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Autor: Fittler, Robert
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More nonstandard quantum electrodynamics

By Robert Fittler

Freie Universität Berlin, Fachbereich Mathematik, Arnimallee 3, 1000 Berlin 33, RFA

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Abstract. Nonstandard quantum electrodynamics, a rigorous field theoretical approximation of classical quantum electrodynamics, is developed further in lowest nontrivial order of perturbation theory. The material covers the vacuum polarization counterterm, the fermion selfenergy and its counterterm, a Ward identity, and the vertex contribution, yielding the anomalous magnetic moment of the electron, the form factor, and leading up to the (lowest order) Lamb shift. Thus a manifestly consistent alternative form of quantum electrodynamics yielding correct numerical results in lowest nontrivial order of pertubation theory is exhibited.

Introduction

Nonstandard quantum electrodynamics arises from the usual quantum electrodynamics by replacing the initial free fermion and photon fields by appropriate nonstandard operatorvalued functions (cf. [1], 0 and [8] pg. 5) using the following modifications.

The basic standard model $\mathbb{M}$ of analysis is twice expanded into $\mathbb{M} \subsetneqq \mathbb{M}_{(0)} \subsetneqq \mathbb{M}_{(1)}$ where $\mathbb{M}_{(0)}$ contains infinite and infinitesimal elements with respect to $\mathbb{M}$, and $\mathbb{M}_{(1)}$ again contains such elements with respect to $\mathbb{M}_{(0)}$ (cf. [1], 1 and 2.5).

The photons are assumed to have a restmass $m \in \mathbb{M}_{(0)}$ which is infinitesimal with respect to $\mathbb{M}$ (cf. [1], 2.5). There are particle number cutoffs for photons and electrons resp. (cf. [1], 3.2 and 4.4) both being infinite and belonging to $\mathbb{M}_{(0)}$. There is a space cutoff Q belonging to $\mathbb{M}_{(1)}$ which is infinite with respect to $\mathbb{M}_{(0)}$ and a UV cutoff P (cf. [1], 2.8 and 2.11). The nonfiniteness of the space cutoff Q with respect to $\mathbb{M}_{(0)}$ is necessary if one wants Q to induce (by Fouriertransformation) an internal norm approximation of the δ-function over $\mathbb{R}^3_{(0)} \in \mathbb{M}_{(0)}$ in the sense of [1], 1.11.

At first sight these modifications effect the initial fields only. However they bear upon other times since the initial fields constitute the building blocks of the (interaction) Hamiltonian, which governs the time dependence through Heisenberg's equation (cf. [1], 0). Such is the complete fundamental principle on which the dynamics is deductively based.

In order to allow a sensible multiplicative charge renormalization in connection with the vacuum polarization the UV cutoff P was assumed to be finite in [1] (cf. [1], 6.23 and 6.24). We are going to remove this obstacle to relativistic invariance and replace the old UV cutoff by a new P which is infinite with respect to $\mathbb{M}$ and belongs to $\mathbb{M}_{(0)}$. The break-down of the multiplicative charge renormalization will be compensated by using the counterterm approach of Gupta (cf. [2], [3]).

Notice that all results of [1], except 6.23/24 remain valid and some are actually sharpened by the infinite value of the UV cutoff P. In particular the standard first order perturbation theory and our nonstandard modified form of it coincide infinitesimally closely (cf. [1], 5.12).

The present approach starts with the nonstandard (internal) Q.E.D. corresponding to the above mentioned Hamiltonian (without counterterms) which yields a Dyson expansion that is convergent in the nonstandard sense (cf. [1], 5.7). Its sum may be highly nonstandard. The same already happens to its lowest nontrivial order summands. In order to rectify this the approprite counterterms are introduced into the original Hamiltonian. The resulting counterterm Hamiltonian again gives rise to a welldefined nonstandard Q.E.D., now yielding finite and even numerically correct contributions in lowest nontrivial order of the corresponding Dyson expansion. Higher order renormalizations will be considered elsewhere. (Notice that the sum of a convergent nonstandard series of finite terms need not necessarily be finite). The actual computations resemble the classical ones rather closely. The classical divergencies appear in the form of (possibly) infinite but welldefined nonstandard numbers which obey the same laws as standard numbers. They depend on the initial choices of the infinite UV cutoff or the infinitesimal photon mass (or both). The same holds for the determination of the counterterms.

Chapter 1 deals with vacuum polarization on the base of [1], chapter 6, where the vacuum polarization term appears in 3rd order. Its 'infinite part' will be compensated by the appropriate 2nd order contribution (cf. 1.1, 1.2 and 1.9) induced by the counterterm

$$-\int :\tfrac{1}{4}\delta F \cdot F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}((\delta F)^2 + (\delta F)^3 + \cdots)F_{i0}F^{i0}: d^3x$$

(cf. 1.1). Any choice of an infinite (0-finite) UV cutoff P determines a unique $\delta F(\sim 1)$ doing the job as long as

$$m^2 \cdot \frac{\alpha}{3\pi}\left(\ln\frac{P^2}{M^2} + \ln 4 - \frac{5}{3}\right) \sim 0.$$

The latter relation between the photonmass m and the UV cutoff P follows from the precondition that $\ln(m^2P)$ be finite (cf. 1.7, 1.8 and 3.16).

Chapter 2 deals with the fermion mass renormalization in lowest (3rd) order by splitting the usual selfenergy contribution (cf. 2.1) into a sum of two integrals (cf. 2.6), one containing Σ as it is commonly used (cf. 2.5), the remaining one containing a function Ξ. The Σ behaves as expected by contributing just a constant A of the 'approximate size' of $\ln P$ (up to finite factors and summands, cf. 2.17). In order to compensate the Σ-summand of the selfenergy integral the usual mass renormalization counterterm

$$-\delta M\int :\bar{\Theta}(0,x)\Theta(0,x):d^3x$$

is introduced into the interaction hamiltonian (cf. 2.18). It contributes in second order a term which just compensates the Σ-summand of the selfenergy contribution if δM is appropriately chosen (cf. 2.20). So far everything goes according to the textbooks (cf. e.g. [4], §9.4).

The remaining Ξ-summand of the selfenergy contribution disappears in the limit $t\to\infty$ (cf. 2.9).

In Chapter 3 the vertex part is dealt with (cf. 3.1) rather closely along the lines of [5], Appendix E, yielding first the expected contribution to the anomalous magnetic moment of the electron (cf. 3.19). This appears in the usual fashion incorporated to the Λ_0-function (cf. 3.3). The 'constant part' L of Λ_0 (cf. (3.16) essentially equals the difference of a UV-infinite and a IR-infinite contribution (cf. 3.15). According to our precondition on the finiteness of $\ln(m^2P)$, L itself is finite. The appropriate choices for m and p even imply that L vanishes (cf. 3.17). Thus one may avoid the socalled spurious charge renormalization (to this order).

Applying the usual Taylor development yields the expected form factor (cf. 3.19). Thus the preparations needed for the lowest order approximation of the Lamb shift (cf. [6], 15E.) are ready (cf. 3.21).

Finally, in Chapter 4, a nonstandard version of the usual Ward identity (cf. 4.7) is developed.

1. The charge renormalization

1.1. *Vacuum polarization counterterm*

In order to compensate the summand $-C(\phi, s\Lambda)$ of the vacuum polarization $(\phi, \sigma\Lambda)$ (cf. [1], 6.21–23) we introduce an additive counterterm

$$-\int :\tfrac{1}{4}\delta F\sum_{\mu,\nu=0}^{3}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}\sum_{n=2}^{\infty}(\delta F)^n\sum_{i=1}^{3}F_{i0}F^{i0}:d^3x$$

into the former interaction hamiltonian

$$-e\int :\sum_{\mu=0}^{3}\bar{\Theta}\gamma_\mu\Theta(B_\mu+C_\mu):d^3x \quad \text{(cf. [1], 6.3)}$$

in the spirit of Gupta [1] (cf. also Dyson [3]). $F_{\mu\nu}$, $F^{\mu\nu}$ are defined according to

$$F_{\mu\nu}(x_0,x):=\frac{\partial(B_\mu(x_0,x)+C_\mu(x_0,x))}{\partial x_\nu}-\frac{\partial(B_\nu(x_0,x)+C_\nu(x_0,x))}{\partial x_\nu}$$

and $F^{\mu\nu}=\sum_{\alpha,\beta=0}^{3}g_{\mu\alpha}g_{\nu\beta}F_{\alpha\beta}$. The constant δF, $|\delta F|<1$, is to be determined in the following. The new interaction hamiltonian, being again a bounded (1-internal) operator on $\mathbb{A}^{4\Omega} \otimes \mathbb{B}^{4\omega}$ (cf. [1], 5.7) yields a corresponding Dyson expansion (cf. [1], 5.8). Its Wick representation carries in second order a summand $w(-t, t)$:

$$= -(-i)^2 eF \int_{-t}^{t} dx_0 \int d^3x \int_{-t}^{t} dy_0 \int d^3y$$

$$\sum_{j=1}^{3} :\bar{\Theta}(x_0, x)\gamma_0\Theta(x_0, x)B_0(x_0, x)\frac{\partial}{\partial y_j} B_0(y_0, y)$$

$$-\bar{\Theta}(x_0, x)\gamma_j\Theta(x_0, x)B_j(x_0, x)\frac{\partial}{\partial y_0} B_j(y_0, y):\frac{\partial}{\partial y_j} C_0(y)$$

where $F := \sum_{n=1}^{\infty} (\delta F)^n$. In analogy to classical Q.E.D. one has

1.2 Theorem. *For any two one-electron states $\phi, \Lambda \in \mathbb{L} \otimes \mathbb{D}$, and any 0-finite value t we have*

$$(\phi, w(-t, t)\Lambda) \mathrel{\tilde{\circ}} eF\frac{i}{(2\pi)^{5/2}} \int_{-t}^{+t} dx_0 \int_{-t}^{+t} dy_0$$

$$\times \int d^3p\, d^3q\, dk_0 h(p-q) \frac{e^{i(\Omega_p - \Omega_q - k_0)x_0} e^{ik_0 y_0}}{(p-q)^2 - k_0^2 + m^2 - i\varepsilon}$$

$$(\phi, :\bar{\Theta}^+(0, -p)\gamma_0\Theta^-(0, q)(p-q)^2$$

$$-\sum_{j=1}^{3} \bar{\Theta}^+(0, -p)\gamma_j\Theta^-(0, q)(p-q)_j k_0 : C_0(p-q)\Lambda)$$

In order to evaluate 1.2 we need the following technical

1.3 Lemma

a) $$\int_{-t}^{t} dx_0 \int_{-t}^{t} dy_0 \frac{e^{i(\Omega_p - \Omega_q - k_0)x_0} e^{ik_0 y_0}}{k^2 - k_0^2 + m^2 - i\varepsilon}$$

$$\mathrel{\tilde{\circ}} \frac{2\pi}{\omega_k^2} \int_{-t}^{t} dx_0 e^{i(\Omega_p - \Omega_q)x_0} - \frac{\pi}{\omega_k^2} \int_{-t}^{t} dx_0 e^{-i\omega_k t}(e^{i(\Omega_p - \Omega_q - \omega_k)x_0} + e^{i(\Omega_p - \Omega_q + \omega_k)x_0})$$

b) $$\int_{-t}^{t} dx_0 \int_{-t}^{t} dy_0 \frac{k_0 e^{i(\Omega_p - \Omega_q - k_0)x_0} e^{ik_0 y}}{k^2 - k_0^2 + m^2 - i\varepsilon}$$

$$\mathrel{\tilde{\circ}} \frac{\pi}{\omega_k} \int_{-t}^{t} dx_0 e^{-i\omega_k t}(e^{i(\Omega_p - \Omega_q + \omega_k)x_0} - e^{i(\Omega_p - \Omega_q - \omega_k)x_0})$$

Proof. By breaking up $\int_{-t}^{t} dy_0$ into $\int_{-t}^{x_0} dy_0 + \int_{x_0}^{t} dy_0$.

1.4 *Remark.* We state the following facts for further use.
a) Any $p, q \in \mathbb{R}^3$ fulfill $\Omega_p - \Omega_q + \omega_{p-q} > 0$ and $\Omega_p - \Omega_q - \omega_{p-q} < 0$

$$\text{b)} \int_{-t}^{+t} dx_0 e^{-i\omega_{p-q}t} e^{i(\Omega_p-\Omega_q-\omega_{p-q})x_0} = i\frac{e^{-i(\Omega_p-\Omega_q)t} - e^{-(\Omega_p-\Omega_q-2\omega_{p-q})t}}{\Omega_p-\Omega_q-\omega_{p-q}}$$

$$\int_{-t}^{+t} dx_0 e^{-i\omega_{p-q}t} e^{i(\Omega_p-\Omega_q+\omega_{p-q})x_0} = i\frac{e^{i(\Omega_p-\Omega_q)t} - e^{-i(\Omega_p-\Omega_q+2\omega_{p-q})t}}{\Omega_p-\Omega_q+\omega_{p-q}}$$

1.5 Corollary. *Under the assumptions of Theorem* 1.2 *we find*

$$\begin{aligned}
(\phi, w(-t, t)\Lambda) &\underset{0}{\sim} F\frac{ei}{(2\pi)^{3/2}}\int_{-t}^{+t} dx_0 \int d^3p\, d^3q h(p-q) e^{i(\Omega_p-\Omega_q)x_0} \\
&\times \left(\phi, \bar{\Theta}^+(0,-p)\gamma_0\Theta^-(0,q)\frac{(p-q)^2}{\omega_{p-q}^2}C_0(p-q)\Lambda\right) + F\frac{ei}{(2\pi)^{5/2}}\int d^3p\, d^3q h(p-q) \\
&\times \left(\phi, \bar{\Theta}^+(0,-p)\gamma_0\Theta^-(0,q)(-i)\pi\frac{(p-q)^2}{\omega_{p-q}^2}\right. \\
&\times \left.\left(\frac{e^{-i(\Omega_p-\Omega_q)t} - e^{i(\Omega_p-\Omega_q-2\omega_{p-q})t}}{\Omega_p-\Omega_q-\omega_{p-q}} - \frac{e^{i(\Omega_p-\Omega_q)t} - e^{-i(\Omega_p-\Omega_q+2\omega_{p-q})t}}{\Omega_p-\Omega_q+\omega_{p-q}}\right)C_0(p-q)\Lambda\right) \\
&+ F\frac{ei}{(2\pi)^{5/2}}\int d^3p\, d^3q h(p-q)\left(\phi, \sum_{j=1}^{3}\bar{\Theta}^+(0,-p)\gamma_j\Theta^-(0,q)i\pi\frac{p_j-q_j}{\omega_{p-q}}\right. \\
&\times \left.\left(\frac{e^{-i(\Omega_p-\Omega_q)t} - e^{i(\Omega_p-\Omega_q-2\omega_{p-q})t}}{\Omega_p-\Omega_q-\omega_{p-q}} + \frac{e^{i(\Omega_p-\Omega_q)t} - e^{-i(\Omega_p-\Omega_q+2\omega_{p-q})t}}{\Omega_p-\Omega_q+\omega_{p-1}}\right)C_0(p-q)\Lambda\right)
\end{aligned}$$

1.6 Theorem. *For any two one-electron states* $\phi, \Lambda \in \mathbb{L} \otimes \mathbb{D}$ *the vacuum polarization counterterm yields*

$$\begin{aligned}
(\phi, w\Lambda) &:= \lim_{t\to\infty} 0\text{-}st(\phi, w(-t,t)\Lambda) \\
&= F\frac{ie}{\sqrt{2\pi}} 0\text{-}st \int d^3p\, d^3q h(p-q)\delta^1(\Omega_p-\Omega_q) \\
&\quad \times \left(\phi, \bar{\Theta}^+(0,-p)\gamma_0\Theta^-(0,p)\frac{(p-q)^2}{\omega_{p-q}^2}C_0(p-q)\Lambda\right)
\end{aligned}$$

Proof. It is to be shown that all the summands in Corollary 1.5, except the first one, disappear in the limit $t \to \infty$. This follows from the Riemann–Lebesgue Lemma (using Remark 1.4) c.f. e.g. [7] Lemma 4.1 pg. 216. As to the first summand see [1], 6.5. q.e.d.

1.7 Corollary. *For any two normed one-electron states* $\phi, \Lambda \in \mathbb{L} \otimes \mathbb{D}$ *the vacuum polarization counterterm yields*

$$(\phi, w\Lambda) \sim F\frac{ie}{\sqrt{2\pi}}\int d^3p\, d^3q \delta^1(\Omega_p-\Omega_q)(\phi, \bar{\Theta}^+(0,-p)\gamma_0\Theta^-(0,q)C_0(p-q)\Lambda)$$

assuming that $F \cdot m^2 \sim 0$.

Proof. The difference between the two expressions has absolute value

$$\tilde{\circ} \left| \frac{ie}{\sqrt{2\pi}} \int d^3p\, d^3q \frac{F \cdot m^2}{(p-q)^2 + m^2} \delta^1(\Omega_p - \Omega_q)(\phi, \bar{\Theta}^+(0, -p)\gamma_0 \bar{\Theta}(0, q) C_0(p-q)\Lambda) \right|$$

$$\leq F \cdot m^2 \left| \frac{ie}{\sqrt{2\pi}} \int d^3p\, d^3q \delta^1(\Omega_p - \Omega_q)(\phi, \bar{\Theta}^+(0, -p)\gamma_0 \Theta^-(0, q) C_0(p-q)\Lambda) \right|$$

$= F \cdot m^2(\phi, s\Lambda)$ (cf. [1] 6.6)

~ 0 since ϕ, Λ are normed and $F \cdot m^2 \sim 0$. q.e.d.

1.8 Lemma. *The condition that $P > 0$ be infinite and $\ln(m^2 \cdot P)$ be finite implies $m^2 \ln P$ to be infinitesimal.*

Proof. m^2P is finite since $\ln(m^2P)$ is finite. The infiniteness of P then implies $\ln P/p \sim 0$. Hence $m^2 \ln P = m^2P \cdot \ln P/P \sim 0$. q.e.d.

1.9 *Remark.* Setting

$$F := C = \frac{\alpha}{3\pi}\left(\ln \frac{P^2}{M^2} + \ln 4 - \frac{5}{3}\right)$$

yields $(\phi, w\Lambda) - C(\phi, s\Lambda) \sim 0$, assuming that $\ln(m^2P)$ be finite (cf. 1.8 and 3.18). Since F is positive it determines a unique δF such that $F = \sum_{n=1}^{\infty} (\delta F)^n$ i.e. $\delta F = F/1 + F$. With this choice of δF the vacuum polarization counterterm (cf. 1.1) induces the compensation of $-C(\phi, s\Lambda)$ by $(\phi, w\Lambda)$ (for suitable ϕ, Λ cf. 1.7).

2. The fermion mass renormalization

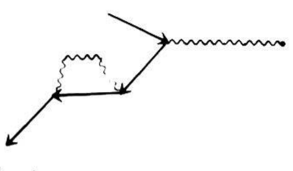
2.1. *A fermion selfenergy contribution*

Let

$$\varepsilon(-t, t) := (ie)^3 \int_{-t}^{t} dx_0 \int_{-t}^{t} dy_0 \int_{-t}^{t} dz_0 \int d^3x\, d^3y\, d^3z$$

$$: \bar{\Theta}(x_0, x)\gamma_\mu \Theta(x_0, x) B_\mu(x_0, x) B_\nu(y_0, y) \bar{\Theta}(y_0, y)$$
$$\times \gamma_\nu \Theta(y_0, y) \bar{\Theta}(z_0, z) \gamma_\rho C_\rho(z) \Theta(z_0, z):$$

be the summand of the S-matrix $U(-t, t)$ (cf. [1], 5.7 and 5.8) corresponding to the Feynman graph.



A straightforward evaluation leads to

2.2 Theorem. *For any 0-finite time interval $[-t, t]$ and any two one-electron* *states $\phi, \Lambda \in \mathbb{L} \otimes \mathbb{D}$ we get the selfenergy contribution*

$$
\begin{aligned}
(\phi, \varepsilon(-t, t)\Lambda) \,\tilde{\circ}\, -e^3 &\int_{-t}^{t} dx_0 \int_{-t}^{t} dy_0 \int_{-t}^{t} dz_0 \int d^3p\, d^3w\, d^3q \int dk_0 \int dq_0 \int du_0 \\
&\times \frac{1}{2\pi} e^{i(\Omega_p - q_0 - u_0)x_0} \frac{1}{2\pi} e^{-i(-q_0-u_0+k_0)y_0} \frac{1}{2\pi} e^{i(k_0-\Omega_w)z_0} \\
&\times \left(\phi, \frac{1}{(2\pi)^{3/2}} \bar{\Theta}^+(0, -p) \frac{h(q)}{q^2 - q_0^2 + m^2 - i\varepsilon} g_{\mu\mu}\gamma_\mu \right. \\
&\times \frac{M + \cancel{u_0, p - q}}{(p-q)^2 - u_0^2 + M^2 - i\varepsilon} h(p-q)\gamma_\mu \frac{M + \cancel{k_0, p}}{p^2 - k_0^2 + M - i\varepsilon} \\
&\left. \times h(p)\gamma_\rho \frac{1}{(2\pi)^{3/2}} \Theta^-(0, w) \frac{1}{(2\pi)^{3/2}} C_\rho(p-w)\Lambda \right)
\end{aligned}
$$

2.3 *Remark.* Theorem 1.12 of [1] can be refined in the following way:

$$
\int \frac{e^{-ip_0(x_0-y_0)}}{p^2 - p_0^2 + m^2 - i\varepsilon} dp_0 = \pi i \frac{e^{-i\omega_p' |x_0 - y_0|}}{\omega_p'}
$$

where $\omega_p' := \sqrt{p^2 + m^2 - i\varepsilon}$ is the root with positive real part ($\varepsilon > 0$). This is proved using the residues calculus.

In analogy to 1.3 one has

2.4 Lemma

$$
\begin{aligned}
&\int_{-t}^{t} dx_0 \int_{-\infty}^{\infty} \frac{dq_0}{q^2 - q_0^2 + m^2 - i\varepsilon} e^{i(\Omega_p - q_0 - u_0)x_0} e^{-i(-q_0-u_0+k_0)y_0} \\
&= \left\{ \frac{2\pi}{q^2 - (\Omega_p - u_0)^2 + m^2 - i\varepsilon} + \frac{\pi}{\omega_q'} \right. \\
&\quad \left. \times \left(\frac{e^{i(-\omega_q' + \Omega_p - u_0)(t - y_0)}}{(-\omega_q' + \Omega_p - u_0)} - \frac{e^{-i(\omega_q' + \Omega_p - u_0)(t + y_0)}}{(\omega_q' + \Omega_p - u_0)} \right) \right\} e^{i(\Omega_p - k_0)y_0}
\end{aligned}
$$

In order to further evaluate the result of Theorem 2.2 we need the following straightforward consequence

2.5 Corollary and definition.

$$
\begin{aligned}
&\int_{-t}^{t} dx_0 \int d^3q \int du_0 \int dq_0 e^{i(\Omega_p - q_0 - u_0)x_0} e^{-i(-q_0-u_0+k_0)y_0} \\
&\quad \times \frac{h(q)h(p-q)}{q^2 - q_0^2 + m^2 - i\varepsilon} g_{\mu\mu}\gamma_\mu \frac{M + \cancel{u_0, p-q}}{(p-q)^2 - u_0^2 + M^2 - i\varepsilon} \gamma_\mu \\
&= \{2\pi\Sigma(\Omega_p, p) + \pi\Xi(\Omega_p, p, t, y_0)\} e^{i(\Omega_p - k_0)y_0}
\end{aligned}
$$

where we define

$$\Sigma(p_0, p) := \int d^3q\, dq_0 \frac{h(q)h(p-q)}{2\omega'_q}\left(\frac{1}{\omega'_q + q_0} - \frac{1}{-\omega'_q + q_0}\right) g_{\mu\mu}\gamma_\mu$$

$$\times \frac{M + \cancel{p_0 - q_0, p - q}}{(p-q)^2 - (p_0 - q_0)^2 + M^2 - i\varepsilon}\gamma_\mu$$

$$= \int d^3q\, dq_0 \frac{h(q)h(p-q)}{q^2 - q_0^2 + m^2 - i\varepsilon} g_{\mu\mu}\gamma_\mu \frac{M + \cancel{p_0 - q_0, p - q}}{(p-q)^2 - (p_0 - q_0)^2 + M^2 - i\varepsilon}\gamma_\mu$$

and

$$\Xi(p_0, p, t, y_0) = \int d^3q\, dq_0 \frac{h(q)h(p-q)}{\omega'_q}\left(\frac{e^{i(-\omega'_q + q_0)(t - y_0)}}{-\omega'_q + q_0} - \frac{e^{-i(\omega'_q + q_0)(t + y_0)}}{\omega'_q + q_0}\right)$$

$$\times g_{\mu\mu}\gamma_\mu \frac{M + \cancel{p_0 - q_0, p - q}}{(p-q)^2 - (p_0 - q_0)^2 + M^2 - i\varepsilon}\gamma_\mu$$

(for ω'_q *cf. Remark* 2.3).

2.6 *Remark.* Theorem 2.2 and Corollary 2.5 yield

$$(\phi, \varepsilon(-t, t)\Lambda) \underset{\circ}{\sim} -e^3 \int_{-t}^{t} dy_0 \int_{-t}^{t} dz_0 \int d^3p\, d^3w \frac{1}{(2\pi)^3} e^{i(\Omega_p y_0 - \Omega_w z_0)} e^{-i\Omega'_p |y_0 - z_0|}$$

$$\times \Bigg(\phi, \frac{1}{(2\pi)^{3/2}} \bar{\Theta}^+(0, p)\{2\pi\Sigma(\Omega_p, p) + \pi\Xi(\Omega_p, p, t, y_0\}$$

$$\times i\pi \frac{M + \cancel{\Omega'_p \cdot sg(y_0 - z_0), p}}{\Omega'_p} h(p)\gamma_\rho \frac{1}{(2\pi)^{3/2}} \bar{\Theta}(0, w) \frac{1}{(2\pi)^{3/2}} C(p - w)\Lambda\Bigg)$$

For the evaluation of $\Xi(\Omega_p, p, t, y_0)$ we need

2.7 Lemma. *For $-t < y_0 < t$ we have*

a) $$\int_{-\infty}^{\infty} dq_0 \frac{e^{i(-\omega'_q + q_0)(t - y_0)}}{-\omega'_q + q_0} g_{\mu\mu}\gamma_\mu \frac{M + \cancel{\Omega_p - q_0, p - q}}{(p-q)^2 - (\Omega_p - q_0)^2 + M^2 - i\varepsilon}\gamma_\mu$$

$$= 2\pi i \frac{e^{i(-\omega'_q + \Omega_p - \Omega'_{p-q})(t - y_0)}}{-\omega'_q + \Omega_p - \Omega'_{p-q}} g_{\mu\mu}\gamma_\mu \frac{M + \cancel{\Omega'_{p-q}, p - q}}{2\Omega'_{p-q}}\gamma_\mu$$

b) $$\int_{-\infty}^{\infty} dq_0 \frac{e^{-i(\omega'_q + q_0)(t + y_0)}}{\omega'_q + q_0} g_{\mu\mu}\gamma_\mu \frac{M + \cancel{\Omega_p - q_0, p - q}}{(p-q)^2 - (\Omega_p - q_0)^2 + M^2 - i\varepsilon}\gamma_\mu$$

$$= 2\pi i \frac{e^{-i(\omega'_q + \Omega'_{p-q} + \Omega_p)(t + y_0)}}{\omega'_q + \Omega'_{p-q} + \Omega_p} g_{\mu\mu}\gamma_\mu \frac{M + \cancel{-\Omega'_{p-q}, p - q}}{2\Omega'_{p-q}}\gamma_\mu$$

Proof. By residue calculus.

2.8. Corollary. *For $-t < y_0 < t$ we have*

$$\Xi(\Omega_p, p, t, y_0) = 2\pi i \int d^3q \frac{h(q)h(p-q)}{\omega'_q} \left\{ \frac{e^{i(-\omega'_q+\Omega_p-\Omega'_{p-q})(t-y_0)}}{-\omega'_q+\Omega_p-\Omega'_{p-q}} g_{\mu\mu}\gamma_\mu \right.$$
$$\times \frac{M + \cancel{\Omega'_{p-q}, p-q}}{2\Omega'_{p-q}} \gamma_\mu - \frac{e^{-i(\omega'_q+\Omega'_{p-q}+\Omega_p)(t+y_0)}}{\omega'_q+\Omega_{p-q}+\Omega_p} g_{\mu\mu}\gamma_\mu$$
$$\left. \times \frac{M + \cancel{-\Omega'_{p-q}, p-q}}{2\Omega'_{p-q}} \gamma_\mu \right\}$$

A similar argument yields

$$\Sigma(\Omega_p, p) = \pi i \int d^3q \frac{h(q)h(p-q)}{\omega'_q}$$
$$\times \left\{ -\frac{1}{-\omega'_q+\Omega_p-\Omega'_{p-q}} g_{\mu\mu}\gamma_\mu \frac{M + \cancel{\Omega'_{p-q}, p-q}}{2\Omega'_{p-q}} \right.$$
$$\left. + \frac{1}{\omega'_q+\Omega_p+\Omega'_{p-q}} g_{\mu\mu}\gamma_\mu \frac{M + \cancel{-\Omega'_{p-q}, p-q}}{2\Omega'_{p-q}} \gamma_\mu \right\}$$

2.9 *Remark.* The summand of $(\phi, \varepsilon(-t, t)\Lambda)$ containing Ξ (cf. Remark 2.6) thus becomes

$$-e^3 \int_{-t}^{t} dz_0 \int_{-t}^{t} dy_0 \int d^3p\, d^3w \frac{1}{(2\pi)^3} e^{i(\Omega_p y_0 - \Omega_w z_0)} e^{-i\Omega'_p |y_0 - z_0|}$$
$$\times \left(\phi, \frac{1}{(2\pi)^{3/2}} \bar{\Theta}^+(0, -p)\pi 2\pi i \int d^3q \right.$$
$$\times \left\{ \begin{array}{l} \dfrac{e^{i(-\omega'_q+\Omega_p-\Omega'_{p-q})(t-y_0)}}{-\omega'_q+\Omega_p-\Omega'_{p-q}} g_{\mu\mu}\gamma_\mu \dfrac{M + \cancel{\Omega'_{p-q}, p-q}}{2\Omega'_{p-q}} \gamma_\mu \\ -\dfrac{e^{-i(\omega'_q+\Omega'_{p-q}+\Omega_p)(t+y_0)}}{\omega'_q+\Omega'_{p-q}+\Omega_p} g_{\mu\mu}\gamma_\mu \dfrac{M + \cancel{-\Omega'_{p-q}, p-q}}{2\Omega'_{p-q}} \gamma_\mu \end{array} \right\}$$
$$\times \frac{h(q)h(p-q)}{\omega'_p} i\pi \frac{M + \cancel{\Omega'_p sg(y_0 - z_0), p}}{\Omega'_p}$$
$$\left. \times h(p)\gamma_\rho \frac{1}{(2\pi)^{3/2}} \Theta^-(0, w) \frac{1}{(2\pi)^{3/2}} C_\rho(p-w)\Lambda \right)$$

which after breaking up the integral $\int_{-t}^{t} dy_0$ into $\int_{-t}^{z_0} dy_0 + \int_{z_0}^{t} dy_0$ can be integrated elementarily yielding an explicit but rather complicated elementary formula. Applying residue calculus with respect to the d^3q integration yields a 0-finite value (depending of t) whose 0-standard part vanishes in the $\mathbb{M}_0$-limit $t \to \infty$, by the Riemann–Lebesgue Lemma.

Now we turn to the summand of $(\phi, \varepsilon(-t, t)\Lambda)$ (cf. Remark 2.6) containing $\Sigma(\Omega_p, p)$.

2.10 Lemma.

$$\Sigma(\Omega_p, p) = \int_0^1 dt \int d^3v\, dv_0 h(v - tp)h(v + (1-t)p)$$

$$\times g_{\mu\mu}\gamma_\mu \frac{M + \cancel{v_0, v} + (1-t)\cancel{\Omega_p, p}}{[v^2 - v_0^2 + t^2M^2 + (1-t)m^2 - i\varepsilon]^2}\gamma_\mu$$

Proof. A straightforward computation using in turn Feynmans identity $1/ab = \int_0^1 dt(1/[a + (b-a)t]^2)$ and the substitution $v = -q + tp$. q.e.d.

2.11 *Remark.* Using the facts $\Sigma g_{\mu\mu}\gamma_\mu\gamma_\mu = 4$ and $\Sigma g_{\mu\mu}\gamma_\mu\gamma_\nu\gamma_\mu = -2\gamma_\nu$ yields

$$\Sigma(\Omega_p, p) = \int_0^1 dt \int d^3v\, dv_0 h(v - tp)h(v + (1-t)p)$$

$$\times \frac{4M - 2(\cancel{v_0, v}) - 2(1-t)(\cancel{\Omega_p, p})}{[v^2 - v_0^2 + t^2M^2 + (1-t)m^2 - i\varepsilon]^2},$$

which can be rewritten in the form

$$\Sigma(\Omega_p, p) = \int_0^1 dt \int_{G(t,p)} d^3v \int dv_0 \frac{2(1-t)(M - \cancel{\Omega_p, p}) + 2(M(1+t) - \cancel{v_0, v})}{(v^2 - v_0^2 + t^2M^2 + (1-t)m^2 - i\varepsilon)^2}$$

where $G(t, p) \subseteq \mathbb{R}_1^3$ is the support of the function

$v \mapsto h(v - tp)h(v + (1-t)p)$ (cf. [1], 6.17).

Since $\bar{\Theta}^+(0, -p)(M - \cancel{\Omega_p, p}) = 0$ the operator $\Sigma(\Omega_p, p)$ contributes in $(\phi, \varepsilon(-t, t)\Lambda)$ (cf. Remark 2.6) only by

$$c(p) := 2\int_0^1 dt \int_{G(t,p)} d^3v \int dv_0 \frac{M(1+t) - \cancel{v_0, v}}{(v^2 - v_0^2 + t^2M^2 + (1-t)m^2 - i\varepsilon)^2}$$

which in turn can be split into

$$c(p) = 2\int_0^1 (c_1(t, p) - c_2(t, p))\, dt$$

where

$$c_1(t, p) = \int_{G(t,p)} d^3v \int_{-\infty}^{+\infty} dv_0 \frac{M(1+t)}{(v^2 - v_0^2 + t^2M^2 + (1-t)m^2 - i\varepsilon)^2}$$

and

$$c_2(t, p) = \int_{G(t,p)} d^3v \int_{-\infty}^{+\infty} dv_0 \frac{\cancel{v_0, v}}{(v^2 - v_0^2 + t^2M^2 + (1-t)m^2 - i\varepsilon)^2}$$

Residue calculus yields

$$\int_{-\infty}^{\infty} \frac{dv_0}{(v^2 - v_0^2 + t^2M^2 + (1-t)m^2 - i\varepsilon)^2} = 4\pi i \frac{1}{(2\omega_v'(t))^3}$$

where $\omega_v'(t) = \sqrt{t^2M^2 + (1-t)m^2 + v^2 - i\varepsilon}$ is the root with positive real part (cf. also 2.3), and

$$\int_{-\infty}^{\infty} \frac{v_0\, dv_0}{(v^2 - v_0^2 + t^2M + (1-t)m^2 - i\varepsilon)^2} = 0$$

Hence

$$c_1(t, p) = 4\pi i \int_{G(t,p)} d^3v \frac{M(1+t)}{(2\omega_v'(t))^3}$$

$$c_2(t, p) = 4\pi i \int_{G(t,p)} d^3v \frac{\cancel{0, v}}{(2\omega_v'(t))^3}$$

2.12 Lemma. *For a finite $p \in \mathbb{R}^3_{(1)}$ we have*

$c_1(t, p) \sim c_1(t, 0) \quad (0 \leq t \leq 1).$

Proof.

$$|c_1(t, p) - c_1(t, 0)| = \left| 4\pi i \int_{G(t,p) \dot{-} G(t,0)} d^3v \frac{M(1+t)}{(2\omega_v'(t))^3} \right|$$

where $G(t, p) \dot{-} G(t, 0)$ is the symmetrical difference

$$\leq 4\pi \int_{\mathbb{B}^0_{P+\|p\|} \setminus \mathbb{B}^0_{P-\|p\|}} d^3v \frac{M(1+t)}{(2\,\|v\|)^3}$$

since $G(t, p) \dot{-} G(t, 0) \subseteq \mathbb{B}^0_{P+\|p\|} \setminus \mathbb{B}^0_{P-\|p\|}$ (for $\mathbb{B}^0_r$ cf. 3.7)

$$= 4\pi M(1+t) \int_{P-\|p\|}^{P+\|p\|} 4\pi \frac{r^2\, dr}{(2r)^3} = 2\pi^2 M(1+t) \ln \frac{P + \|p\|}{P - \|p\|} \sim 0 \quad \text{q.e.d.}$$

In analogy to classical Q.E.D. one gets

2.13 Lemma. *For finite $p \in \mathbb{R}^3_{(1)}$ we have*

$$\int_0^1 dt c_1(t, p) \sim 4\pi^2 iM(\ln P + R)$$

where P is the UV cutoff and R is a finite real remainder, independent of p.

2.14 Lemma. *For finite $p \in \mathbb{R}^3_{(1)}$, $0 \leq t \leq 1$, we have $c_2(t, p) \sim -c_2(1-t, p)$*

Proof. Using the symmetry between $G(t, p)$ and $G(1-t, p)$.

2.15 Corollary. *For finite $p \in \mathbb{R}^3_{(1)}$, we get*

$$\int_0^1 c_2(t, p)\, dt \sim 0$$

2.16 Theorem. *For finite $p \in \mathbb{R}^3_{(1)}$ the only contribution $c(p)$ of $\Sigma(\Omega_p, p)$ in $(\phi, \varepsilon(-t, t)\Lambda)$ (cf. 14) amounts to $c(p) \sim 8\pi^2 iM(R + \ln P)$ where P is the UV cutoff and R a finite real constant.*

Proof. Cf. 2.11, 13 and 15.

2.17 Corollary. *For any finite time interval $[-t, t]$ and any two one-electron states $\phi, \Lambda \in \mathbb{L} \otimes \mathbb{D}$ with finite norm and finitely bounded support the selfenergy contribution (cf. 2.1, 2) amounts to*

$$(\phi, \varepsilon(-t, t)\Lambda) \sim -e^3 \int_{-t}^{t} dy_0 \int_{-t}^{t} dz_0 \int d^3p \int d^3w \int dk_0 \frac{1}{(2\pi)^3} e^{i(\Omega_p - k_0)y_0} e^{i(k_0 - \Omega_w)z_0}$$

$$\times \left(\phi, \frac{1}{(2\pi)^{3/2}} \bar{\Theta}^+(0, -p)\{A + \pi\Xi(\Omega_p, p, t, y_0)\}\right.$$

$$\left.\times \frac{M + \cancel{k_0, p}}{p^2 - k_0^2 + M^2 - i\varepsilon} h(p)\gamma_\rho \frac{1}{(2\pi)^{3/2}} \Theta^-(0, w) \frac{1}{(2\pi)^{3/2}} C_\rho(p - w)\Lambda\right)$$

where $A := 2\pi \cdot 8\pi^2 iM(R + \ln P)$ is infinite but 0-finite,

Proof. Cf. 2.6, 9, 16.

2.18. *Selfenergy counterterm*

In order to compensate the infinite summand arising from A in $\Sigma(\Omega_p, p)$ to the fermion selfenergy contribution $(\phi, \varepsilon(-t, t)\Lambda)$ (cf. 2.6, 16 and 17) we introduce an additive 'selfenergy counterterm'

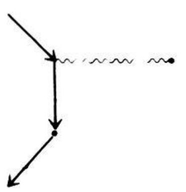
$$-\delta M : \int \bar{\Theta}(0, x)\Theta(0, x)\, d^3x :$$

into the interaction hamiltonian. The constant (0-finite) factor δM is to be determined in the following (cf. 2.20). The new Dyson expansion contains in second order the summand

$$\delta(-t, t) = (-i)^2 e\delta M \int_{-t}^{t} dx_0 \int d^3x \int_{-t}^{t} dy_0\, d^3y$$

$$: \bar{\Theta}(x_0, x)\Theta(x_0, x)\bar{\Theta}(y_0, y)\gamma_\mu\Theta(y_0, y) : C_\mu(y)$$

corresponding to the Feynman graph.



A straightforward evaluation leads to

2.19 Theorem. *For any two one-electron states $\phi, \Lambda \in \mathbb{L} \otimes \mathbb{D}$ the selfenergy counterterm induces in the* 0*-finite time interval* $[-t, t]$

$$(\phi, \delta(-t, t)\Lambda) \underset{0}{\sim} \frac{ie}{(2\pi)^{5/2}} \delta M \int_{-t}^{t} dy_0 \int_{-t}^{t} dz_0 \int d^3p \int d^3w \int dk_0 e^{i(\Omega_p - k_0)y_0} e^{i(k_0 - \Omega_w)z_0}$$

$$\times (\phi, \bar{\Theta}^+(0, -p) \frac{M + \cancel{k_0, p}}{p^2 - k_0^2 + M^2 - i\varepsilon} h(p)\gamma_\rho \Theta^-(0, w) C_\rho(p - w)\Lambda)$$

2.20. *Fermion mass renormalization*

Fixing δM at the positive 0-finite, but infinite value

$\delta M = 4\pi e^2 M(\ln P + R)$

causes the infinite part of the selfenergy contribution to be infinitesimally closely compensated by the counterterm contribution, for normed one-electron states in $\mathbb{L} \otimes \mathbb{D}$ with finitely bounded support (cf. 2.17 and 19), i.e. it remains

$$(\phi, (\varepsilon(-t, t) + \delta(-t, t))\Lambda) \underset{0}{\sim} \text{0-standard part of}$$

$$-e^3 \int_{-t}^{t} dy_0 \int_{-t}^{t} dz_0 \int d^3p \int d^3w \int dk_0 \frac{1}{(2\pi)^3} e^{i((\Omega_p - k_0)y_0 + (k - \Omega_w)z_0)}$$

$$\times \Bigg(\phi, \frac{1}{(2\pi)^{3/2}} \bar{\Theta}^+(0, -p)\pi\Xi(\Omega_p, p, t, y_0) \frac{M + \cancel{k_0, p}}{p^2 - k_0^2 + M^2 - i\varepsilon} h(p)\gamma_\rho \frac{1}{(2\pi)^{3/2}}$$

$$\times \Theta^-(0, w) \frac{1}{(2\pi)^{3/2}} C_\rho(p - w)\Lambda\Bigg)$$

which in the $\mathbb{M}_0$-limit $t \to \infty$ disappears (cf. 2.9).

3. The vertex contribution

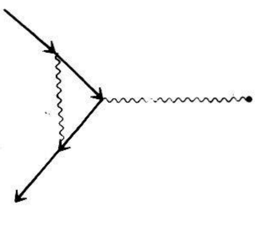
3.1. *The vertex part*

is the summand

$$v(-t, t) = (ie)^3 \int_{-t}^{+t} dx_0 \int_{-t}^{+t} dy_0 \int_{-t}^{+t} dz_0 \int d^3x \int d^3y \int d^3z$$

$$:\bar{\Theta}(x_0, x)\gamma_\mu \Theta(x_0, x) B_\mu(x_0, x) \bar{\Theta}(y_0, y)\gamma_\rho \Theta(y_0, y) C_\rho(y) \bar{\Theta}(z_0, z)\gamma_\nu \Theta(z_0, z) B_\nu(z_0, z):$$

of the S-matrix $U(-t, t)$ (cf. [1], 5.7 and 8) corresponding to the Feynman graph.



A straightforward evaluation leads to

3.2 Theorem. *For any two one-electron states $\phi, \Lambda \in \mathbb{L} \otimes \mathbb{D}$ the vertex part yields*

$$(\phi, v(-t, t)\Lambda) \mathrel{\tilde{\circ}} \Big(\phi, \frac{e^3}{(2\pi)^{9/2}} \int_{-t}^{+t} dx_0 \int_{-t}^{+t} dy_0 \int_{-t}^{+t} dz_0 \int d^3p\, d^3q\, d^3w$$

$$\times \int du_0\, dk_0\, dq_0 \frac{1}{2\pi} e^{i(\Omega_p - q_0 - u_0)x_0} \frac{1}{2\pi} e^{i(u_0 - k_0)y_0} \frac{1}{2\pi} e^{i(q_0 + k_0 - \Omega_w)z_0}$$

$$\times \bar{\Theta}^+(0, -p)(-g_{\mu\mu}) \frac{h(q)}{q^2 - q_0^2 + m^2 - i\varepsilon} \gamma_\mu \frac{M + \cancel{u_0, p - q}}{(p-q)^2 - u_0^2 + M^2 - i\varepsilon} h(p-q)\gamma_\nu$$

$$\times \frac{M + \cancel{k_0, w - q}}{(w-q)^2 - k_0^2 + M^2 - i\varepsilon} h(w-q)\gamma_\mu \Theta^-(0, w) C_\nu(p-w)\Lambda\Big)$$

3.3 Corollary. *For any two one-electron states $\phi, \Lambda \in \mathbb{L} \otimes \mathbb{D}$ the vertex part yields*

$$(\phi, v\Lambda) := 0\text{-}\lim_{t \to \infty} 0\text{-}st(\phi, v(-t, t)\Lambda)$$

$$= \frac{e^3}{(2\pi)^{9/2}} 0\text{-}st\Big(\phi, \int d^3p\, d^3w\, \delta^1(\Omega_p - \Omega_w) \bar{\Theta}^+(0, -p)$$

$$\times \Lambda_\nu(p, w) \Theta^-(0, w) C_\nu(p-w)\Lambda\Big)$$

where

$$\Lambda_\nu(p, w) = \int d^3q \int dq_0 (-g_{\mu\mu}) \frac{h(q)}{q^2 - q_0^2 + m^2 - i\varepsilon}$$

$$\times \gamma_\mu \frac{M + \cancel{\Omega_p - q_0, p - q}}{(p-q)^2 - (\Omega_p - q_0)^2 + M^2 - i\varepsilon} h(p-q)$$

$$\times \gamma_\nu \frac{M + \cancel{\Omega_w - q_0, w - q}}{(w-q)^2 - (\Omega_w - q_0)^2 + M^2 - i\varepsilon} h(w-q)\gamma_\mu$$

3.4 *Remark.* Similarly to classical Q.E.D. we get

$$\Lambda_\nu(p, w) = (-2) \int_0^1 dx \int_0^{1-x} dy \int d^3q \int dq_0 h(q) h(p-q) h(w-q)$$

$$\frac{-2\not{p}\gamma_\nu\not{w} + 2(\not{q}\gamma_\nu\not{p} + \not{w}\gamma_\nu\not{q}) + 2M(\gamma_\nu(\not{p} + \not{w} - 2\not{q}) + (\not{p} + \not{w} - 2\not{q})\gamma_\nu) - 2\not{q}\gamma_\nu\not{q} - 2M^2\gamma_\nu}{[q^2 - q_0^2 + 2(q(px + wy)) + m^2(1 - x - y) - i\varepsilon]^3}$$

The computation of $\Lambda_\nu(p, w)$ follows as far as this is possible the standard versions, cf. [5], Appendix E pg. 315–321. Let $K(p, w)$ for fixed $p, w \in \mathbb{R}^3$ be the support of $q \mapsto h(q)h(p-q)h(w-q)$. Thus $\Lambda_\nu(p, w)$ can be written in the form

$$\Lambda_\nu(p, w) = -2 \int_0^1 dx \int_0^{1-x} dy \int_{K(p,w)} d^3q \int_{-\infty}^{+\infty} \frac{\mathcal{P}_\nu(\not{p}, \not{w}, \not{q})}{(q^2 - q_0^2 + 2(qr) + s - i\varepsilon)^3} dq_0$$

where $\mathcal{P}_\nu$ is a polynomial (quadratic in q), $r = px + wy$ and $s = m^2(1 - x - y) \geq 0$. Notice that the denominator is equal to $(-(q_0 - r_0)^2 + (q - r)^2 + |r|^2 + s - i\varepsilon)^3$ where $|r|^2 = r_0^2 - \|r\|^2 = (r, r)$. Using residue calculus one proves:

3.5 Lemma. *For $s > 0$, it follows*

$$a)\ \int_{-\infty}^{+\infty} \frac{dq_0}{(q^2 - q_0^2 + 2(qr) + s - i\varepsilon)^3} = 2\pi i \frac{3}{16} \frac{1}{((q-r)^2 + |r|^2 + s - i\varepsilon)^{5/2}}$$

$$b)\ \int_{-\infty}^{+\infty} \frac{q_0\, dq_0}{(q^2 - q_0^2 + 2(qr) + s - i\varepsilon)^3} = 2\pi i \frac{3}{16} \frac{r_0}{((q-r)^2 + |r|^2 + s - i\varepsilon)^{5/2}}$$

$$c)\ \int_{-\infty}^{+\infty} \frac{q_0^2\, dq_0}{(q^2 - q_0^2 + 2(qr) + s - i\varepsilon)^3} = 2\pi i \left\{ \frac{3}{16} \frac{r_0^2}{((q-r)^2 + |r|^2 + s - i\varepsilon)^{5/2}} - \frac{1}{16} \frac{1}{((q-r)^2 + |r|^2 + s - i\varepsilon)^{3/2}} \right\}$$

Which yields the following

3.6 Corollary. *For finite $p, w, r \in \mathbb{R}^4_{(1)}$, $s > 0$, $|r|^2 = r_0^2 - r^2 > 0$ we have*

$$a)\ \int_{K(p,w)} d^3q \int_{-\infty}^{+\infty} dq_0 \frac{1}{(q^2 - q_0^2 + 2(qr) + s - i\varepsilon)^3} \sim \frac{i}{2} \pi^2 \frac{1}{s + |r|^2}$$

$$b)\ \int_{K(p,w)} d^3q \int_{-\infty}^{+\infty} dq_0 \frac{q_\nu}{(q_0^2 - q_0^2 + 2(qr) + s - i\varepsilon)^3} \sim \frac{i}{2} \pi^2 \frac{r_\nu}{s + |r|^2}, \qquad \nu = 0, 1, 2, 3$$

$$c)\ \int_{K(p,w)} d^3q \int_{-\infty}^{+\infty} dq_0 \frac{q_\nu q_\mu}{(q^2 - q_0^2 + 2(qr) + s - i\varepsilon)^3} \sim \frac{i}{2} \pi^2 \frac{r_\nu r_\mu}{s + |r|^2}, \qquad \mu \neq \nu$$

$$d)\ \int_{K(p,w)} d^3q \int_{-\infty}^{+\infty} dq_0 \frac{q_j^2}{(q^2 - q_0^2 + 2(qr) + s - i\varepsilon)^3} \sim \frac{i}{2} \pi^2 \frac{r_j^2}{s + |r|^2} + iL_j(p, w, r, s), \qquad j = 1, 2, 3$$

$$e)\ \int_{K(p,w)} d^3q \int_{-\infty}^{+\infty} dq_0 \frac{q_0^2}{(q^2 - q_0^2 + 2(qr) + s - i\varepsilon)^3} \sim \frac{i}{2} \pi^2 \frac{r_0^2}{s + |r|^2} - \frac{i\pi}{6} - iL_0(p, w, r, s)$$

where

$$L_j(p, w, r, s) = 2\pi \frac{3}{16} \int_{K(p,w)} \frac{(q_j - r_j)^2 d^3q}{((q-r)^2 + |r|^2 + s)^{5/2}}$$

and

$$L_0(p, w, r, s) = 2\pi \frac{1}{16} \int_{K(p,w)} \frac{(q-r)^2\, d^3q}{((q-r)^2 + |r|^2 + s)^{5/2}}$$

3.7 *Remark.* For finite values of p, w, r, s with $s>0$, $|r|^2>0$ we have

$$L_\nu(p, w, r, s) \sim 2\pi \frac{3}{16} \int_{\mathbb{B}_P^0} \frac{k_1^2\, d^3k}{(k^2+|r|^2+s)^{5/2}} \quad \text{for} \quad \nu = 0, 1, 2, 3$$

which we will denote by $L(r, s)$. (Here $\mathbb{B}_P^0$ is the ball with radius $P=$ UV cutoff around 0.)

Proof. First notice that

$$\int_{\mathbb{B}_{P+1}^0 \setminus \mathbb{B}_{P-1}^0} \frac{(q_i - r_i)^2\, d^3q}{((q-r)^2+|r|^2+s)^{5/2}} \sim 0$$

for any finite u of $1 \geqslant 0$.

This implies

$$L_2(p, w, r, s) \sim 2\pi \frac{3}{16} \int_{\mathbb{B}_P^0} \frac{q_1^2\, d^3q}{(q^2+|r|^2+s)^{5/2}}$$

The case $\nu=0$ follows since $(q-r)^2 = (q_1-r_1)^2 + (q_2-r_2)^2 + (q_3-r_3)^2$ q.e.d.

From 3.6 and 3.7 we can get

3.8 Lemma. *For finite values of p, w, r, s with $|r|^2>0$, $s>0$ we have*

$$\int_{K(p,w)} d^3q \int dq_0 \frac{\mathcal{P}_\nu(\not p, \not w, \not q)}{(q^2 - q_0^2 + 2(q, r) + s - i\varepsilon)^3} \quad \text{(cf. 3.4)}$$

$$\sim i\frac{\pi^2}{2}\frac{1}{s+|r|^2}\left\{\begin{array}{l} -2\not w\gamma_\nu\not p + 2(\not r\gamma_\nu\not p + \not w\gamma_\nu\not r) \\ +2M(\gamma_\nu(\not p + \not w - 2\not r) + (\not p + \not w - 2\not r)\gamma_\nu) \\ -2\not r\gamma_\nu\not r - 2M^2\gamma_\nu \end{array}\right\}$$

$$-2i\gamma_\nu\left(-g_{\nu 0}\frac{\pi^2}{6} + 2L(r, s)\right).$$

Notice that replacing $K(p, w)$ by $\mathbb{B}_P^0$ would not change this infinitesimal approximation.

3.9 Corollary. *Setting $Q := p - w$ and using $r = xp + yw$, $s = m^2(1-x-y)$ for $0 \le y \le 1-x \le 1$, $0 \le x \le 1$ (cf. 3.4) Lemma 3.8 yields*

$$\bar u(p)\int_{K(p,w)} d^3q \int dq_0 \frac{\mathcal{P}_\nu(\not p, \not w, \not q)}{(q^2 - q_0^2 + 2(q, xp+yq) + m^2(1-x-y) - i\varepsilon)^3} u(w)$$

$$\sim i\frac{\pi^2}{2}\frac{1}{m^2(1-x-y)+|xp+yw|^2}$$

$$\times\, \bar u(p)\left\{\begin{array}{l} \gamma_\nu(4M^2 - 4(x+y)M^2 - 2(x+y)^2M^2 + 2\,|Q|_\nu^2\,(1-(x+y)+xy)) \\ +\gamma_\nu \not Q \cdot 2M(y - x(x+y)) \\ -\not Q\gamma_\nu \cdot 2M(x - y(x+y)) \end{array}\right\} u(w)$$

$$-2i\gamma_\nu\left(-g_{\nu 0}\frac{\pi^2}{6}+2L(xp+yw,\, m^2(1-x-y))\right)\bar{u}(p)u(w)$$

where

$$|Q|_0^2 = Q_0^2+Q_1^2+Q_2^2+Q_3^2$$
$$|Q|_1^2 = -Q_0^2-Q_1^2+Q_2^2+Q_3^2$$
$$|Q|_2^2 = -Q_0^2+Q_1^2-Q_2^2+Q_3^2$$
$$|Q|_3^2 = -Q_0^2+Q_1^2+Q_2^2-Q_3^2$$

Proof. Apply $\not{p}u(w) = (\not{w}+\not{Q})u(w) = (M+\not{Q})u(w)$ and $\bar{u}(p)\not{w} = \bar{u}(p)(\not{p}-\not{Q})$
$= \bar{u}(p)(M-\not{Q})$.

3.10 *Remark.* a) Using $\sigma_{\mu\nu} := \frac{1}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$ it follows

$$\gamma_\nu\not{Q} = -\Sigma Q_\mu\sigma_{\mu\nu} + Q_\nu g_{\nu\nu}$$
$$\not{Q}\gamma_\nu = +\Sigma Q_\mu\sigma_{\mu\nu} + Q_\nu g_{\nu\nu}$$

b) Using $p_0 = \Omega_p$, $w_0 = \Omega_w$ it follows

$$|xp+yw|^2 = (x+y)^2M^2 - xy\,|p-w|^2$$

3.11 Theorem. *For $\nu = 0$, p, w finite, $0 \le x \le 1$ and $0 \le y \le 1-x \le 1$ we have*

$$\bar{u}(p)\int_{K(p,w)} d^3q \int dq_0 \frac{\mathscr{P}_0(\not{p},\not{w},\not{q})}{(q^2-q_0^2+2(q,xp+yq)+m^2(1-x-y)-i\varepsilon)^3}u(w)$$
$$\sim i\frac{\pi^2}{2}\frac{1}{m^2(1-x-y)+(x+y)^2M^2-xy\,|Q|^2}$$
$$\times\,\bar{u}(p)\left\{\begin{matrix}\gamma_0(4M^2-4(x+y)M^2-2(x+y)^2M^2-2\,|Q|^2\,(1-(x+y)+xy))\\ -2M\Sigma Q_\mu\sigma_{\mu 0}(x+y)(1-(x+y))\end{matrix}\right\}u(w)$$
$$-2i\gamma_0\left(-\frac{\pi^2}{6}+2L(xp+yw,\, m^2(1-(x+y)))\right)\bar{u}(p)u(w)$$

assuming that $\Omega_p = \Omega_w$, where $Q := p - w$,

$$|Q|^2 = -Q_1^2-Q_2^2-Q_3^2 = -((p_1-w_1)^2+(p_2-w_2)^2+(p_3-w_3)^2).$$

Proof. Straightforward from 3.9, 10.

In order to approximate $(\phi, v\Lambda)$ (cf. Corollary 3.3) one uses the Taylor development of the above expression as a function of $|Q|^2$, regarding $\Sigma Q_\mu\sigma_{\mu 0}$ as an independent parameter; then one has to integrate over $dx\,dy$ (cf. 3.4) and $d^3p\,d^3w$.

We start with the summand of the 'constant term' containing $\sum_{\mu=0}^3 Q_\mu\sigma_{\mu 0}$, corresponding to an additional magnetic moment. In analogy to classical Q.E.D. (where $m = 0$) we have

3.12 Lemma.

$$\int_0^1 dx \int_0^{1-x} dy \frac{(x+y)(1-(x+y))}{m^2(1-x-y)+(x+y)^2M^2} \sim \frac{1}{2M^2}.$$

3.13 Corollary. *The 'constant' summand of* $(\phi, v\Lambda)$ *depending upon* $\Sigma(p_\mu - w_\mu)\sigma_{\mu 0}$ *is given by*

$$\frac{ie}{\sqrt{2\pi}}\left(\phi, \int d^3p\, d^3w \delta^1(\Omega_p - \Omega_w)\bar{\Theta}^+(0, -p) \sum_{\mu=0}^{3} (p_\mu - w_\mu)\sigma_{\mu 0} \right.$$
$$\left. \times \frac{1}{2M}\frac{\alpha}{2\pi}\Theta^-(0, w)C_0(p-w)\Lambda\right)$$

(*where* $\alpha = e^2/4\pi$) *which gives rise to the anomalous magnetic moment of the electron* (*cf.* [5] *pg.* 320).

3.14 *Remark.* Now we turn to the remaining summands in the constant term ($|Q|^2 = 0$) of the above mentioned Taylor development:

$$\gamma_0 i \frac{\pi^2}{2}\int_0^1 dx \int_0^{1-x} dy \frac{4M^2 - 4(x+y)M^2 - 2(x+y)^2M^2}{m^2(1-(x+y))+(x+y)^2M^2} - 2\gamma_0 i \int_0^1 dx \int_0^{1-x}$$
$$\left(-\frac{\pi^2}{6} + 2\cdot 2\pi \frac{1}{16}\int_{\mathbb{B}_P^0} d^3q \frac{q^2}{(q^2 + m^2(1-(x+y)) + (x+y)^2M^2)^{5/2}}\right) dy$$

(cf. 3.11 and 7).

Introducing polar coordinates instead of q_1, q_2, q_3 and substituting $x = (u/2) + v$, $y = (u/2) - v$ and integrating over dv (cf. 3.12) yields

$$\gamma_0 i \frac{\pi^2}{2}\int_0^1 du \frac{4M^2u(1-u) + 2M^2u^3}{m^2(1-u) + M^2u^2}$$
$$- 2\gamma_0 i \pi^2 \int_0^1 du \int_0^P d\rho \frac{\rho^4 u}{(\rho^2 + m^2(1-u) + u^2M^2)^{5/2}} + \gamma_0 i \frac{\pi^2}{6}$$
$$= i\gamma_0\left(L_+ + L_- + \frac{\pi^2}{6}\right)$$

where L_+ and L_- denote the first two factors of $\gamma_0 i$ respectively. Explicit integrations yield

3.15 Lemma

$$L_+ + L_- + \frac{\pi^2}{6} \sim \pi^2\left(2\ln\frac{M}{m} - \tfrac{1}{2}\ln 2P + \tfrac{1}{4}\ln M^2 - \tfrac{3}{4}\right)$$

3.16 *Remark.* Given any finite value L and any infinite (infinitesimal) value for P (for m^2) then there is a infinitesimal m^2 (infinite P) such that

$$-L \sim L_+ + L_- + \frac{\pi^2}{6}.$$

That L be finite is equivalent to $\ln(m^2P)$ being finite. The latter condition we want to hold once and for all.

3.17. *Spurious charge renormalization*

Choosing P and m such that L is small or even infinitesimal there is quite little or no corresponding charge renormalization at all necessary.

In order to evaluate the term which is linear in $|Q|^2$ in the above mentioned Taylor series we need.

3.18 Lemma

a) $$2\int_0^1 dx \int_0^{1-x} dy \frac{1-(x+y)+xy}{m^2(1-x-y)+(x+y)^2M^2} \sim \frac{1}{M^2}\left(2\ln\frac{M}{m}-\frac{11}{6}\right)$$

b) $$\int_0^1 dx \int_0^{1-x} dy \frac{4M^2-4(x+y)M^2-2(x+y)^2M^2}{(m^2(1-x-y)+(x+y)^2M^2)^2} xy \sim \frac{1}{M^2}\left(\frac{2}{3}\ln\frac{M}{m}-\frac{7}{6}\right)$$

c) $$2\frac{2\pi}{16}\frac{5}{2}\int_0^1 dx \int_0^{1-x} dy \int_{\mathbb{B}_P^0} \frac{q^2xy\, d^3q}{(q^2+m^2(1-x-y)+(x+y)^2M^2)^{7/2}} \sim \frac{\pi^2}{2\cdot 2\cdot 6M^2}$$

Proof. By a chain of straightforward integrations. Thus one gets

3.19 Proposition. *The coefficient of all contributions in $\bar{u}(p)\Lambda_0(p, w)u(w)$ linear in $|Q|^2$ amounts to*

$$\begin{aligned}
&-2\Big\{i\gamma_0\frac{\pi^2}{2}\int_0^1 dx\int_0^{1-x} dy \\
&\quad\times\left(\frac{4M^2-4(x+y)M^2+2(x+y)^2M^2}{(m^2(1-x-y)+(x+y)^2M^2)^2}xy - 2\frac{1-(x+y)+xy}{m^2(1-x-y)+(x+y)^2M^2}\right) \\
&\quad -2i\gamma_0 2\int_0^1 dx\int_0^{1-x} dy 2\pi\frac{1}{16}\cdot\frac{5}{2}\int_{\mathbb{B}_P^0}\frac{q^2xy\, d^3q}{(q^2+m^2(1-x-y)+(x+y)^2M^2)^{7/2}}\Big\} \\
&\quad \sim -i\gamma_0\pi^2\frac{1}{M^2}\left(-\frac{4}{3}\ln\frac{M}{m}+\frac{11}{6}-\frac{7}{6}-\frac{1}{6}\right) \\
&= i\gamma_0\pi^2\frac{1}{M^2}\frac{4}{3}\left(\ln\frac{M}{m}-\frac{3}{8}\right)
\end{aligned}$$

This yields

3.20 Theorem. *The combined contributions of the 'lowest terms' in $p-w$ and*

$|p-w|^2$ of $(\phi, v\Lambda)$ are

$$\sim \frac{ie}{\sqrt{2\pi}}\Big(\phi, \int d^3p\, d^3w\,\delta^1(\Omega_p-\Omega_w)\bar{\Theta}^+(0,-p)$$

$$\times \sum_{\mu=0}^{3}(p_\mu - w_\mu)\sigma_{\mu 0}\frac{1}{2M}\frac{\alpha}{2\pi}\bar{\Theta}(0,w)C_0(p-w)\Lambda\Big)$$

$$+\frac{ie}{\sqrt{2\pi}}\Big(\phi, \int d^3p\, d^3w\,\delta^1(\Omega_p-\Omega_w)\bar{\Theta}^+(0,-p)$$

$$\times\frac{\alpha}{3\pi}\frac{|p-w|^2}{M^2}\left(\ln\frac{M}{m}-\frac{3}{8}\right)\gamma_0\Theta^-(0,w)C_0(p-w)\Lambda\Big)$$

assuming that ϕ and Λ are normed and have finitely bounded support and that $L\sim 0$ (*cf.* 3.19).

3.21. *The Lamb shift*

We now have accumulated the theoretical and numerical results (cf. 1.9, 2.20, 3.20, [1], 6.21, 5.13) required to carry out the approximation to the Lamb shift according to [6], 15.E.

4. A Ward identity

4.1. *Definition*

In order to compare $\Sigma(p)$ and $\Lambda_\nu(p,w)$ (cf. 2.5 and 3.3 resp.) we introduce their respective slightly modified 'integrands'

$$\sigma(p,u):=\frac{1}{-|u|^2+m^2-i\varepsilon}g_{\mu\mu}\gamma_\mu\frac{M+\not{p}-\not{u}}{-|p-u|^2+M^2-i\varepsilon}\gamma_\mu$$

and

$$\lambda_\nu(p,w,u)$$

$$:=\frac{-1}{-|u|^2+m^2-i\varepsilon}g_{\mu\mu}\gamma_\mu\frac{M+\not{p}-\not{u}}{-|p-u|^2+M^2-i\varepsilon}\gamma_\nu\frac{M+\not{w}-\not{u}-i\varepsilon}{-|w-u|^2+M^2-i\varepsilon}\gamma_\mu.$$

By straightforward differentiation one gets

4.2 Lemma

$$\frac{\partial\sigma(p,u)}{\partial p_\nu}=\lambda_\nu(p,p,u)\quad\text{for}\quad \nu=0,1,2,3.$$

4.3 Lemma

$$\Sigma(p)=\int_0^1 dt\int_{G(t,p)}d^3v\int_{-\infty}^{\infty}dv_0\frac{4M-2\not{v}-2(1-t)\not{p}}{(-|v|^2+t(M^2-|p|^2)+t^2M^2-(1-t)m^2-i\varepsilon)^2}$$

Proof. A straightforward generalization of the case $M^2 - |p|^2 = 0$ (cf. 2.10). Notice that in $G(t, p)$ only the space-like component of p is considered.

4.4 *Remark.* The contribution of

$$\int_{\mathbb{B}_P^0} d^3u \int du_0 \sigma(p, u) \quad \text{to} \quad \Sigma(p) = \int d^3u h(u) h(u-p) \int du_0 \sigma(p, u)$$

corresponds in Lemma 4.3 to the contribution

$$\int_0^1 dt \int_{H(t,p)} d^3v \int_{-\infty}^{\infty} dv_0 \frac{4M - 2\not{v} - 2(1-t)\not{p}}{(-|v|^2 + t(M^2 - |p|^2) + t^2M^2 - (1-t)m^2 - i\varepsilon)^2}$$

where $H(t, p)$ is the support of $v \mapsto h(v - tp)$ (cf. also 2.10 and 11).

4.5 *Remark.* Residue calculus yields

a) $$\int_{-\infty}^{\infty} \frac{dv_0}{(\|v\|^2 - v_0^2 + t(M^2 - |p|^2) + t^2M^2 + (1-t)m^2 - i\varepsilon)^2}$$

$$= \begin{cases} 4\pi i \dfrac{1}{(2\omega'(v, p, t))^3} & \text{for} \quad \|v\|^2 + t(M^2 - |p|^2) + t^2M^2 + (1-t)m^2 \geqslant 0 \\ 4\pi \dfrac{1}{(2\omega'(v, p, t))^3} & \text{otherwise} \end{cases}$$

where $\omega'(v, p, t) := \sqrt{\|v\|^2 + t(M^2 + |p|^2) + t^2M^2 + (1-t)m^2 - i\varepsilon)^2}$ is the square root with positive real part.

b) $$\int_{-\infty}^{\infty} \frac{v_0 dv_0}{(\|v\|^2 - v_0^2 + t(M^2 - |p|^2) + t^2M^2 + (1-t)m^2 - i\varepsilon)^2} = 0.$$

4.6 Lemma. *For any finite $p \in \mathbb{R}^4$ and for any 0-infinitesimal $p - q \neq 0$ the expression*

$$\frac{1}{|p-q|} \int_0^1 dt \left\{ \int_{G(t,p) \dot{-} H(t,p)} d^3v \int_{-\infty}^{\infty} dv_0 \frac{4M - 2\not{v} - 2(1-t)\not{p}}{(-|v|^2 + t(M^2 - |p|^2) + t^2M^2 + (1-t)m^2 - i\varepsilon)^2} \right.$$
$$\left. - \int_{G(t,q) \dot{-} H(t,q)} d^3v \int_{-\infty}^{\infty} dv_0 \frac{4M - 2\not{v} - 2(1-t)\not{q}}{(-|v|^2 + t(M^2 - |q|^2) + t^2M^2 + (1-t)m^2 - i\varepsilon)^2} \right\}$$

is infinitesimal.

Sketch of the Proof

Notice that $G(t, p) \dot{-} H(t, p)$ is the 'symmetrical difference' with the appropriate signs for the contributions of the corresponding integrals.

According to Remark 4.5 we have

$$\int_{-\infty}^{\infty} dv_0 \frac{4M - 2\not{v} - 2(1-t)\not{p}}{(-|v|^2 + t(M^2 - |p|^2) + t^2M^2 + (1-t)m^2 - i\varepsilon)^2}$$
$$= \frac{4\pi i}{(2\omega'(v, p, t))^3} (4M - 2(\gamma_1 v_1 + \gamma_2 v_2 + \gamma_3 v_3) - 2(1-t)\not{p}) =: I(v, p, t)$$

Now

$$I(t) := \int_{G(t,p) \dot{-} H(t,p)} I(v, p, t)\, d^3v - \int_{G(t,q) \dot{-} H(t,q)} I(v, p, t)\, d^3v$$

can be viewed as an integral over part of the $|p-q|$-thin 'surfaces' of two balls both of radius P but with different (finite) centers. $1/\omega'(v, p, t)^3$ can be approximated by $1/P^3$ with error 'of order' $1/P^4$. This implies that the $4M$-part and the $2(1-t)\not{p}$-part contribute in $I(t)$ 'of order' $|p-q|$. (P^2/P^3). Thus their contribution in $1/|p-q| \int_0^1 dtI(t)$ is of order $1/P \sim 0$.

As to the $\gamma_i v_i$-parts ($i = 1, 2, 3$), they cancel each other in $I(t)$ and $I(1-t)$ up to an error 'of order' $|p-q|\, P^3/P^4$. Thus again they contribute at most of order $1/P \sim 0$ in

$$\frac{1}{|p-q|} \int_0^1 dtI(t). \quad \text{q.e.d.}$$

4.7. *A Ward identity*

For finite $p \in \mathbb{R}^4_{(1)}$ we have

$$\frac{\partial \Sigma(p)}{\partial p_\nu} \sim \Lambda_\nu(p, p)$$

Proof. Since

$$\Sigma(p) = \int_{\mathbb{B}_P^0} d^3u \int_{-\infty}^{\infty} du_0 \sigma(p, u) + \int_{G(1,p) \dot{-} \mathbb{B}_P^0} d^3u \int_{-\infty}^{\infty} du_0 \sigma(p, u)$$

we have

$$\frac{\partial \Sigma(p)}{\partial p_\nu} = \int_{\mathbb{B}_P^0} d^3u \int_{-\infty}^{\infty} du_0 \frac{\partial \sigma(p, u)}{\partial p_\nu} + \lim_{\Delta_\nu p \to 0} \frac{1}{\Delta_\nu p}$$

$$\times \left\{ \int_{G(1,p+\Delta p_\nu) \dot{-} \mathbb{B}_P^0} d^3u \int_{-\infty}^{\infty} du_0 \sigma(p + \Delta p_\nu, u) - \int_{G(t,p) \dot{-} \mathbb{B}_P^0} d^3u \int_{-\infty}^{\infty} du_0 \sigma(p, u) \right\}$$

$$\sim \int_{\mathbb{B}_P^0} d^3u \int_{-\infty}^{\infty} du_0 \lambda_\nu(p, p, u) \quad \text{(because of Lemma 4.2 and 4.6)}$$

$\sim \Lambda_\nu(p, p)$ since $\varepsilon \underset{\circ}{\sim} 0$ (cf. 3.3 and 4.1). q.e.d.

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