but that of the quantized motions of a relativistic string [2], the simplest generalization of a point particle. For a point particle, the action is the length of the space–time path between initial and final positions. Minimizing this action naturally gives straight line propagation with constant velocity. Similarly, for a string, the action is the area of the surface swept by the string as it moves in space–time (Fig. 4). For a string with fixed ends, separated by a distance L, the minimal action over a time interval T is just LT, the area of a rectangle with sides L and T. Hence the energy of a string with fixed ends is proportional to its length, which means that the only dimensional parameter of the theory is the (constant) tension T_0 of the string at rest (if quarks are pictured as quantum numbers at the end of the string, confinement is trivial). In general, minimizing the area (the Minkowskian analogue of the

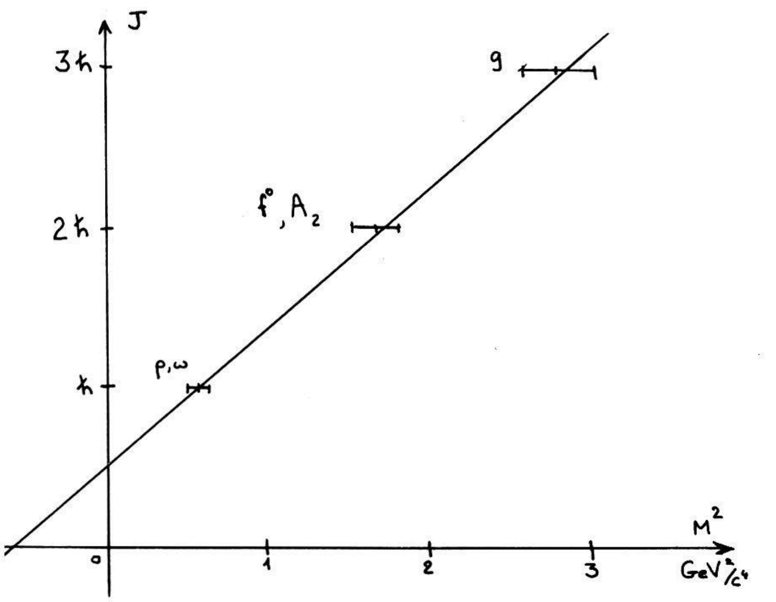

Fig. 3 Example of linear Regge trajectories

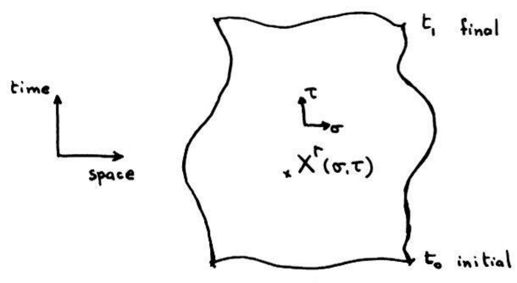

Fig. 4 The space–time worldsheet of the string

soap-bubble problem), one obtains in an appropriate coordinate choice [2], precisely the same equation,

$$[(\partial^2/\partial\tau^2)X^\mu] - [(\partial^2/\partial\sigma^2)X^\mu] = 0 \quad .$$

One also obtains boundary conditions. For an open free string, these mean that the ends move at the velocity of light c. The simplest motion of an open free string is the solid rotation of a line segment of length 2L around its centre, with constant angular velocity such that the ends move with velocity c; L can vary, only T_0 is constant. The energy of such a motion is found equal to

$$E = \pi T_0 L$$

and its angular momentum

$$J = (\pi T_0/2c)L^2 \quad .$$

Quantizing J in integer units of $\hbar$, one finds a discrete spectrum of particles, with spin $n\hbar$, and mass

$$M_n^2 = E_n^2/c^4 = (2\pi\hbar T_0/c^3)n \quad .$$

Numerically, for strongly interacting particles, a slope (Fig. 3)

$$\alpha' \equiv J/\hbar E^2 = 1/2c\pi\hbar T_0$$

of 1 GeV^{-2} corresponds to a tension of 13 tons.

Interactions between strings are described in a geometrical fashion (Fig. 5): two open strings can join by the ends to form a single one, which can later break in two. This process can

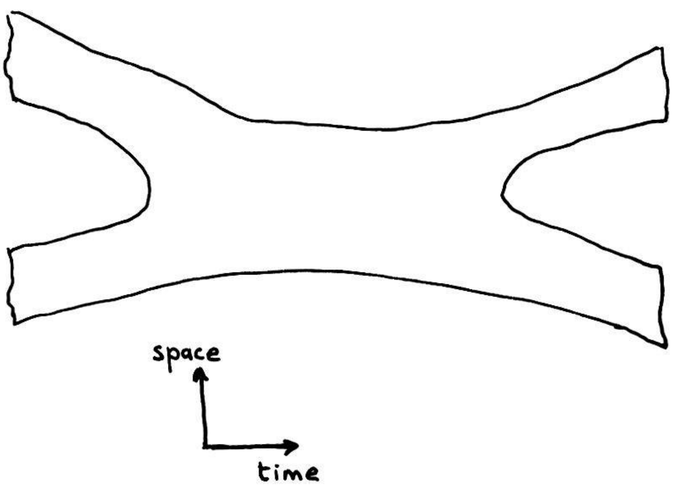

Fig. 5 The surface corresponding to the amplitude of Fig. 2a, considered as the interaction of strings in space-time

repeat itself. Twelve years ago, it was shown that integration over all intermediate string configurations corresponds exactly to the summation over the resonances of Fig. 2a. In this sense, the theory is purely geometrical. This is the source of its beauty and conceptual simplicity. It also makes it very rigid, any modification being generally the source of intractable complications.

The string just described, the simplest possible one, is also called the bosonic string, because all the particles of its spectrum have integer spin. The problem of half-integer spin has been solved by the so-called Neveu–Schwarz–Ramond model [3], or fermionic string. The bosonic string has only orbital degrees of freedom, corresponding to its position in space. Together with these, a fermionic string also has spin degrees of freedom. These can be interpreted as a distribution of half-integer spins along the string. Depending on the boundary conditions, one may have an odd number of these spins, and one obtains a fermion (Ramond sector), or an even number, and one obtains a boson (Neveu–Schwarz sector). To preserve the geometrical simplicity of the theory, and its compatibility with relativity, it is necessary that orbital and spin degrees of freedom be connected through a special symmetry, called supersymmetry, using anticommuting numbers. Supersymmetry, which appeared for the first time on this occasion, later had applications in other branches of physics and in mathematics. Like bosonic strings, fermionic strings have linear Regge trajectories, and interact in the same fashion (Fig. 5). Superstrings are fermionic strings for which only states of a given chirality have been kept.

Strings have remarkable properties not shared by ordinary point-particle field theories. The most spectacular is probably that their quantization is possible in a natural and consistent way in space–times of fixed and *a priori* rather mysterious dimensions: 26 for the bosonic string and 10 for fermionic strings. Unfortunately, there exists no simple explanation of these numbers which we could propose here. They also all have in their spectrum massless particles of spin one (for open strings) and two (for closed strings) (Fig. 6). Together with other features, this had made it clear about 12 years ago that they would not form the basis of a fundamental theory of strong interactions. Nevertheless, they still provide excellent phenomenological

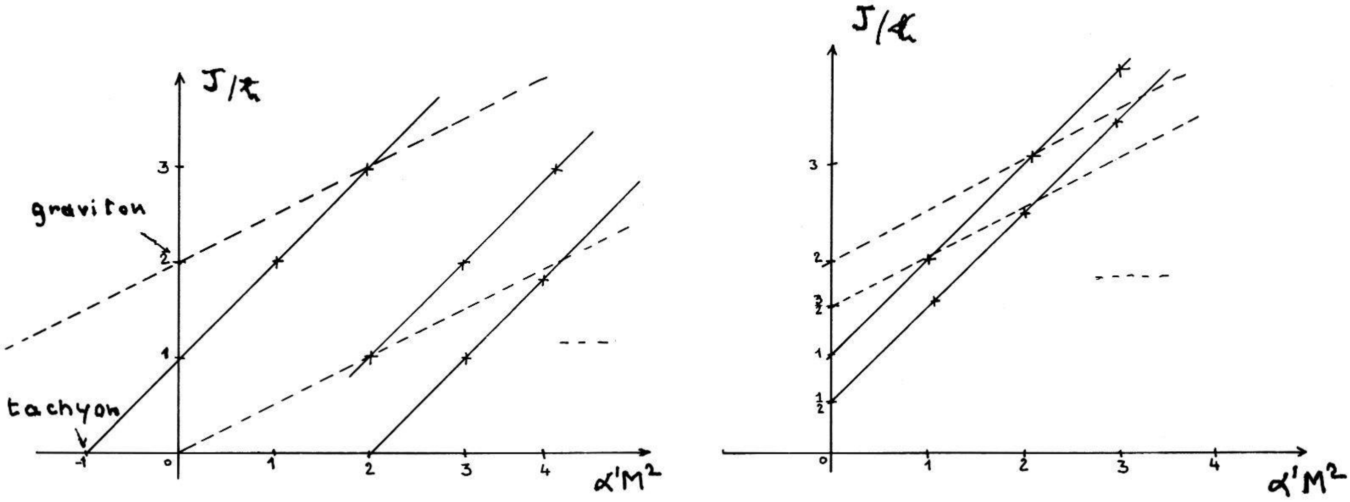

Fig. 6 a) Spectrum of open (solid lines) and closed (dotted lines) bosonic strings. b) Spectrum of open and closed fermionic strings.

parametrizations. The present candidate for a fundamental theory of strong interactions is quantum chromodynamics; its numerical simulations are judged on their ability to reproduce the linear Regge trajectory spectrum characteristic of strings.

Scherk and Schwarz [4] then turned the above drawbacks of strings as theories of strong interactions, into crucial assets towards a quite different and much more ambitious aim. They suggested to reinterpret them and turn them into a unified theory of all interactions, including gravitation, thanks to the massless spin-two particle identified with the graviton. This is anologous to what happened for non-Abelian gauge theories, initially invented by Yang and Mills for strong interactions, which now form the basis of the unification of weak and electromagnetic interactions (Weinberg–Salam model). The mass scale of the theory is then changed from 1 GeV—characteristic of strong interactions—to 10^{17} GeV, which is the Planck mass, an energy where the gravitational interaction of two electrons becomes comparable with their electromagnetic interaction (according to Einstein, gravitation is coupled to the matter *energy*-momentum tensor, whilst electromagnetism is coupled to the charge, a Lorentz-invariant quantity).

Scherk and Schwarz also proposed that the extra dimensions, from 4 to 10 (or 26), are actually of very small extent (compactified), and hence invisible at presently available energies. The only observable particles are the massless ones. A given string model uniquely predicts a spectrum of massless particles and their interactions, and among them there is always gravitation and the graviton. These suggestions are interesting not only because they unify gravity with the other interactions (which supergravity also does), but also because it seems to tame the non-renormalizable infinities which appear when general relativity, and even supergravity, are quantized. Some string theories have indeed no infinity, order by order in perturbation theory.

Although the above-mentioned nice properties of string models had been known or suspected for some time, they were not the trigger of their present revival. It was the discovery in September 1984 [5] that superstrings in 10 dimensions, which naturally distinguish left from right (like weak interactions; they are called chiral theories), do not necessarily present the inconsistencies generally plaguing chiral theories. These inconsistencies, also called chiral anomalies, which appear in first order of perturbation theory, are absent in superstrings if and only if their internal symmetry group is either SO(32) (the rotation group in 32 Euclidean dimensions) or the direct product $E_8 \times E_8$ of two exceptional Lie groups. It is the first time that the cancellation of chiral anomalies selects in a natural way one (or two) internal symmetry group(s) which, in the case of $E_8 \times E_8$, contains most groups already proposed for unified theories of strong, weak, and electromagnetic interactions.

Another very interesting development concerns the compactification procedure, and is due to two mathematicians, Frenkel and Kac [6]. The initial suggestion of Scherk and Schwarz may seem *ad hoc:* what principle could one invoke to fix the size and shape of the compactified dimensions? Also, compactification would break the group of rotations in the compactified directions to (at best) some much smaller, discrete, crystallographic group. Frenkel and Kac have shown that actually this need not be so, and that one can actually generate a rank d continuous internal symmetry group by cleverly compactifying d spatial directions. This is illustrated in the case $d = 2$ and the group SU(3) in Fig. 7: space is taken to be the rhombus with sides $\vec{L}_1$ and $\vec{L}_2$ ($|\vec{L}_1| = |\vec{L}_2|$), with periodic boundary conditions. Imagine a bosonic open string

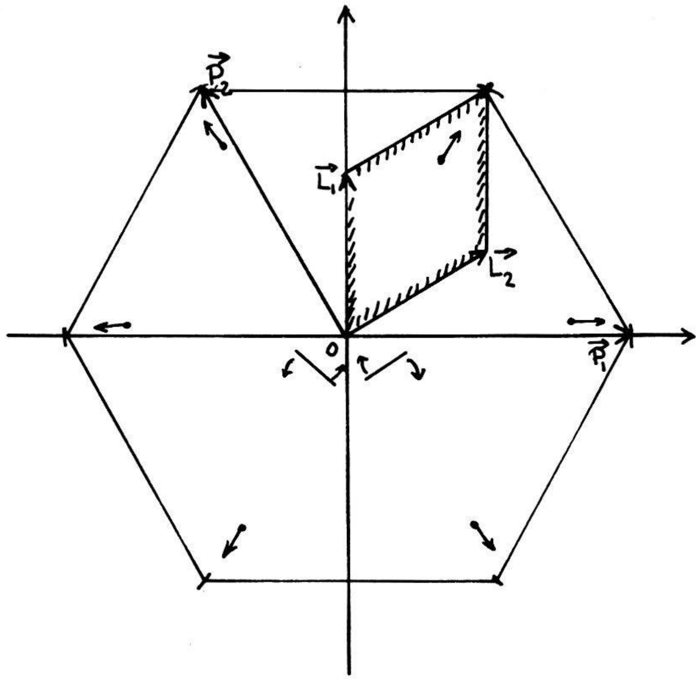

Fig. 7 The Frenkel-Kac construction for the group SU(3)

moving in the two-dimensional compact space, together with the remaining non-compact 24 dimensions. If the size of the rhombus is chosen correctly in terms of α', one may have, according to the mass formula of Fig. 6a, eight massless scalar states, consisting, as indicated in Fig. 7, of the clockwise and counterclockwise rotations in the plane, with angular momentum one, and of six possible directions for a tachyon moving in the plane. Frenkel and Kac have shown that these eight states actually fall into the adjoint representation of SU(3), and that not only the massless states but also the full string spectrum has this SU(3) symmetry. The Frenkel–Kac construction is actually a crucial ingredient for the $E_8 \times E_8$ superstring mentioned above.

At present, investigations are proceeding in different directions. On the formal side, string theories remain poorly understood, and one does not yet have at one's disposal a Lagrangian formulation that is as explicit and powerful as for ordinary quantum field theories. In particular, one just begins to understand their gauge invariances, which allow for the existence of high spins without ghosts; and an eventual fundamental geometric principle, analogous to the principle of equivalence in general relativity, remains to be discovered. Phenomenologically, one must understand how the dimension is effectively reduced from 10 to 4 at low energies. The reduction determines the spectrum of observable particles (leptons, quarks, gluons, intermediate bosons, etc.) and hence the predictive power of the theory. The Frenkel–Kac construction, which requires the existence of the tachyon, does not work for superstrings, and most of the other proposed schemes seem rather artificial, compared with the elegant simplicity and uniqueness of the theory to which they are applied.

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